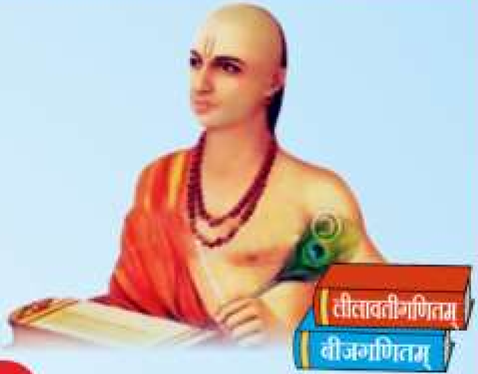
आज़ादी का अमृत महोत्सव



MATHEMATICS TEXTBOOK

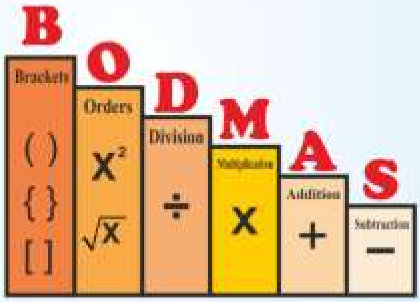
(On the basis of vedic Mathematics and Sanskrit Literature of Mathematics)

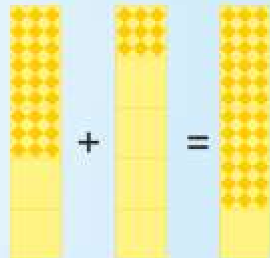
Veda Bhushan II Year / Prathama - II Year / Class VII

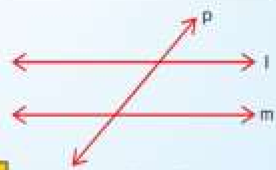
MAHARSHI SANDIPANI RASHTRIYA VEDA SANSKRIT SHIKSHA BOARD

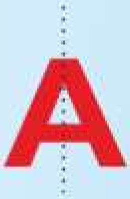
(Established and Recognized by the Ministry of Education, Government of India)

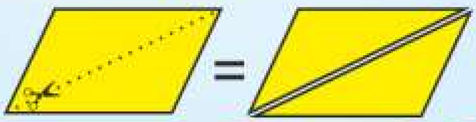
योगे युतिः स्यात् क्षययोः स्वयोर्वा धनर्णयोरन्तरमेव योगः ।
शून्यर्णयोः स्वधनयोः खशून्ययोर्वा वधः शून्यम् ।
योगोऽन्तरं तुल्यहरांशकानां कल्प्यो हरो रूपमहारराशेः।
अन्योन्यहाराभिहतौ हरांशौ राश्योः समच्छेदविधानमेवम् ।
मिथो हराभ्यामपवर्त्तिताभ्यां यद्वा हरांशौ सुधियात्र गुण्यौ॥
छेदं लवं च परिवर्त्य हरस्य शेषः कार्योऽथ भागहरणे गुणनाविधिश्च।
यावत्तावत्कल्प्यमव्यक्तराशेर्मानं तस्मिन् कुर्वतोद्दिष्टमेव ।
तुल्यौ पक्षौ साधनीयौ प्रयत्नात् त्यक्त्वा क्षिप्त्वा वापि संगुण्य भक्त्वा ॥
त्रैराशिकफलराशिं तमथेच्छाराशिना हतं कृत्वा।
लब्धं प्रमाणभाजितं तस्मादिच्छाफलमिदं स्यात्॥
वर्गस्समचतुरश्रः फलञ्च सदृशद्वयस्य संवर्गः॥
यो अक्रन्दयत् सलिलं महित्वा योनिं कृत्वा त्रिभुजं शयानः।
वत्सः कामदुघो विराजः स गुहा चक्रे तन्वः पराचैः॥
त्रिभुजस्य फलशरीरं समदलकोटीभुजार्धसंवर्गः ।
द्वादश प्रधश्चक्रमेकं त्रीणि नभ्यानि क उ तच्चिकेत ।
तस्मिन्त्साकं त्रिशता न शङ्कवोऽर्पिता षष्टिर्न चलाचलासः।।
चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥











ēkādhikēna pūrvēṇa | ekanyūnena pūrveṇa | Vinakulam | Vilokanam

pūraṇāpūraṇābhyām | saṅkalana-vyavakalanābhyām | nikhilaṃ navataścaramaṃ daśataḥ

MAHARSHI SANDIPANI RASHTRIYA VEDA VIDYA PRATISHTHAN, UJJAIN (M.P.)

(Ministry of Education, Government of India)

Phone : (0734) 2502266, 2502254, E-mail : msrvvpujn@gmail.com, website - www.msrvvp.ac.in

Note : -

Any Suggestion may be sent in email: msrvvp.textbooks@gmail.com, along with the proposed texts for replacement/ addition/correction etc., and sender's identification details. The texts may be sent in Unicode font only.

MATHEMATICS TEXTBOOK

(On the basis of vedic Mathematics and Sanskrit Literature of Mathematics)

Veda Bhushan II Year / Prathama - II Year / Class VII

MAHARSHI SANDIPANI RASHTRIYA VEDA SANSKRIT SHIKSHA BOARD

(Established and Recognized by the Ministry of Education, Government of India)

MAHARSHI SANDIPANI RASHTRIYA VEDA VIDYA PRATISHTHAN, UJJAIN (M.P.)

(Ministry of Education, Government of India)

Veda Vidya Marg, Chintaman, Jawasiya, Ujjian, Madhya Pradesh 456006

Phone : (0734) 2502266, 2502254, E-mail : msrvvpujn@gmail.com, website - www.msrvvp.ac.in

Text book developed by :

Cover and decoration :

Drawing :

Technical Support :

Composed by :

Reviewers :

 ISBN :

Price :

Edition : 1st Edition. (2023)

Published Copies :

Paper usage : Printed on 80 GSM paper with RSTB Watermark

Publisher : Maharshi Sandipani Rashtriya Vedavidya Pratishthan (Under the Ministry of Education, Govt. of India) Vedavidya Marg, Chintaman Ganesh, Post Jawasiya, Ujjain (M.P.) 456006

email : msrvvpujn@gmail.com,

Web : msrvvp.ac.in

Phone : (0734) 2502255, 2502254

PREFACE

(In the light of NEP 2020)

The Ministry of Education (Department of Higher Education), Government of India established Rashtriya Veda Vidya Pratishthan in Delhi under the Chairmanship of Hon'ble Education Minister (then Minister of Human Resource Development) under the Societies Registration Act, 1860 (XXI of 1860) on 20th January, 1987. The Government of India notified the resolution in the Gazette of India vide no 6-3/85- SKT-IV dated 30-3-1987 for establishment of the Pratishthan for preservation, conservation, propagation and development of oral tradition of Vedic studies (Veda Samhita, Padapatha to Ghanapatha,Vedanga, Veda Bhashya etc), recitation and intonation of Vedas etc and interpretation of Vedas in scientific lines. In the year 1993 the name of the organization was changed to Maharshi Sandipani Rashtriya Veda Vidya Pratishthan (MSRVVP) and it was shifted to Ujjain, Madhya Pradesh.

The National Education Policy of 1986 and Revised Policy Formulations of 1992 and also Programme of Action (PoA) 1992 have mandated Rashtriya Veda Vidya Pratishthan for promoting Vedic education throughout the country. The importance of India's ancient fund of knowledge, oral tradition and employing traditional Guru's for oral education was also emphasized in the PoA.

In accordance with the aspirations of the nation, national consensus and policy in favour of establishing a Board for Veda and Sanskrit Education at national level, the General Body and the Governing Council of MSRVVP under the Chairmanship of Hon'ble Education Minister, Government of India, have set up "Maharshi Sandipani Rashtriya Veda

Sanskrit Shiksha Board" (MSRVSSB) in tune with the mandate of the Pratishthan and its implementation strategies. The Board is necessary for the fulfillment of the objectives of MSRVVP as envisioned in the MoA and Rules. The Board has been approved by the Ministry of Education, Government of India and recognized by the Association of Indian Universities, New Delhi. The bye-laws of the Board have been vetted by Central Board of Secondary Education and curriculum structure have been concurred by the National Council of Educational Research and Training, New Delhi.

It may also be mentioned here that the committee "Vision and Roadmap for the Development of Sanskrit - Ten year perspective Plan", under the Chairmanship of Shri N. Gopalaswamy, former CEC, constituted by the Ministry of Education Govt. of India in 2015 recommended for establishment of a Board of Examination for standardization, affiliation, examination, recognition, authentication of Veda Sanskrit education up to the secondary school level. The committee was of the opinion that the primary level of Vedic and Sanskrit studies should be inspiring, motivating and joyful. It is also desirable to include subjects of modern education into Vedic and Sanskrit Pathashalas in a balanced manner. The course content of these Pathashalas should be designed to suit to the needs of the contemporary society and also for finding solutions to modern problems by reinventing ancient knowledge.

With regard to Veda Pathashala-s it is felt that they need further standardization of recitation skills along with introduction of graded materials of Sanskrit and modern subjects so that the students can ultimately acquire the capabilities of studying Veda bhashya-s and mainstreaming of students is achieved for their further studies. Due emphasis may also be given for the study of Vikriti Patha of Vedas at an appropriate level. The members of the committee have also expressed their concern that the Vedic recitation studies are not uniformly spread all over India; therefore, due steps may be taken to improve the situation without in anyway interfering with regional variations of recitation styles and teaching method of Vedic recitation.

It was also felt that since Veda and Sanskrit are inseparable and complementary to each other and since the recognition and affiliation problems are same for all the Veda Pathashalas and Sanskrit Pathashalas throughout the country, a Board may be constituted for both together. The committee observed that the examinations conducted by the Board should have legally valid recognition enjoying parity with modern Board system of education. The committee observed that the Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain may be given the status of Board of Examinations with the name “Maharshi Sandipani Rashtriya Veda Sanskrita Vidya Parishat with headquarters in Ujjain which will continue all programs and activities which were being conducted hitherto in addition to being a Board of Examinations.

The promotion of Vedic education is for a comprehensive study of India’s glorious knowledge tradition and encompasses multi-layered oral tradition of Vedic Studies (Veda Samhita, Padapatha to Ghanapatha,Vedanga, Veda Bhashy aetc), recitation and intonation, and Sanskrit knowledge system content. In view of the policy of mainstreaming of traditional students and on the basis of national consensus among the policy making bodies focusing on Vedic education, the scheme of study of Veda stretching up to seven years in Pratishthan also entails study of various other modern subjects such as Sanskrit, English, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, etc. as per the syllabus and availability of time. In view of NEP 2020, this scheme of study is with appropriate inputs of Vedic knowledge and drawing the parallels of modern knowledge in curriculum content focusing on Indian Knowledge System.

In Veda Pathashala-s, GSP Units and Gurukula-s of MSRVVP, affiliated to the Board transact the curriculum primarily based on oral tradition of a particular complete Veda Shakha with perfect intonation and memorization, with additional subsidiary modern subjects such as English, Sanskrit, Mathematics, Science, Social Science and SUPW. Gradually, the Veda Pathashala-s will also introduce other skill and vocational subjects as per their resources.

It is a well-known fact that there were 1131 shakha-s or recensions of Vedas; namely 21 in Rigveda, 101in Yajurveda, 1000 in Samaveda and 9 in Atharva Veda. In course of time, a large number of these shakhas became extinct and presently only 10 Shakhas, namely, one in Rigveda, 4 in Yajurveda, 3 in Samaveda and 2 in Atharvaveda are existing in recitation form on which Indian Knowledge System is founded now. Even in regard to these 10 Shakhas, there are very few representative Vedapathis who are continuing the oral Vedic tradition/ Veda recitation/Veda knowledge tradition in its pristine and complete form. Unless there is a full focus for Vedic learning as per oral tradition, the system will vanish in near future. These aspects of Oral Vedic studies are neither taught nor included in the syllabus of any modern system of school education, nor do the schools/Boards have the systemic expertise to incorporate and conduct them in the conventional modern schools.

The Vedic students who learn oral tradition/ recitation of Veda are there in their homes in remote villages, in serene and idyllic locations, in Veda Gurukulas, (GSP Units), in Veda Pathashala-s, in Vedic Ashrams etc. and their effort for Veda study stretches to around 1900 – 2100 hours per year; which is double the time of other conventional school Board's learning system. Vedic students have to have complete Veda by-heart and recite verbatim with intonation *(udatta, anudatta, swaritaetc),* on the strength of memory and guru parampara, without looking at any book/pothi. Because of unique ways of chanting the Veda mantras, unbroken oral transmission of Vedas and its practices, this has received the recognition in the UNESCO-World Oral Heritage in the list of Intangible Cultural Heritage of Humanity. Therefore, due emphasis is required to be given to maintain the pristine and complete integrity of the centuries old Vedic Education (oral tradition/ recitation/ Veda knowledge Tradition). Keeping this aspect in view the MSRVVP and the Board have adopted unique type of Veda curriculum with modern subjects like Sanskrit, English, Vernacular language, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture etc. as well as skill and vocational subjects as prescribed by NEP 2020.

As per Vedic philosophy, any person can become happy if he or she learns both *Para-Vidya and Apara-Vidya.* The materialistic knowledge from the Vedas, their auxiliary branches and subjects of material interest were called *Apara-Vidya*. The knowledge of supreme reality, the ultimate quest from Vedas, Upanishads is called *Para-Vidya.* In all the total number of subjects to be studied as part of Veda and its auxiliaries are fourteen. There are fourteen branches of learning or *Vidyas* - four Vedas, Six Vedangas, Mimamsa (Purva Mimamsa and Uttara Mimamsa), Nyaya, Puranas and Dharma shastra. These fourteen along with Ayurveda, Dhanurveda, Gandharvaveda and Arthashastra become eighteen subjects for learning. All curriculum transaction was in Sanskrit language, as Sanskrit was the spoken language for a long time in this sub-continent.

Eighteen Shilpa-s or industrial and technical arts and crafts were mentioned with regard to the Shala at Takshashila. The following 18 skills/Vocational subjects are reported to be subjects of the study– (1) Vocal music (2) Instrumental music (3) Dancing (4) Painting (5) Mathematics (6) Accountancy (7) Engineering (8) Sculpture (9) Cattle breeding (10) Commerce (11) Medicine (12) Agriculture (13) Conveyancing and law (14) Administrative training (15) Archery and Military art (16) Magic (17) Snake charming (18) Art of finding hidden treasures.

For technical education in the above mentioned arts and crafts an apprenticeship system was developed in ancient India. As per the Upanishadic vision, the vidya and avidya make a person perfect to lead contented life here and liberation here-after.

Indian civilization has a strong tradition of learning of shastra-s, science and technology. Ancient India was a land of sages and seers as well as of scholars and scientists. Research has shown that India had been a Vishwa Guru, contributing to the field of learning (vidya-spiritual knowledge and avidya- materialistic knowledge) and learning centers like modern universities were set up. Many science and technology based advancements of that time, learning methodologies, theories and techniques discovered by the ancient sages have created and strengthened the fundamentals of our knowledge on many aspects, may it be on astronomy, physics, chemistry, mathematics, medicine, technology, phonetics, grammar etc. This needs to be essentially understood by every Indian to be proud citizen of this great country!

The idea of India like "Vasudhaiva Kutumbakam" quoted at the entrance of the Parliament of India and many Veda Mantra-s quoted by constitutional authorities on various occasions are understood only on study of the Vedas and true inspiration can be drawn only by pondering over them. The inherent equality of all beings as embodiment of "sat, chit, ananda" has been emphasized in the Vedas and throughout the Vedic literature.

Many scholars have emphasized that Veda-s are also a source of scientific knowledge and we have to look into Vedas and other scriptural sources of India for the solution of modern problems, which the whole world is facing now. Unless students are taught the recitation of Vedas, knowledge content of Vedas and Vedic philosophy as an embodiment of spiritual and scientific knowledge, it is not possible to spread the message of Vedas to fulfill the aspiration of modern India.

The teaching of Veda (Vedic oral tradition/ Veda recitation/ Veda knowledge Tradition) is neither only religious education nor only religious instruction. It will be unreasonable to say that Vedic study is only a religious instruction. Veda-s are not religious texts only and they do not contain only religious tenets; they are the corpus of pure knowledge which are most useful to humanity as whole. Hence, instruction or education in Veda-s cannot be construed as only "religious education/religious instruction."

Terming "teaching of Veda as a religious education" is not in consonance with the judgment of the Hon'ble Supreme Court (AIR 2013: 15 SCC 677), in Civil Appeal no. 6736 of 2004 (Date of judgment-3rd July

2013). The Vedas are not only religious texts, but they also contain the knowledge in the disciplines of mathematics, astronomy, meteorology, chemistry, hydraulics, physics, science and technology, agriculture, philosophy, yoga, education, poetics, grammar, linguistics etc. which has been brought out in the judgment by the Hon'ble Supreme Court of India.

Vedic education through establishment of Board in compliance with NEP-2020

The National Education Policy-2020 firmly recognizes the Indian Knowledge Systems (also known as 'Sanskrit Knowledge Systems'), their importance and their inclusion in the curriculum, and the flexible approach in combining various subjects. Arts' and Humanities' students will also learn science; try to acquire vocational subjects and soft skills. India's special heritage in the arts, sciences and other fields will be helpful in moving towards multi-disciplinary education. The policy has been formulated to combine and draw inspiration from India's rich, ancient and modern culture and knowledge systems and traditions. The importance, relevance and beauty of India's classical languages and literature is also very important for a meaningful understanding the national aspiration. Sanskrit, being an important modern language mentioned in the Eighth Schedule of Indian Constitution, its classical literature that is greater in volume than that of Latin and Greek put together, contains vast treasures of mathematics, philosophy, grammar, music, politics, medicine, architecture, metallurgy, drama, poetry, storytelling, and more (known as 'Sanskrit Knowledge Systems').These rich Sanskrit Knowledge System legacies for world heritage should not only be nurtured and preserved for posterity but also enhanced through research and put in to use in our education system, curriculum and put to new uses. All of these literatures have been composed over thousands of years by people from all walks of life, with a wide range of socio-economic background and vibrant philosophy. Sanskrit will be taught in engaging and experiential as well as contemporary relevant methods. The use of Sanskrit knowledge system is exclusively through listening to sound and pronunciation. Sanskrit textbooks at the Foundation and Middle School level will be available in Simple Standard Sanskrit (SSS) to teach Sanskrit through Sanskrit (STS) and make its study enjoyable. Phonetics and pronunciation prescriptions in NEP 2020 apply to the Vedas, the oral tradition of the Vedas and Vedic education, as they are founded upon phonetics and pronunciation.

There is no clear distinction made between arts and science, between curricular and extra-curricular activities, between vocational and academic streams, etc. The emphasis in NEP 2020 is on the development of a multi-disciplinary and holistic education among the sciences, social sciences, arts, humanities and sports for a multi-disciplinary world to ensure the unity and integrity of all knowledge. Moral, human and constitutional values like empathy, respect for others, cleanliness, courtesy, democratic spirit, spirit of service, respect for public property, scientific temper, freedom, responsibility, pluralism, equality and justice are emphasized.

The NEP-2020 at point no. 4.23 contains instructions on the pedagogic integration of essential subjects, skills and abilities. Students will be given a large amount of flexible options in choosing their individual curriculum; but in today's fast-changing world, all students must learn certain fundamental core subjects, skills and abilities to be a well-grounded, successful, innovative, adaptable and productive individual in modern society. Students must develop scientific temper and evidence based thinking, creativity and innovation, aesthetics and sense of art, oral and written expression and communication, health and nutrition, physical education, fitness, health and sport, collaboration and teamwork, problem solving and logical thinking, vocational exposure and skills, digital literacy, coding and computational thinking, ethics and moral reasoning, knowledge and practice of human and constitutional values, gender sensitivity, fundamental duties, citizenship skills and values, knowledge of India, environmental awareness etc. Knowledge of these skills include conservation, sanitation and hygiene, current affairs and important issues facing local communities, the states, the country and the world, as well as proficiency in multiple languages. In order to enhance the linguistic skills of children and to preserve these rich languages and their artistic treasures, all students in all schools, public or private, shall have the option of learning at least two years in one classical language of India and its related literature.

The NEP-2020 at point no. 4.27 states that -"Knowledge of India" includes knowledge from ancient India and its contributions to modern India and its successes and challenges, and a clear sense of India's future aspirations with regard to education, health, environment, etc. These elements will be incorporated in an accurate and scientific manner throughout the school curriculum wherever relevant; in particular, Indian Knowledge Systems, including tribal knowledge and indigenous and traditional ways of learning, will be covered and included in mathematics, astronomy, philosophy, yoga, architecture, medicine, agriculture, engineering, linguistics, literature, sports, games, as well as in governance, polity, conservation. It will have informative topics on inspirational personalities of ancient and modern India in the fields of medicinal practices, forest management, traditional (organic) crop cultivation, natural farming, indigenous sports, science and other fields.

The NEP-2020 at point no. 11.1 gives directions to move towards holistic and multidisciplinary education. India emphasizes an ancient tradition of learning in a holistic and multidisciplinary manner, including the knowledge of 64 arts such as singing and painting, scientific fields such as chemistry and mathematics, vocational fields such as carpentry, tailoring; professional work such as medicine and engineering, as well as the soft skills of communication, discussion and negotiation etc. which were also taught at ancient universities such as Takshashila and Nalanda. The idea that all branches of creative human endeavour, including mathematics, science, vocational subjects and soft skills, should be considered 'arts', has a predominantly Indian origin. This concept of 'knowledge of the many arts' or what is often called 'liberal arts' in modern times (i.e., a liberal conception of the arts) will be our part of education system.

At point No. 11.3 the NEP-2020 further reiterates that such an education system "would aim to develop all capacities of human beings - intellectual, aesthetic, social, physical, emotional, and moral in an integrated manner. Such an education will help develop well-rounded individuals that possess critical 21st century capacities in fields across the arts, humanities, languages, sciences, social sciences, and professional, technical, and vocational fields; an ethic of social engagement; soft skills, such as communication, discussion and debate; and rigorous specialization in a chosen field or fields. Such a holistic education shall be, in the long term, the approach of all undergraduate programmes, including those in professional, technical, and vocational disciplines."

The NEP-2020 at point no. 22.1 contains instructions for the promotion of Indian languages, art and culture. India is a rich storehouse of culture – which has evolved over thousands of years, and is reflected in its art, literary works, customs, traditions, linguistic expressions, artifacts, historical and cultural heritage sites, etc. Traveling in India, experiencing Indian hospitality, buying beautiful handicrafts and handmade clothes of India, reading ancient literature of India, practicing yoga and meditation, getting inspired by Indian philosophy, participating in festivals, appreciating India's diverse music and art and watching Indian films are some of the ways through which millions of people around the world participate in, enjoy and benefit from this cultural heritage of India every day.

In NEP-2020 at point no. 22.2 there are instructions about Indian arts. Promotion of Indian art and culture is important for India and to all of us. To inculcate in children a sense of our own identity, belonging and an appreciation of other culture and identity, it is necessary to develop in children key abilities such as cultural awareness and expression. unity, positive cultural identity and self-esteem can be built in children only by developing a sense and knowledge of their cultural history, art, language and tradition. Therefore, the contribution of cultural awareness and expression is important for personal and social well-being.

The core Vedic Education (Vedic Oral Tradition / Veda Path / Veda Knowledge Tradition) of Pratishthan along with other essential modern subjects- Sanskrit, English, Mother tongue, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, Indian Art, Socially useful productive work etc., based on the IKS inputs are the foundations/sources of texts books of Pratishthan and Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board. These inputs are in tune with the NEP 2020. The draft books are made available in pdf form keeping in view the NEP 2020 stipulations, requirements of MSRVVP students and the advice of educational thinkers, authorities and policy of Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain. These books will be updated in line with NCFSE in future and finally will be made available in print form.

The Teachers of Veda, Sanskrit and Modern subjects in Rashtriya Adarsh Veda Vidyalaya, Ujjain and many teachers of Sanskrit and modern subjects in aided Veda Pathshalas of Pratishthan have worked for last two years tirelessly to prepare and present Sanskrit and modern subject text books in this form. I thank all of them from the bottom of my heart. Many eminent experts of the national level Institutes have helped in bringing quality in the textbooks by going through the texts from time to time. I thank all those experts and teachers of the schools. I extend my heartfelt gratitude to all my co-workers who have worked for DTP, drawing the sketches, art work and page setting.

All suggestions including constructive criticism are welcome for the improvement of the quality of the text books.

आपरितोषाद् विदुषां न साधु मन्ये प्रयोगविज्ञानम्।
बलवदपि शिक्षितानाम् आत्मन्यप्रत्ययं चेतः॥

(Abhijnanashakuntalam 1.02)

Until the scholars are fully satisfied about the content, presentation, attainment of objective, I do not consider this effort to be successful, because even the scholars are not fully confident in the presentation without feedback from the stakeholders.

Prof. ViroopakshaV Jaddipal
Secretary
Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain
Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board, Ujjain

FOREWORD

Since the beginning of the Vedic period, mathematics has been given the highest place. India has a rich tradition of mathematics. Since the ancient times, Indian mystics and mathematicians have done excellent work in this field. In the Indian tradition, Lord Ganesh has defined mathematics in his book Buddhivilasini as follows:

गण्यते संख्यायते तद्गणितम्। तत्प्रतिपादकत्वेन तत्संज्ञ शास्त्रम् उच्यते

Meaning, the science that explains the basis of calculation is called mathematics. In Paniniya metallurgy numbers metals represent (Ganana), which means to calculate. The word mathematics is derived from the suffix in Gana Dhatu, so mathematics was special in Indian thought since ancient times.

The purpose of this book is to use ancient knowledge and upgrade the methods of modern mathematics with the help of ancient achievements, by incorporating mathematical concepts available in Sanskrit literature along with Vedic mathematics. An attempt has been made to simplify calculations through Vedic mathematics.

To improve learning level skills in mathematics of Vedic students across India by harmonizing the teaching of mathematics in concurrence with changing environment in the current global scenario, curriculum and textbooks have been designed keeping in mind the main principles like discussion, analysis of examples and application, as per the guide lines given in the National Education Policy 2020.

The language used in the textbook is very simple and intuitive, which will make it easier for the students to understand. The Ved Bhushan II (seventh equivalent) year textbook has a total of 10 chapters, which is almost equivalent to the Class VII mathematics syllabus across India. In the textbook, many references have been included, take and from Vedic and o Sanskrit literature as well as sutra (formula) from Brahmsfoot Theory, ShulbaSutra, Aryabhtiyam, Lilavati and Beejganitam etc. Through this, Vedic students will be able to understand ancient mathematical concepts along with modern mathematics and appreciates the dignity of their Indian tradition. Revising the concepts of the previous class, the chapters of the textbook have been composed as per the requirement of the Ved Bhushan second year course of Ved Vidalia.

Chapter 1- discusses in detail the properties of integers and the operations on integers. In Chapter 2- the relationship between fractions and decimal, operations such as sum, difference, multiplication and division are presented in detail. Chapter3- describes the interaction of sum, difference, multiplication and division using the formulas of Vedic mathematics. Chapter 4- explains about data handling, arrangement of data as well as finding the mean, median and mode of the data. Chapter 5- describes how to solve simple equations and write sentences in the form of equations. Chapter 6- describes in detail about the angles formed by the intersecting lines, angles made by a transversal on two parallel lines and relation between them. Chapter 7- describes the concept of ratio between two similar quantities, proportion of quantities, percentages and its inter-conversion, cost and selling prices and percentage of a number. Chapter 8- explains about the concept of rational numbers in detail. Chapter 9- covers the area and perimeter of square, rectangle, parallelogram, and triangle, and circumference, area of a circle. The method of remembering the formulas is also given in detail. In Chapter10- linear symmetry, reflection symmetry, rotational symmetry, etc. have been introduced under the concept of symmetry.

The textbook has various activities to develop the knowledge of mathematics of Vedic students as well as to develop the skills of rediscovering facts. Important concepts and outcomes are given as "We Learned" at the end of each chapter to enhance the level of understanding and help the students to attain perfection.

In order to make students aware the rich traditions of India and the contribution made by Indian mathematicians in mathematics, same has been mentioned at the end of the textbook.

By understanding the mathematical concepts available in the textbook, Vedic students will be able to prepare for competitive exams. After studying the said book, students should study NCERT of class VII and books related to the subject.

The author will be grateful for the positive suggestions sent for error correction of the textbook.

CONTENTS

Chapter – 1

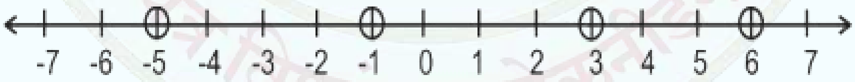
INTEGER

Dear students! We have studied about whole numbers and integers in our last class. You must be aware that on adding negative numbers $-1, -2, -3, -4$ …… with the group of whole numbers 0, 1, 2, 3, … we get the group of integers. You must we aware, how to add and subtract integers. We must we very careful of signs while adding and subtracting integers. In this chapter, we will learn about the properties of integers, how to multiply and divide integers with the help of our previous knowledge.

Let us now, revise the concepts of integers which we have studied already.

Revision:

In the following diagram, some of the integers are represented on a number line.



In the above diagram, the integers $-5, -1$, 3 and 6 are represented on a number line. Can you find the least of the integers represented on the number line and write them in their ascending order?

We know that, on moving from left to right the value of integers increase and on moving from right to left, the value of integers decrease.

Hence, the ascending order of given integers is - $-5 < -1 < 3 < 6$

Can you add and subtract integers?

We get the following results while adding and subtracting integers:

1. The result of addition of two positive integers is a positive integer.

Remember:

1. 0 is less than every positive integers.
2. 0 is greater than every negative integer.
3. Every positive integer is greater than every negative integer.
4. among the negative integers -1 is the greatest negative integer.
5. On a number line, greater the distance between 0 and an integer on its right, greater is its value.
6. On a number line, greater the distance between 0 and an integer on its left, lesser is its value.
7. Greater the positive integer, lesser is its opposite (it's negative).

Example: (+ 2) + (+ 4) = + 6

2. The result of addition of two negative integers is a negative integer.

Example: (– 2) + (– 4) = – 6

3. The result of addition of two integers with opposite signs is the difference between their absolute values and the sign of the result is the sign of integer whose absolute value is greater.

Example: (+ 6) + (– 2) = + 4

4 Negative of a negative integer is positive.

Example: – (– 4) = + 4

Properties of integer:

1. Closure Property

ADDITION:

1 The sum or difference between two integers is always an integer. Hence, the

Integers are closed under the operations of addition and subtraction.

Example:

Addition $17(\text{an integer}) + 13(\text{an integer}) = 30(\text{an integer})$

$(-17)(\text{an integer}) + 3(\text{an integer}) = (-14)(\text{an integer})$

Subtraction: $7(\text{an integer}) - 9(\text{an integer}) = (-2)(\text{an integer})$

$7(\text{an integer}) - (-3)(\text{an integer}) = 10(\text{an integer})$

We can conclude from above examples, that for any two integers *a* and *b*, $(a + b)$ and $(a - b)$ are integers.

2 COMMUTATIVE PROPERTY:

For any two integers, *a* and *b*, the order of addition of integers namely $(a + b)$ and $(b + a)$ have the same result.

Thus, we can conclude that the integer satisfy the commutative property under the operation of addition.

Example: $8 + (-9) = (-1) = (-9) + 8$

$(-13) + 10 = (-3) = 10 + (-13)$

From the above examples, for any two integers *a* and *b*

$$a + b = b + a$$

ASSOCIATIVE PROPERTY:

For any three integers *a*, *b* and *c*, while adding, the order of grouping of integers does not change the result.

Explanation: $a + (b + c) = (a + b) + c$

Example: $(-5) + [(-4) + (-3)] = [(-5) + (-4)] + (-3)$

$(-5) + (-7) = (-9) + (-3)$

$(-12) = (-12)$

3 ADDITIVE IDENTITY:

Additive identity of integers is an integer, when added to any given integer does not change its value.

We know that, on adding 0 with any integer, its value does not change.

Example: $7 + 0 = 7$

$(-7) + 0 = (-7)$

Hence, 0 is the additive identity of integer.

In general, for any integer a

$$\boldsymbol{a + 0 = a = 0 + a}$$

FOR MULTIPLICATION:

1. Closure Property:

For any two or more integers, the order of multiplication does not change the result of the product. Hence, we can conclude that, integers satisfy the closure property under the operation of multiplication.

Example: $(-20) \times (-10) = 200$ an integer

$(-3) \times (2) = (-6)$ an integer

In general, for any two integers a and b $(a \times b)$as an integer.

2. **Commutative Property**:

In the product of integers, the order of multiplication does not change the value of the result. Hence, for integers the commutative property is satisfied under the operation of multiplication.

Example: $(-20) \times 3 = 3 \times (-20)$

$(-60) = (-60)$

In general, for any two integers *a* and *b*, we can say that

$$a \times b = b \times a.$$

3. **ASSOCIATIVE Property:**

In multiplication of three are more integers, the order of grouping of integers does not change the result of the product.

Example: $(-3) \times [(-4) \times (-2)] = [(-3) \times (-4)] \times (-2)$

$(-3) \times (8) = (12) \times (-2)$

$(-24) = (-24)$

In general, for any three integers *a*, *b* and *c* we can say that

$$(a \times b) \times c = a \times (b \times c)$$

4. **Multiplication of 0 and an integer:**

The product of an integer and 0 is always 0.

Example: $5 \times 0 = 0 = 0 \times 5$

$(-4) \times 0 = 0 = 0 \times (-4)$

In general, for any integer *a*,

$$a \times 0 = 0 = 0 \times a$$

5. **Multiplicative identity:**

The product of any integer and 1 is the same integer. Hence, we can say that the multiplicative identity of all the integers is 1.

Example: $5 \times 1 = 5 = 1 \times 5$

$(-4) \times 1 = (-4) = 1 \times (-4)$

In general, for any integer *a*

$$a \times 1 = a = 1 \times a$$

Remember: 0 is the additive identity and 1 is the multiplicative identity of integers.

On multiplying an integer with (-1), we get negative of the integer.

$$a \times (-1) = (-1) \times a = -a$$

Addition of integers: In the following mantra of beejganitam composed by Bhaskaracharya ji, the sum and difference between two integers is explained.

योगे युतिः स्यात् क्षययोः स्वयोर्वा धनर्णयोरन्तरमेव योगः ।

(बीजगणितम्, धनर्णसङ्कलनम्, पृ. 5)

The above sholka means, the sum of two negative integers is negative and sum of two positive integers is positive. The sum of a positive and a negative integer is the difference between them.

Example:

1. (+ 3) + (4) = + 7 **2.** (– 4) + (– 3) = – 7

3. (+ 7) + (– 2) = (+ 5) **4.** – 2 + (+ 8) = (+ 6)

Difference between two integers:

संशोध्यमानं स्वमृणत्वमेति स्वत्वं क्षयस्तद्युतिरुक्तवच्च ॥

(बीजगणितम्, धनर्णव्यवकलनम्, पृ. 7)

The above shloka means, the sign of subtrahend (the number which is subtracted) is changed (from positive to negative and vice versa), and follow the rules of addition of two integers two evaluate.

Example:

1. (– 3) – (– 3) = – 3 + 3 = 0

2. (+ 4) – (+ 5) = 4 – 5 = – 1

3. (+10) – (+ 4) = 10 – 4 = 6

4. – 12 – (– 4) = – 12 + 4 = – 8

Complete the following table

Addition of Integers				Subtraction of Integers		
3	4			3	– 5	
–3	12			4	4	
5	6			4	– 5	
4	–1			–5	1	
–3	4			3	6	

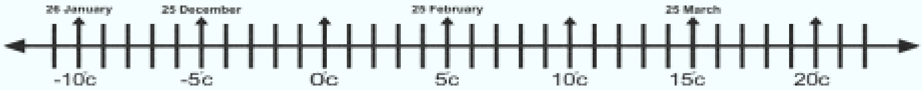
Comparison of integers:

You must be aware that we compare the numbers using (<, > and =) sings.

1.	– 14	**<**	14	**4.**	13	**>**	– 15
2.	13	>	10	**5.**	20	>	10
3.	– 13	<	10	**6.**	30	>	– 15

Exercise 1.1

1. The following number line we presents the temprature (°C) at LEH during different months of a year.



   1) Use the number line given above to find the temperature at Amarnath on different dates

      (a) 26^{th} January:______________

      (b) 25^{th} December:______________

      (c) 25^{th} February:______________

      (d) 25^{th} March: ______________

   2) Was the temperature on 26^{th} January more than the temperature on 25^{th} March?

   3) In which month the temperature was highest?

2. prashant deposits Rs. 5oo in a bank and withdraws Rs. 100 after a month. If withdrawals are represented by negative sign (–), how are deposits represented? Find the balance amount in this account after the withdrawal?

3. Solve the following:

1.	$(-4) + (+3)$	2.	$15 - 8 + 2$
3.	$400 + (-100) + 200$	4.	$23 - 7 - 5$
5.	$-27 + (-3) + 30$	3.	$400 - 100 + 3$

4. Compare the following integers using the signs (<, = and >)

1.	$-4 + 3 + 2$	$1 + 3 + 4$
2.	$30 + (-4) + (-9)$	$4 + (-9) + 30$

3. $6 + 7 - 13$ $13 - 7 - 6$

4. $0 + 1 - 3$ $1 - (-3) + 0$

5. $18 - 12 - 1$ $20 - 10 - 5$

5. Fill in the following blanks:

1. $15 - 3 = 8 -$ ……
2. $18 +$…… $= 18$
3. $8 -$ …… $= 0$
4. $12 + 4 = 13 +$……
5. $15 +$…… $= 18 + 12$

> - **Think?**
>   1. At what temprature will the water kept in the refridgerator freeze?
>   2. To what temprature should the water be heated to boil?

Multiplication of integers:

According to the following words, composed by Brahmagupth ji in his Brahmasphutasiddhanta,Kuttakadhyaya

ऋणमृणधनयोर्घातो धनमृणयोर्धनवधो धनं भवति।

शून्यर्णयोः खधनयोः खशून्ययोर्वा वधः शून्यम् ॥

(ब्रह्मस्फुटसिद्धान्तःकुट्टकाध्याय,33)

In the above verse, it is mentioned that The product of a positive and negative integer is negative, the product of two positive and negative integers is positive and the product of 0 with any integer is 0.

We find the references about product of integers in beejganit

(beejganit P. 8) other than Brahmasphutasiddhanta.

1 Product of positive and negative integer:

$3 \times 2 = 2 + 2 + 2 = 6$

$3 \times (-2) = (-2) + (-2) + (-2) = -6$

Similarly,

$4 \times (-3) = (-3) + (-3) + (-3) + (-3) = -12$

Multiply:

Example: $3 \times (-5) = 5 \times (-3)$

Solution : $3 \times (-5) = (-5) + (-5) + (-5) = -15$

Or $5 \times (-3) = (-3) + (-3) + (-3) + (-3) + (-3) = -15$

Do and Learn-

1. $-5 \times 4 =$......	2. $3 \times -4 =$......
3. $2 \times -4 =$......	4. $2 \times -8 =$......
5. $-5 \times 6 =$......	6. $3 \times -3 =$......
7. $3 \times -6 =$......	8. $7 \times -3 =$......

On solving the above problems, we can say that the "product of a positive and negative integer is always negative integer".

(ii) Product of two negative integers:

The product of two negative integers is positive.

While finding the product of two negative integers, we find the product of their absolute values and wright positive sign before it.

Example: (i) $(-10) \times (-7)$

Solution : In this, both (-10) and (-7) are negative integer.

Hence, $(-10) \times (-7) = +70 = 10 \times 7$

(ii) $(-5) \times (-2)$

Solution $= (+10)$ or 10

In general for any two positive integers *a* and *b*

$$a \times b = (-a) \times (-b) = (-a).(-b) .$$

❖ Do and Learn

Find the product of the following integers

(i) $(-6) \times (-7)$ (ii) $(-25) \times (-4)$ (iii) $(-9) \times (-7)$

(iii) Product of three or more negative integers

We know that the product of two negative integers is positive. Can we find the product of three or more negative integers?

Commutative property of multiplication for integers:

We know that the whole number satisfy the commutative proparty of multiplication. Is this proparty satisfy given for integers?

Complete the following table-

s. No.	Statement 1	Statement 2	Result
1	$7 \times 5 = 35$	$5 \times 7 = 35$	$7 \times 5 = 5 \times 7$
2	$(-1) \times 3 = -3$	$3 \times (-1) =$......	
3	$4 \times (-5) = -20$	$(-5) \times 4 = -20$	
4	$6 \times 11 =$......	$11 \times$..... $=$......	
6	$10 \times -12 =$......	$-12 \times$..... $=$	

We can obsurve that the product of two integers is always an integer.

Hence, integers satisfy the closure property under the opration of multiplication. Further, from the above table the order in which the integers are multiply does not change the result. Hence, integers satistfy the commutative property under the opration of multiplication.

In general, for any two integers a and b,

$$\boldsymbol{a \times b = b \times a}$$

Amazing Facts:

Comment about the following statements and results

$+1 = (-1) \times (-1)$

$-1 = (-1) \times (-1) \times (-1)$

$+1 = (-1) \times (-1) \times (-1) \times (-1)$

$-1 = (-1) \times (-1) \times (-1) \times (-1) \times (-1)$

From the about results, we can observe that when (-1) is multiplied for even number of time the result is 1 and when multiplied for odd number of times the result is (-1).

Think –

what is the result when (-1) is multiplied for 15 times?

Let us now observe the following examples.

(i) $(-3)\times(-4)$

Solution : $(-3)\times(-4=12)$

(ii) $(-3) \times (-2) \times (-2)$

Solution $[(-3) \times (-2)] \times (-2)$

$= (6) \times (-2)$

$= -12$

(iii) $(-3) \times (-3) \times (-2) \times (-1)$

Solution $[(-3) \times (-3)] \times [(-2) \times (-1)]$

$= 9 \times 2$

$= 18$

(iv) $(-1) \times (-1) \times (-2) \times (-3) \times (-4)$

Solution

$[(-1) \times (-1)] \times [(-2) \times (-3)] \times (-4)$

$= 1 \times 6 \times (-4)$

$= [1 \times 6] \times (-4)$

$= 6 \times (-4)$

$= -24$

From the above examples, we can say that

1. The result of the product of two negative integers is positive.
2. The result of the product of three negative integers is negative.
3. The result of the product of four negative integers is positive.
4. The result of the product of five negative integers is negative.

In other words, the product of even number of negative integers is positive and the product of odd number of negative integers is negative.

Think:-

what will be the sign of the product of fifve negative and three positive integers?

Do and Learn

Solve the following

(1) $(-3) \times (-2) \times (-1)$ = ... (2) $(-2) \times (-3) \times (-1)$ =.......

(3) $(-4) \times (-5)$ =........ (4) $(-2) \times (-9)$ = ...

Product of 0 with Integers-

The following words which explains about the product of 0 with integers is found in Brahmasphutasiddhanta

शून्यर्णयोः खधनयोः खशून्ययोर्वा वधः शून्यम् ॥

(ब्रह्मस्फुटसिद्धान्तःकुट्टकाध्याय,33 ब)

In the above vers, it is mentioned that the product of either positive or negative integer and 0 is 0. Tho product of two zeros is 0. Other than this vers we find the refereces in beejganitam about the rule of product of zeros.

In genral for any integer a we can say that

$$a \times 0 = 0 = 0 \times -a$$

$$2 \times 0 = 0 = 0 \times (-2)$$

Example: Multiply: $(-3) \times (-2) \times (-1) \times 0$

Solution $(-3) \times (-2) \times (-1) \times 0$

$= [-3 \times -2] \times [(-1) \times 0]$

$= 6 \times 0$

$= 0$

Division of integers

the following vers which explains the division of integers is found in beejganitam.

भागहारेऽपि चैवं निरुक्तम् ।

(बीजगणितम्, भजनम्, पृ. 11)

We can understand from the above vers that the rule of sign for multiplication is also applicable for division of integers. Let us learn about the division of integers using the following example we know that the division is opposite opration of multiplication.

$$4 \times 5 = 20$$

$$20 \div 4 = 5 \quad \text{or} \quad 20 \div 5 = 4$$

From the above example we can say that for every multiplicative statement $(4 \times 5 = 20)$ there are two division statements $(20 \div 4 = 5, 20 \div 5 = 4)$ associated with it.

S. No.	Multiplicative Statement	Corresponding Dinvision Statements
1.	$3 \times -5 = -15$	$(-15) \div 3 = -5$, $(-15) \div (-5) = 3$
2.	$-2 \times -7 = 14$	$14 \div (-2) = -7$, $14 \div (-7) = (-2)$
3.	$3 \times -2 = -6$	...................., ...
4.	$(+6) \times (+2) =$.........	...................., ...

Note: We divide the integers using the method of division of whole numbers. We need to be careful about signs(positive or negative) of the result.

In genral, $a \div (-b) = (-a) \div b$

Where b or $(-b)$ is ont 0

Example: Divide 6 by 3

Solution : 6 ÷ 3

$$= \frac{6}{3} = 2$$

We find following shloka in Brahmasphutasiddhanta explaining the division of integers.

धनभक्तं धनमृणहृतमृणं धनं भवति खं खभक्तं खम् ।

भक्तमृणेन धनमृणं धनेन हृतमृणमृणं भवति ॥

(ब्रह्मस्फुटसिद्धान्तः,कुट्टकाध्याय: 34)

The above shloka mentions about the division of integers. The quotient of two negative or two positive integers is positive. On dividing a positive integer by negative integer or negative integer by a positive integer is negative.

1. **Negative integer ÷ Positive integer = Negative integer**

Example: (–10) ÷ 2

Solution : (–10) ÷ 2

$$= \frac{(-10)}{2} = (-5)$$

2. **Positive integer ÷ Negative integer = Negative integer**

Example: 20 ÷ (– 2)

Solution : 20 ÷ (– 2)

$$= \frac{20}{(-2)} = (-10)$$

3. **Positive integer ÷ Positive integer = Positive integer**

Example: 60 ÷ 2

Solution : 60 ÷ 2

$= \frac{60}{2} = 30$

4. Negative integer ÷ Negative integer = Positive integer

Example: (–100) ÷ (– 50)

Solution :

(–100) ÷ (– 50)

$= \frac{(-100)}{(-50)}$

$= 2$

Do and Learn:

(i) $\frac{-25}{5}$ (ii) –25 ÷ –5 (iii) 40 ÷ – 20

BODMAS

का करके पुनि भाग कर, फिर गुण लेह सुजान ।

ता पीछे धन ऋण कर, भिन्न रीति यह जान ॥

BODMAS is one of the ruls in mathametics which helps us two solve problems easily.

BODMAS

B =	stands for bracket	(), { }, [ ]
O =	stands for of,	$(\ )^2$ or $\sqrt{\ }$
D =	stands for division	÷
M =	stands for multiplication	×
A =	stands for addition	+
S =	stands for subtraction	–

Solve the following

Example 1- 3 × (– 2) + 1

Solution : $[3 \times (-2)] + 1$

$= \quad -6 + 1$

$= \quad -5$

Example 2 -

$(-4) \div 2 + 3 - 1$

Solution :

$-2 + 3 - 1$

$= \quad -3 + 3$

$= \quad 0$

Example 3

$15 \div 3 \times 4 + 3 - 1$

Solution : following the rules of BODMAS $(\div, \times, +, -)$ $(\div, \times, +, -)$

$15 \div 3 \times 4 + 3 - 1$

Than

$5 \times 4 + 3 - 1$

$= \quad 20 + 3 - 1$

$= \quad 23 - 1$

$= \quad 22$

Example 4: $2 - 3 \times 4 + 18 \div 3$

Solution : following the rules of BODMAS $(\div, \times, +, -)$

$2 - 3 \times 4 + 6$

$= \quad 2 - 12 + 6$

$= \quad 2 - 6$

$= \quad -4$

Exercise 1.2

1. Multiply the following

(i) $4 \times (-3)$ (ii) $12 \times (-1)$

(iii) 30×10 (iv) $4 \times (-3) \times (-2)$

(v) $(-5) \times (-3) \times (-2)$ (vi) $(-5) \times 0 \times (-1)$

(vii) $(-124) \times 0$ (viii) $(-1) \times (-1)$

(ix) $(-1) \times (-1) \times (-1)$ (x) $(-1) \times (-5) \times (-1)$

2. Complete the following table

×	4	–2	–1	3
1	....	....	....	....
–3	....	....	....	....
4	....	....	....	....
–2	....	....	+2	– 6

3. Write the pairs of integers such that

(i) Difference is (-5)

(ii) Sum is 0

(iii) Sum is (-3)

4. Write the Largest and the smallest integer between (-5) and 10

5. Divide the following

(i) $(-25) \div 5$ (ii) $(-35) \div (-35)$

(iii) $15 \div (-3)$ (iv) $(-28) \div (7)$

(v) $[-6 \div 2] \div [3 \div 1]$ (vi) $[30 \div 5] \div [6 \div (-3)]$

6. complete the following table

$\div$	-20	100	-60
-1	....	....	....
-4	....	-25	....
2	....	....	....
5	....	....	....

7. Use the rules of BODMAS and solve

(1) $2 \times (-4) + 3$ (2) $(-8) \div 2 + 3$

(3) $4 \times (-2) + 3 \times (-1)$ (4) $5 \times (-4) - 20 + 10$

(5) $2 \times 4 \div (-2) + 2$ (6) $12 \div 2 \times 4 + 5 - 3$

(7) $20 \div 5 \times 3 + 4 - 8$ (8) $20 \div 5 \times 2 - 8$

➢ **We Learnt**

1 The group of integers consists of whole number (0,1,2,3,…..) and negative numbers (–1, – 2, – 3,……).

2 Sum of two positive integers is a positve integer and sum of two negative integers is a negative integer.

3 Negative of a negative integer is a positive integer

Example: $-(-9) = 9$ or $+9$

4 In multiplication of integers

1 Product of a positive and negative is a negative integer.

$$3 \times (-2) = -6 \text{ or } (-3) \times 2 = -6$$

2 Product of a negative integer and the negative integer is a positive integer.

$$(-2) \times (-3) = 6 \text{ or } +6$$

5 In the division of integer

1 On dividing a positive integer by a negative integer or a negative integer by a positive integer the quotient is a negative integer.

$$\frac{-15}{3} = -5$$

3 On dividing a negative integer by a negative integer the quotient is a positive integer

$$\frac{(-15)}{(-5)} = 3$$

4 For any integer a

(i) $a \div 0 = \frac{a}{0}$ is not defined.

(ii) $a \div 1 = a$.

(iii) $0 \div a = \frac{0}{a}$ $(if\ \mathrm{a} \neq 0)$

(iv) In BODMAS rule we evaluate the expression in the order of brackets, of, division, multiplication, addition and subtraction.

CHAPTER – 2

FRACTIONS AND DECIMALS

Dear vedic students! You must have studied about fractions and decimals in your last class. You must be aware that there are three types of fractions namely proper, improper and mix fractions. Do you remember the method of finding equivalent fractions, like and addition, subtraction of fraction?

We have learnt about reading, comparing, adding and subtracting decimal numbers. In this chapter, we will revise the concepts which we know already, further learn about how to multiply, divide fractions and decimals.

In vedic literature, the following verse is found in yajurveda mentioning about fractions.

त्र्यविश्च मे त्र्यवी च मे दित्यवाट् च मे दित्यौही च मे

पञ्चाविश्च मे पञ्चावी च मे त्रिवत्सश्च मे त्रिवत्सा

च मे तुर्यवाट् च मे तुर्यौही च मे यज्ञेन कल्पन्ताम् ॥

(यजुर्वेद : 18/26)

In the above verse, the word Avi (अवि) refers to time. Using the above vedmantra of yajurveda fractions and decimals can we formed. The words such as trayavi (त्र्यवि) and pacchavi (पञ्चावि) meaning one and half, two and half are found in yajurveda about shloka. These numbers can be written in the fractional form of $1\frac{1}{2}$ or $\frac{3}{2}$, $2\frac{1}{2}$ or $\frac{5}{2}$, and in decimal forms 1.5, 2.5 respectively.

Let us now revise the concepts that we already we know. Classify the following fractions proper and improper fractions.

$$\frac{8}{9}, \frac{5}{3}, \frac{19}{6}, \frac{25}{3}, \frac{11}{6}, \frac{3}{4}, \frac{19}{12}$$

$$\frac{5}{9}, \frac{7}{3}, \frac{19}{16}, \frac{9}{3}, \frac{1}{6}, \frac{5}{4}, \frac{19}{17}$$

Convert the following improper fractions into their mixed forms.

Example: Write three equivalent fractions of $\frac{3}{5}$.

Solution : equivalent fractions of $\frac{3}{5}$

(i) $\frac{3}{5} = \frac{3\times2}{5\times2} = \frac{6}{10}$

(ii) $\frac{3}{5} = \frac{3\times3}{5\times3} = \frac{9}{15}$

(iii) $\frac{3}{5} = \frac{3\times4}{5\times4} = \frac{12}{20}$

Hence, $\frac{6}{10} = \frac{9}{15}, \frac{12}{20} = \frac{3}{5}$

Sum and difference between like fractions:

In the following verse taken from Leelavati ganit, we find references about the addition and subtraction of fractions.

योगोऽन्तरं तुल्यहरांशकानां कल्प्यो हरो रूपमहारराशेः।

(लीलावती,भिन्न सङ्कलितव्यवकलिते : 3)

It means, on adding or subtracting the like fractions, the denominator remains the same. In case if there is no digit in the denominator we consider it as 1. Like $3=\frac{3}{1}, 4=\frac{4}{1}$ and $5=\frac{5}{1}$.

Example: Shubham painted $\left(\frac{3}{5}\right)^{th}$ of the wall and his sister Bulbul painted $\left(\frac{1}{5}\right)^{th}$ of the wall. Find the total part of the wall painted by both of them.

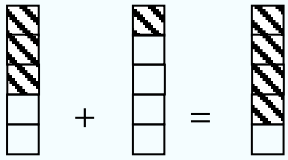
Solution : The part of the wall painted by Shubham: $\left(\frac{3}{5}\right)$

The part of the wall painted by Bulbul: $\left(\frac{1}{5}\right)$

Hence, the total part of the wall painted by both is

$= \frac{3}{5} + \frac{1}{5}$

$= \frac{3+1}{5} = \frac{4}{5}$

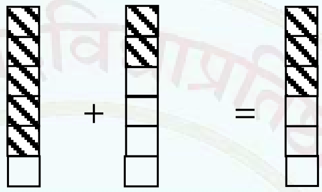


Thus, both of them together painted $\frac{4}{5}$ of the wall.

Example: Subtract $\frac{2}{6}$ from $\frac{5}{6}$

Solution :

$\frac{5}{6} - \frac{2}{6}$



$= \frac{5-2}{6} = \frac{3}{6}$

You must have learnt how to add and subtract like fractions from the above examples. Let us now learn how to add and subtract unlike fractions. Start with we convert unlike fractions in to their equivalent like fractions.

Addition and subtraction of fractions:

In Leelavati ganit the following verse is found explaining the method of adding or subtracting unlike fractions.

अन्योन्यहाराभिहतौ हरांशौ राश्योः समच्छेदविधानमेवम् ।

मिथो हराभ्यामपवर्त्तिताभ्यां यद्वा हरांशौ सुधियाऽत्र गुण्यौ॥

(लीलावती,भिन्नपरिकर्माष्टकम् 1,पृ.35)

It means, to find the equivalent like fractions of given to unlike fractions, multiply both numerator and denominator of the first fraction with denominator of the second fraction, and similarly multiply the numerator and denominator of the second fraction with the denominator of first fraction.

After converting them in to like fraction, **add** both the numerator to get the numerator the sum and keep the denominator of the like equivalent obtained. Similarly we find the **difference** between the numerator and with the denominator of the like fractions.

Example: find the sum of $\frac{2}{5}$ and $\frac{1}{4}$.

Solution : $\frac{2}{5} + \frac{1}{4}$

LCM (4, 5) = 20

Hence,

$$\frac{2}{5} + \frac{1}{4}$$

$$= \frac{2\times4}{5\times4} + \frac{1\times5}{4\times5}$$

$$= \frac{8}{20} + \frac{5}{20} = \frac{8+5}{20} = \frac{13}{20}$$

Example: Find the sum of $1\frac{2}{3}$ or $2\frac{1}{5}$

Solution : $1\frac{2}{3} + 2\frac{1}{5}$

$$= \frac{5}{3} + \frac{11}{5}$$

LCM (3, 5) =15

Converting mix fraction

$$1\frac{2}{3} = \frac{5}{3}$$

$$2\frac{1}{5} = \frac{11}{5}$$

Hence,

$$\frac{5\times5}{3\times5} + \frac{11\times3}{5\times3}$$

$$= \frac{25}{15} + \frac{33}{15} = \frac{25+33}{15} = \frac{58}{15}$$

Example: Subtract: $\frac{1}{5}$ from $\frac{5}{6}$

Solution : It is necessary to convert the given unlike fraction into their equivalent like fractions.

Solution :

$$\frac{5}{6} - \frac{1}{5}$$

LCM (5, 6) = 30

Heance,

$$\frac{5\times5}{6\times5} - \frac{1\times6}{5\times6}$$

$$= \frac{25}{30} - \frac{6}{30}$$

$$= \frac{25-6}{30}$$

$$= \frac{19}{30}$$

Example

Find $1\frac{1}{6} - 2\frac{1}{4}$

Solution :

$$1\frac{1}{6} - 2\frac{1}{4}$$

or $\frac{7}{6} - \frac{9}{4}$

LCM (6, 4) = 12

$$= \frac{7\times2}{6\times2} - \frac{9\times3}{4\times3}$$

$$= \frac{14}{12} - \frac{27}{12}$$

$$= \frac{14-27}{12}$$

$$= \frac{-13}{12}$$

Multiplication of Fractions

To find the product of two fractions, multiply the numerators and denomirators seperately and write in fractional form

Product of two fractions = $\frac{\text{product of numirators}}{\text{product of denominators}}$

you are aware that the repeted sum is difind as multiplaction.

$$3 \times 5 = 5 + 5 + 5 = 15$$

Product of a whole number and a fraction:

Example: If we want to multiply 2 and $\frac{1}{5}$, we add $\frac{1}{5}$ for two times.

$$2 \times \frac{1}{5}$$

$$= \frac{1}{5} + \frac{1}{5}$$

$$= \frac{1+1}{5} = \frac{2}{5}$$

Note: in mathamatics, 'of ' stands for multiplication $(\times)$

example: what is $\frac{1}{2}$ of 20 ?

Soluation:

$\frac{1}{2}$ of 20

$= \frac{1}{2} \times 20$

$= \frac{1}{2} \times \frac{20}{1}$

$= \frac{20}{2} = 10$

Example: what is $3\frac{1}{2}$ of 20

Solution: $3\frac{1}{2}$ of 20

$= \frac{7}{2} \times 20$ (converting mixed fraction into improper fraction)

$= \frac{7}{2} \times \frac{20}{1}$

$$= \frac{140}{2}$$
$$= 70$$

Product of two fractions:

The rule to find the product of two fractions is found in Leelavati's ganit in the form of a verse which is given below -

अंशाहतिश्छेदवधेन भक्ता लब्धं विभिन्ने गुणने फलं स्यात्।

(लीलावती,भिन्न गुणनम् : 4 अ)

The above verse says, on dividing the product of the numerators of fractions by the product of the denominators of fractions, we get the result of the product of fractions.

In product of two whole numbers the result is grater than both the numbers which we already know.

Example: 3 × 4 = 12 and 12 > 4, 12 > 3

Using the example given above, find the product of two fraction given below and compare it with the fractions in the individually. Let us complete the following table and discuss about the product of two fractions

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$	$\frac{2}{3} > \frac{8}{15}$, $\frac{4}{5} > \frac{8}{15}$	**Product is less than the both the fractions**
$\frac{1}{5} \times \frac{2}{7} = \frac{\ldots}{\ldots}$	… … … … , … … … …	
$\frac{3}{5} \times \frac{\ldots}{8} = \frac{21}{40}$	… … … … , … … … …	
$\frac{2}{\ldots} \times \frac{4}{9} = \frac{8}{45}$	… … … … , … … … …	

From the above table, the product of two proper fractions is less than both the fraction.

Give some more examples to prove the above statement. Now let us find the product of a proper and an improper fraction.

Do and Learn

Find the product of the following fractions

A. $\frac{6}{5} \times \frac{2}{7}$ B. $\frac{8}{3} \times \frac{4}{5}$

Example: Find the product of $\frac{2}{3}$ and $\frac{7}{5}$.

Soluation: In $\frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$, we can observe that the product $\frac{14}{15}$ is less than the improper fraction $\frac{7}{5}$ and grater than the proper fraction $\frac{2}{3}$.

Try:

Fill in the following blanks.

(A) $\frac{1}{2} \times \frac{1}{7} = \frac{1 \times 1}{2 \times 7} = \frac{....}{....}$ (B) $\frac{1}{5} \times \frac{1}{7} = \frac{....}{....} = \frac{....}{....}$

(C) $\frac{1}{7} \times \frac{1}{2} = \frac{....}{....} = \frac{....}{....}$ (D) $\frac{1}{7} \times \frac{1}{5} = \frac{....}{....} = \frac{....}{....}$

Example: Ram reads $\frac{1}{3}$ of a book on vedic literature in 1 hour. Find the part of the book that he can read in $2\frac{1}{2}$ hours, If he reads with the same speed,

Soluation: The part of the book that Ram compeletes reading in one hour: $\frac{1}{3}$

Hence, the part of the book that he can compelet reading in $2\frac{1}{2}$:

$2\frac{1}{2} \times \frac{1}{3} = \frac{5}{2} \times \frac{1}{3}$ (converting mixed fraction into its improper form)

we know that $\frac{5}{2} = 5 \times \frac{1}{2}$,

hence,

$$\frac{5}{2} \times \frac{1}{3} = 5 \times \frac{1}{2} \times \frac{1}{3}$$

$$= 5 \times \frac{1}{6} = \frac{5}{6}$$

Thus, Ram reads $\frac{5}{6}$ part of the book in $2\frac{1}{2}$ hour.

1 Product of two proper fraction:

(I) $\frac{1}{3} \times \frac{2}{5} = \frac{1\times2}{3\times5} = \frac{2}{15}$

(ii) $\frac{5}{6} \times \frac{2}{5} = \frac{5\times2}{6\times5} = \frac{10}{30}$ or $\frac{1}{3}$

2 product of a proper an d improper fraction:

(i) $\frac{3}{5} \times \frac{7}{2} = \frac{7\times3}{5\times2} = \frac{21}{10}$

(ii) $\frac{11}{2} \times \frac{3}{4} = \frac{11\times3}{2\times4} = \frac{33}{8}$

3 Product of two mixed fractions:

(i) $1\frac{2}{3} \times 3\frac{2}{2}$

$= \frac{5}{3} \times \frac{8}{2} = \frac{5\times8}{3\times2} = \frac{40}{6}$ or $\frac{20}{3}$

(ii) $3\frac{1}{2} \times 1\frac{3}{4}$

$= \frac{7}{2} \times \frac{7}{4} = \frac{7\times7}{2\times4} = \frac{49}{8}$

Example problem taken from Leelavati ganit:

सत्र्यंशरूपद्वितयेन निघ्नं ससप्तमांशद्वितयं भवेत् किम्।

अर्धं त्रिभागेन हतं च विद्धि दक्षोऽसि भिन्ने गुणनाविधौ चेत्।।

(लीलावती गणित भिन्नगुणने करणसूत्रं, अत्रोद्देशक: 42)

Dear friend! If you concur with the rules of product of two fractions, find the product of 2 and two-third $\left(2 + \frac{2}{3} = \frac{8}{3}\right)$, with two and one – seventh $\left(2 + \frac{1}{7} = \frac{15}{7}\right)$,is multiplied with third part one – half $\left(\frac{1}{2}\right)$of it.

Solution : Solve the problem taking inputs from the students

Example: Out of 25 students in a school $\left(\frac{1}{5}\right)$ of the students study Samved and $\left(\frac{3}{5}\right)$ of the students study Atharvaved. Find the....

1. number of students studying Samved,
2. numbet of students studying atharvaved.

Solution : Total number of students in the school: 25

1. number of students studying Samved:

$$\frac{1}{5} \text{ of } 25 = \frac{1}{5} \times 25 = \frac{25}{5} = 5$$

2. number of students studying atharvaved:

$$\frac{3}{5} \text{ of } 25 = \frac{3}{5} \times 25 = \frac{75}{5} = 50$$

Reciprocal (inverse) of a fraction:

the pair of numbers other than 0, whose product is 1 are called reciprocals of each other. On inter changing the numeartor and denominator of $\frac{2}{5}$, we get $\frac{5}{2}$.

Since, $\frac{2}{5} \times \frac{5}{2} = \frac{10}{10} = 1,$

The fractions $\frac{2}{5}$ and $\frac{5}{2}$ are reciprocals of each other.

Similarly, reciprocal of 7 is $\frac{1}{7}$.

Think: what will be reciprocal of $\frac{1}{3}$?

Division of fractions:

In bhaskracharya ji's Leelavati ganit, the following verse explains the division of fractions.

छेदं लवं च परिवर्त्य हरस्य शेष: कार्योऽथ भागहरणे गुणनाविधिश्च।

(लीलावती,भिन्नभागहारा:, 4 अ)

Meaning, to divide fractions find the product of the first fraction and the reciprocal (inter changing numerator and denominator) of the second fraction.

1 **Division of a whole number by a fraction:**

To divied a whole number by a fraction, we multiply the whole number and reciprocal of the fraction.

Example: $4 \div \frac{3}{5}$

Solution : We know that the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

Hence,

$$4 \div \frac{3}{5} = 4 \times \frac{5}{3} = \frac{20}{3}$$

Division of fraction by a whole number:

To divied a fraction by whole number, we first find the reciprocal of the whole number and multiply it with the fraction.

Example: $\frac{3}{2} \div 4$

Solution : We know that the reciprocal of 4 is $\frac{1}{4}$.

Hence, $\frac{3}{2} \div 4 = \frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$.

Division of a fraction by a fraction:

To divied a fraction by another fraction, we keep the first fraction as it is, replace division sign with multiplication sign, followed by the reciprocal (inter changing numerator and denominator) and hence find the product of fractions using the rule given in the following shloka

'अंशाहतिश्छेदवधेन भक्ता लब्धं विभिन्ने गुणने फलं स्यात्।'

Example: $\frac{1}{3} \div \frac{5}{4}$

Solution : We know that the reciprocal of $\frac{5}{4}$ is $\frac{4}{5}$.

Hence, $\frac{1}{3} \div \frac{5}{4} = \frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$.

division of mixed fractions: We convert the mixed fractions into their respective improper forms and divide the fraction.

Solutain: Converting mixed fraction

$$= \quad \frac{7}{2} \div \frac{11}{3}$$

We know that the reciprocal of $\frac{11}{4}$ is $\frac{4}{11}$.

$$\frac{7}{2} \times \frac{3}{11}$$

$$= \frac{7 \times 3}{2 \times 11}$$

$$= \frac{21}{22}$$

Example : problem from Leelavati Ganit

सत्र्यंशरूपद्वितयेन पञ्च त्र्यंशेन षष्ठं वद मे विभज्य।

दर्भीयगर्भाग्रसुतीक्ष्णबुद्धिश्चेदस्ति ते भिन्नहृतौ समर्था।।

(लीलावती गणित भिन्नगुणने करणसूत्रं, अत्रोद्देशक: 43)

Dear friend, if you are proficient, find the result of division of 5 by $\left(2+\frac{1}{3}\right)$ and $\frac{1}{6}$ by $\frac{1}{3}$.

Solution : Discuss the above problem with the students and solve.

Exercise 2. 1

1. Find the sum of the following fractions.

(i) $\frac{2}{3}+\frac{1}{7}$ (ii) $\frac{4}{3}+\frac{2}{3}$

(iii) $\frac{5}{2}+\frac{3}{4}$ (iv) $\frac{4}{5}+\frac{3}{2}$

(v) $\frac{9}{2}+\frac{1}{3}$ (vi) $\frac{2}{4}+\frac{3}{5}$

(vii) $3\frac{1}{2}+2\frac{1}{7}$ (viii) $5\frac{2}{1}+3\frac{5}{2}$

(ix) find the sum of the mixed fractions $2\frac{4}{5}$ and $3\frac{5}{6}$.

2. Solve (subtract):

(i) $\frac{4}{5}-\frac{3}{5}$ (ii) $\frac{5}{3}-\frac{1}{2}$

(iii) $\frac{6}{5}-\frac{5}{6}$ (iv) $\frac{5}{6}-\frac{8}{3}$

(v) $\frac{11}{3}-\frac{5}{6}$ (vi) $\frac{7}{5}-\frac{9}{2}$

(vii) subtract $\frac{3}{4}$ from $\frac{5}{6}$. (viii) $4\frac{2}{5}-1\frac{2}{5}$

3. Multiply and write the result in its simplest form.

(i) $8\times\frac{2}{5}$ (ii) $\frac{2}{3}\times 4$ (iii) $\frac{5}{3}\times 6$

(iv) $\frac{5}{7}\times 3$ (v) $\frac{5}{7}\times\frac{2}{3}$ (vi) $15\times\frac{5}{15}$

(vii) $20\times\frac{3}{4}$ (viii) $\frac{3}{8}\times\frac{3}{4}$ (ix) $\frac{12}{5}\times 3$

(x) $\frac{18}{2}\times\frac{9}{4}$ (xi) $\frac{3}{4}\times\frac{5}{7}$ (xii) $\frac{3}{8}\times\frac{2}{7}$

4. Solve the following:

(i) $\frac{3}{4}$ of $\frac{5}{7}$ (ii) 27 of $\frac{1}{3}$

(iii) 50 of $\frac{1}{5}$ (iv) 24 of $\frac{3}{4}$

5. Find the reciprocals of the following:

(i) $\frac{3}{7}$ (ii) $\frac{7}{2}$ (iii) 3 (iv) $\frac{1}{4}$ (v) $\frac{2}{6}$

6. Divide and write the result in its simplest form:

(i) $18\div\frac{3}{2}$ (ii) $5\div 3\frac{4}{5}$

(iii) $4 \div 1\frac{1}{4}$ (iv) $4 \div \frac{3}{8}$

7. Solve

(i) $\frac{7}{3} \div 2$ (ii) $5\frac{3}{4} \div 4$ (iii) $\frac{6}{15} \div 5$

(iv) $\frac{7}{3} \div 5$ (v) $\frac{2}{7} \div 7$ (vi) $1\frac{1}{2} \div 3$

8. Solve

(i) $\frac{3}{4} \div \frac{5}{3}$ (ii) $\frac{2}{7} \div \frac{3}{7}$ (iii) $\frac{2}{8} \div \frac{3}{5}$

(iv) $2\frac{1}{3} \div \frac{3}{5}$ (v) $4\frac{1}{5} \div 2\frac{1}{3}$ (vi) $1\frac{1}{2} \div 2\frac{1}{2}$

9. If, out of 700 shlokas of Geeta, Pradhyuman learnt $\frac{2}{7}$ part and Vrindavan learnt $\frac{3}{7}$ part, find

(1) The number of shlokas learnt by Pradhyumam and

(2) The number of shlokas learnt by Vrindaven.

10. In all India Ved sammelan, out of 80 students particitated from a vedic school, $\left(\frac{1}{10}\right)$ of the students won in ved mantraantakshri and $\left(\frac{1}{20}\right)$ of the students won in ved shloka competitions. Find the difference between the number of winners of both the competitions.

11. A train covers 469 km in 22 hours. Find the average speed of the train.

12. Find the average of 42, 38 and 124.

13. A person spends $\frac{1}{3}$ of his salary on food, $\frac{2}{5}$ of his salary on rent and $\frac{1}{5}$ of his salary on clothes. If he is left out with Rs. 400, find his salary.

Rivision of Decimals:

you have already learnt about decimal number.

let us know rivise the consepts on decimal numbers which we already know.

Read the following decimal numbers.

25. 2 is read as twenty five point two

3. 34 is read as three point three four

13. 25 = ………………………………

3. 425 = ………………………………

Complete the following table one has been done for you

Hundreds 100	Tens 10	Ones 1	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Decimal
4	2	5	3	2	1	425.321
6	…..	8	4	…..	8	628.428
…..	1	…..	3	2	…..	514.321
…..	2	…..	…..	2	4	128.324

We can write the expanded form of decimal number as follows.

$$425.321 = 4\times100 + 2\times10 + 5\times1 + 3\times\frac{1}{10} + 2\times\frac{1}{100} + 1\times\frac{1}{1000}$$

In the similar way, write the expanded form of the following decimal numbers

Comparison of decimal numbes

the distance between hostel and mandir is 12. 37 m whereas the destance between hostel and the school is 12.48m. which one is far off from the hostel, mandir or school?

1. On compairing the whole number part which lies on the left of the desimal point, they are same.
2. On compairing the desimal part which lies on they right of the desimal point, on tenths digit $3<4$. Hense $12.37 < 12.48$

   Thus, the distance bitween hostel and school is more.

Do and Learn

Compaire the following desimal numbers:

1. 356.12 and 356.02
2. 786.237 and 786.257

Use of Desimal Numbers :

To convert smaller units of currency, measurements such as length and whight in to larger units we use desimal numbers.

Example:

138 paise = Rs. $\frac{138}{100}$ = Rs. 1.38

$341 \text{ m} = \frac{341}{1000} \text{ km} = \_\_\_\_\_$

$13 \text{ g} = \frac{13}{1000} \text{ kg} = 0.013 \text{ kg}$

Remember
Rs1 = 100 paise
1 km = 1000 m
1 kg = 1000 g
1 m = 100 cm

Try:

Complete the following blanks:

1. 53 g = ________kg   2. 188 paise = Rs ______

Addition and Subtraction of Desimals:

You must be aware that while adding and subtraction desimal numbers, we write the desimal point one below the other. We start frome right and move towards left to add or subtract desimal numbers.

To add or subtract whole number and desimal number, we write as many zeros as the number of digits in the desimal number, add the end of whole number after writing the desimal point.

Example: Add: 21.37 and 13.12

Solution : 21.37

+ 13.12

34.49

Example: Add 3.4 and 12.460 -

Solution: 3.400

+ 12.460

15.860

Example: Subtract : 2.78 from 5.34

Solution :

	Ones		Tenths	hunderdths
	5	.	3	4
-	2	.	7	8
	2	.	5	6

Example: Subtract: 0.34 from 3

	Ones		Tenths	hunderdths
	3	.	0	0
-	0	.	3	4
	2	.	6	6

Remember:- On writing any number of zeros at the end of the desimal part, the value of the desimal number does not change.

Example: Krishna and Prashant bought 5 m and 75 cm length of cloth to stitch kurtas.If krishna requires 2 m and 50 cm length of cloth to sititch his kurta, find the length of the cloth left for Prashanth's kurta.

Solution : The total length of the cloth bought : 5 m and 75 cm

Length of the cloth requred to stitch krishna's kurta: 2 m and 50 cm Length of the cloth left for Prashanth's kurta:

$$\begin{array}{r} 5.75 \\ -\ 2.50 \\ \hline \boxed{3.25} \end{array}$$

Thus, the lengt of the cloth left for Prshanth's kurta is 3 m and 25 cm.

Product of Decimal Numbers:

To find the product of two decimal numbers, we multiply the number without considering the decimal point (just like multiplying the whole numbers). Furter we add the number of digits in the decimal parts of both the decimal numbers, move as many digits as the sum obtained from the right and write the decimal point.

Example: Find the product

$$1.1 \times 1.2$$

Solution : Multiplying the number considering them as whole numbers:

$$11 \times 12 = 132$$

Counting number of digits in the decimal parts of numbers and writing the decimal point in the product.

$$1.1 \times 1.2 = 1.32$$

Example: If a house buys 1.5 L of milk per day, find the number of litres bought in a month.

Solution : The amount of milk bought in a day : 1.5L

According to the quesiton

The amount of milk bought in 15 day:

$$\begin{aligned} & 15 \times 1.5 \\ = & \ 1.5 \times 15 \\ = & \ 22.5 \text{ L} \end{aligned}$$

Hense, the amount of milk bought in 15 days is 22.5 L.

Do and Learn

Find the product of the following

(1) 3.2×2.3 (2) 1.2×1.5

Product of Decimal Numbers with powers of 10 (10,000, 1000):

(1) 1.25×10

finding the product of 125 and 10

$125 \times 10 = 1250$

writing the decimal point

$1.25 \times 10 = 12.50$

(2) 1.25×100

finding the product of 125 and 100

$125 \times 100 = 12500$

writing the decimal point

$1.25 \times 100 = 125.00$

(3) 1.25×1000

finding the product of 125 and 1000

$125 \times 1000 = 125000$

writing the decimal point

$1.25 \times 1000 = 1250.00$

From the above examples, observe that on multiplying a decimal number with 10, 100, 1000 the decimal point move towards the right by 1,2, 3 digits respectively. In other words, the decimal point moves as many digits as the number of zeros in the as the multiplier.

From the above examples, we can conclude that

$$0.08 \times 10 = 08, 0.08 \times 100 = 8 \text{ and } 0.08 \times 1000 = 80.$$

Do and Learn

Find the product of the following

1. 5.56×10 =........ 2. 5.56×100 =........
3. 5.56×1000 =........ 4. 5.56×10000 =........

Find the perimeter of an equilateral triangle of side 2.5 cm. (Hint: all the sides of an equilateral triangle or equal in length.)

Division of Decimals:

Sheetal is designingning an art work for her school. She requires 1.7 cm length of ribbons and she has 8.5 cm length of a ribbon. How many piecies of ribbons will she get if she cuts the ribbon that she has ? She things that on dividing 8.5 by 1.7, she will get the required answer.Is she correct?

From the above, we need to divide a decimal number by another.

Hence, we need to know the division of decimal numbers. Let us now, learn how to divide decimal numbers using examples.

Example: Divide 2.4 by 2.

Solusion : We know that

$$2.4 = \frac{24}{10}$$

$\left(\text{the expended form of } 2.4 \text{ is } 2 \times 1 + 4 \times \frac{1}{10}\right).$

Hence,

$2.4 \div 2$

$= \frac{24}{10} \div 2$

$= \frac{24}{10} \times \frac{1}{2}$

$= \frac{24 \times 1}{10 \times 2} = \frac{24}{20} = 1.2$

In division of fractions, we multiply the dividend by the reciprocal of the diviser.

Divison of Decimals by the Powers of 10:

Example: $42.31 \div 10$

$\frac{42.31}{100} \div \frac{10}{1}$

$= \frac{4231}{100} \times \frac{1}{10}$

$= \frac{4231}{1000} = 4.231$

We know

1. How to convert a decimal into decima ftaction

   $4.2 = \frac{4.2}{10}$

2. Convert the decimal fraction into its decimal form.

Example: $42.31 \div 100$

Solution : $\frac{42.31}{100} \div \frac{100}{1}$

$= \frac{4231}{100} \times \frac{1}{100}$

$= \frac{4231}{10000}$

$= 0.4231$

Example : $42.31 \div 1000$

Example: $\frac{42.31}{100} \div 1000$

$= \frac{4231}{100} \times \frac{1}{1000}$

$= \frac{4231}{100000}$

$= 0.04231$

From the above examples, we observe that on dividing a decimal number by 10,100,1000 the decimal point on the number moves towards left by 1, 2, 3 digits respectively.

Note: when we do not have sufficient number of digits in the decimal number while moving the decimal point towards left, we write as many zeros as required before the decimal number.

Division of a Whole by a Decimal:

To divide a whole number by a decimal, we express the decimal as a fraction. Ferther we multiply the whole number with the reciprocal of the decimal fraction.

Example: $32 \div 0.8$

Solution: $32 \div \frac{8}{10}$

$= 32 \times \frac{10}{8}$

$= \frac{32\times10}{8}$

$= \frac{320}{8} = 40$

Division of Decimal Number by a Decimal Number:

To divide a decimal number by another, we write the decimal numbers into their respetive decimal fractions. Then, we multiply dividend with the reciprocal of the divisor to compute.

Example: $3.15 \div 0.5$

Solution : $3.15 \div 0.5$

$= \frac{3.15}{100} \div \frac{0.5}{10}$

$= \frac{315}{100} \times \frac{10}{5}$

$= \frac{315\times10}{100\times5}$

$= \frac{3150}{500} = 6.3$

Example: Divide 40.5 by 0.15

Solution : 40.5 ÷.15

$$= \frac{405}{10} \div \frac{15}{100}$$

$$= \frac{405}{10} \times \frac{100}{15} = \frac{40500}{150} = 270$$

Note: If the number of digits after the decimal point in both the dividend and divisor is the same, we can remove the decimal points and divide the number as we divide the whol numbers.

Exersice: 2.2

1. Choose the correct answer in the following multipal choice qustions:

A. 1.7 × 100 = ?

(I) 0.0017 (ii) 0.017 (iii) 1700 (iv) 17000

B. 30 ÷ 0.05 = ?

(i) 5 (ii) 50 (iii) 500 (iv) 0.05

C. The number that we shall divide 0.0002 such that the quotient is 0.1 is____.

(i) 0.001 (ii) 0.01 (iii) 0.1 (iv) 1

2. find the sum of the following decimal numbers

1) 3.5 + 4.12 = ☐ 2) 3.123 + 31.21 = ☐ 3) 3.45 + 5.26 = ☐

3. Find the product of the following decimal numbers

1) 3.4 × 2.7 = ☐ 2) 4.57 × 4.31 = ☐ 3) 13.45 – 12.20 = ☐

4. Harshil bought 3 kg 250 g of chillies, 12 kg 750 gm potatoes and 500g of tomatoes. Find the total weight of vegetbles the he bought.

5. How much is 40.7 km less than 47 km?

6. Madhav has two kurtas coasting rs.525.50 and rs. 450.75. Find the deffrence bitween costs.

7. Find the product of the following decimal numbers.

1) 7×3.4	2) 81.2×2
3) 0.08×3	4) 3×0.21
5) 3.73×0.10	6) 0.37×10
7) 1.42×100	8) 3.42×100
9) 14.3×1000	10) 1.42×1000

8. Find the area of the rectangle of length 3.4cm and breadth 1.2 cm.

9. Satish has 3 ropes of length 3.4cm each. Find the total length of the rope that he has.

10. The cost 1kg rice is rs.17.85. Find the cost of 25.450kg.

11. The cost of a watch is rs.575.58.Find the cost of 78 such watches.

12. Find the product of following decimals.

1) 3.4×4.2	2) 6.25×2.5	3) 2.4×0.2
4) 12.3×1.2	5) 0.08×3.4	6) 13.4×0.8

13. Divide the following decimals.

1) $0.8 \div 2$	2) $0.42 \div 6$	3) $4.2 \div 10$
4) $98.6 \div 10$	5) $86 \div 100$	6) $8.05 \div 100$
7) $1.3 \div 1000$	8) $4.32 \div 1000$	

14. Divide the following decimals.

1) $1.2 \div 0.4$ 2) $3.6 \div 0.6$

3) $1.25 \div 0.5$ 4) $0.48 \div 0.4$

15. The cost of 87 books is Rs.1189.29. find the cost of each book.

16. Using 25L of petrol, a car covers 490.5km.Find the distance travaled by it using 1L of petrol.

17. The cost of a tin of 5L of refind oil is Rs.1122.15. Find the cost of 1L of refind oil.

18. If the perimeter of an equilateral triangle is 25.5cm, find the length of its each side.

We learnt:

1. In this chapter, we learnt (addition and subtraction) multipication and division of decimal numbers.

3. To find the product of decimal numbers, we convert the decimals into their decimal fraction and use the rules of product of fractions.

$$\text{Product of two fraction} = \frac{\text{Product of numerators}}{\text{Product of denominators}}$$

4. by interchanging the numerator and denominator of a fraction, we get its reciprocal.

   Like: The reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$.

5. In division of decimals

   1. A whole number by a decimal

   We multiply the whole number with the reciprocal of decimal fraction of the decimal number.

   $3 \div \frac{12}{1} = 3 \times \frac{1}{12} = \frac{3}{12}$

   2. Dividing a decimal by a whole number

   We multiply the decimal with the reciprocal of the whole number

   $12 \div 3 = 12 \times \frac{1}{3} = \frac{12}{3} = 4$

   3. Dividing a decimal number by another

   We mulitply the first decimal number and the reciprocal of the second decimal number.

   $\frac{8}{12} \div \frac{4}{3} = \frac{8}{12} \times \frac{3}{4} = \frac{8 \times 3}{12 \times 4} = \frac{24}{48}$

6. In case of addition or subtraction of two unlike decimals, we write as many zeros as required at the end of the decimal which has less number of digits after the decimal point to make them like

decimals and add or subtract.

(writing zeros at the end of a decmal number does not change its value.)

7. We multiply the decimals without considering the decimal points, and write the decimal point in the product such that a number of digits after the decimal point is equal to the sum of the number of digits after the decimal point in multipicand and multiplier.

$3.2 \times 10 = 32.0$

$3.2 \times 100 = 320.0$

$1.32 \times 1000 = 1320.00$

8. In the Division of Decimals : We convert the decimals into their corresponding decimal fractions using 10, 100, …., and follow the rules of division of fractions to compute.

9. To convert a decimal fraction which has 10, 100, 1000…. We move the decimal point in the desimal number, as many digits as the number of zeros in the denominatior.

$24.6 \div 10 = 2.46$

$24.6 \div 100 = 0.246$

$24.6 \div 1000 = 0.0246$

Chapter – 3

VEDIC MATHEMATICS

Dear Vedic students! We learnt in our last class the topics related to ekadhikena purvena, ekanunena purvena, vinkulam number and ananthyayodarshkeshpi rule of multiplication. In this chapter, we will revise the concepts that we already know and further the rules of addition, subtraction, multiplication and division by which the computation becomes easier.

Revision: Let us now revise the concepts learnt in our previous class.

1. the ekadhikena of 18 is 19.
2. the ekadhikena purvena of 3 in 135 is 235.
3. the ekanunen of 105 is 104.
4. on doing ekanunen purvena on 4 is 1345, we get 1245.
5. we know that the difference between the number and the nearest base 10 is called is its deviation. The deviation of a number can either be positive or negative.

In base 10	**In base 100**
The deviation of 13 is + 3	The deviation of 102 is + 02.
The deviation of 95 is -5.	The deviation of 103 is + 03.

6. Vinkulam Numbers:

- Converting the number into its vinkulam form-

  1 the vinkulam form of 7 is $7 = 10 - 3 = 1\bar{3}$.

  2 the vinkulam form of 28 is $28 = 30 - 2 = 3\bar{2}$.

- Converting a vinkulam number into its general form

1 the general form of the vinkulam number $2\bar{4}3 = 200 - 40 + 3 = 163$.

1 the general form of the vinkulam number

$$3\bar{4}5\bar{3} = 3000 - 400 + 50 - 3 = 2647.$$

2 the general form of the vinkulam number

$$5\overline{34} - 30 - 4 = 466.$$

$$5\bar{3}\bar{4} = 500 - 30 + \bar{4} = 47\bar{4} = 400 + 70 - 4 = 466.$$

7. **Multiply**

By vilokanam sutra:

34×11

$= 0\ [\ 3\ \ 4\ ]\ 0$

$0 + 3 \ / \ 3 + 4 \ / \ 4 + 0$

$3 \ / \ 7 \ / \ 4$

3 7 4

Anatyodharshkyapi sutra:

$$\begin{array}{r} 35 \\ \times\ 35 \\ \hline \boxed{1225} \end{array} \qquad \begin{array}{r} 37 \\ \times\ 33 \\ \hline \boxed{1221} \end{array}$$

Do and Learn

1 **the ekanuna purvena of 4 in 345 is ……..**

2 ekadhikena purvena of 4 in 345 is ………..

3 the deviation of 1005 is ……….

4 45×11 =..........

5 47×43 = ...

- Find the sum using the ekadhikena sutra

$$\begin{array}{cccc} & 3 & 4 & 5 \\ & \dot{1} & 8 & 3 \\ + & 3 & \dot{5} & 7 \\ \hline 0 & 8 & 8 & 5 \end{array}$$

[ekadhikena of 5 is 6]

Now, let us learn two more sutras to find the sum of numbers

1. Purnapurnabhyas rule

We rearrange (group) the numbers such that their sum is either powers or multiples of 10 while adding to or more numbers. It's easy to add the multiples of 10.

(the multiples of 10 are 10, 20, 30, 40, 50,)

Example:

Find the sum:

26 +14 + 58 + 13 + 22 +17

Solution :

26 + 14 + 58 + 13 + 22 + 17

= (26 + 14) + (58 + 22) + (13 + 17)

= 40 + 80 + 30 = 150

Example: Find the sum 45 + 67 + 48 + 19 + 11 + 12 + 13 + 15

Solution :

45 + 67 + 48 + 19 + 11 + 12 + 13 + 15

= (45 + 15) + (67 + 13) + (48 + 12) + (19 + 11)

= 60 + 80 + 60 + 30 = 230

Let us now try to group the numbers such that the sum is multiples of 10 using purnapurnabhyas rule.

Example: Add using the purnapurnabhyas rule

	3	2	4
	4	3	6
	4	3	4
+	1	2	3

We can add the numbers orally as follows:

300 + 400 + 400 + 100 = 1200

20 + 30 + 30 + 20 = 100

4 + 6 + 4 + 3 = 17

= 1200 + 100 + 17 = 1317

4. **fine the sum of the following:**

	3	4	2	1
	2	4	5	1
	3	1	1	5
+	3	1	2	1

= 11000 + 1000 + 100 + 8

= 12000 + 100 + 8 = 12108

2. Addition of numbers using subtraction:

This rule has two words namely addition and subtraction. This rule is applied when the addends are not multiples of 10. We write the numbers as the sum or difference of multiples of 10

Example: Find the sum: 26 +17 + 8 + 37 + 11

Solution : We add or subtract required numbers from multiples of 10 to express the given numbers.

= (20 + 6) + (20 – 3) + (10 – 2) + (40 – 3) + (10 + 1)

= (20 + 20 + 10 + 40 + 10) + (6 – 3 – 2 – 3 +1)

= 100 + (–1)

= 100 – 1

= 99

Example: Find the sum : 103+299+ 303 + 597 +13

Solution :

$(100 + 3) + (300 – 1) + (300 + 3) + (600 – 3) + (10 + 3)$

$= (100 + 300 + 300 + 600 + 10) + [3 – 1 + 3 – 3 + 3]$

$= 1310 + 5$ (solving)

$= 1315$

Subtraction (Difference)-

In vedic mathematics, the meaning of **Nikhilam navathah charmaum dashatah.** Every nine last ten, other digits except one's digit is called Nikhil number.

Let us learn how to use the above sutra to evaluate: If we subtract 689 from 7000, we carry forward digits many times while solving in ordinary way. It takes more time and chances of committing errors are more.

In this sutra we calculate starting from right to left. We replace every zero on the left by 9 and the last zero of the number on the right by 10 then, we subtract 1 from the left most digit of the number.

Let us understand using an example.

Example: Subtract 689 from 1000

Solution :

Solution:
$$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ - & 6 & 8 & 9 \\ \hline & & & \end{array} \quad \text{On riwriting the number} \quad \begin{array}{cccc} 0 & 9 & 9 & 10 \\ - & 6 & 8 & 9 \\ \hline 0 & 3 & 1 & 1 \end{array}$$

Exampl: find the diffrence between 7000 and 689

Solution:
$$\begin{array}{cccccc} 7 & 0 & 0 & 0 & 0 & 0 \\ - & 1 & 8 & 3 & 5 & 7 \\ \hline & & & & & \end{array} \quad \square \quad \begin{array}{cccccc} 6 & 9 & 9 & 9 & 9 & 10 \\ - & 1 & 8 & 3 & 5 & 7 \\ \hline 6 & 8 & 1 & 7 & 4 & 3 \end{array}$$

Example: subtract 387 from 856.

Solution: 856 – 387

Step **1:** In this, 6 < 7

Hence, we subtract 6 from 7 and write its supplement (parammitra). 7 – 6 = 1, 9 is the supplyment of one. Which will we written in one's digit.

$$\begin{array}{rrrr} & & & (10) \\ & 8 & 5 & 6 \\ - & 3 & 8 & 7 \\ \hline & & & 9 \end{array}$$

Step – 2:

Ferther 5 smaller than 8 5 < 8.

Find the supplyment of 8 – 5 = 3. Now find 9 – 3 = 6, which is written in tens digit.

$$\begin{array}{rrrr} & & (9) & (10) \\ & 8 & 5 & 6 \\ - & 3 & 8 & 7 \\ \hline & & 6 & 9 \end{array}$$

Step – 3 : subtracting one from the first digit (8) of the number, we get 8 – 1 = 7. Ferther subtracting 3 from 7 (7 – 3 = 4) which is written hundreds place.

$$\begin{array}{rrrr} & 8 & 5 & 6 \\ & 3 & 8 & 7 \\ \hline - & 4 & 6 & 9 \\ & & 6 & 9 \end{array}$$

Multiplication by Vilokanam sutra :

1. Multiplication by 5:

Rule : Write zero at the end of the number and divide the new number by 2.

Example: Solve 42×5

Solution : On writing zero at the end of 42, we get 420.

On dividing it by 2, $\frac{420}{2} = 210$

Or half of 420 is 210

Thus, $42 \times 5 = 210$

Example: Multiply 84 by 5.

Solution : Write zero at the end of 84 to get 840.

On dividing 840 by 2, $\frac{840}{2} = 420$

Thus, $84 \times 5 = 420$

Do and Learn

(1) 38×5 (2) 48×5

Discuss: To multiply a number by 25, we can multiply the number $\frac{100}{4}$.

2. Product of a number and 10-

$6 \times 10 = 60$

$10 \times 10 = 100$

$78 \times 10 = 780$

3. Praduct of a number and 100

$6 \times 100 = 600$

$10 \times 100 = 1000$

$78 \times 100 = 7800$

Observe that there is no change in the multipicands 6, 10 and 78.

- On multiplying the numbers by 10 zero is written at the ones digit and the multiplicand is written before it.
- On multiplying the numbers by 100 zeros are written at the ones and

tens digits, and the multiplicand is written before them.

Product of Numbers Using Eknunena Purvena Rule:

The meaning of Eknunena Purvena Rule is, "One less than the number". This is an important rule in multiplication. This rule is applied only when all the digits of one of the numbers in the product is 9.

If the digits in the multiplier is 9 the following situation arises.

- If the number of digits in both multiplier and multiplicand is the same and the digits in multiplier are 9

You must be aware of the terms multiplicand and multiplier

$$\begin{array}{r l} 12 & \rightarrow \text{Multiplicand} \\ \times\, 4 & \rightarrow \text{Multiplier} \\ \hline 48 & \rightarrow \text{Product} \\ \hline \end{array}$$

- When the number of digits in multiplier is greater than the number of digits in multiplier, all the didits in multiplicand are 9
- When the number of digits in multiplier is less than the number of digits in multiplier, all the didits in multiplicand are 9

Now, let us learn how to use the rule in all the three situations using examples.

Case1: If the number of digits in both multiplier and multiplicand is the same and the digits in multiplier are 9.

Rule: 1. Subtract 1 from the multipicand and write on the left. (left side)

2. Subtract the number obtained in step 1 frome the multiplier.(Right side)

Example: Multiply: 645×999

Solution : Left side = Multiplicand – 1 = 645 – 1 = 644

Right side = Multiplier – left side

= 999 – 644 = 355

Thus, $645 \times 999 =$

Left side 644 / Right side 355

= 644355

Example 2 : Find the product : 4563×9999

Solution : Left side 4563 – 1 = 4562

Right side 9999 – 4562 = 5437

Hense, 4563×9999 = 45625437

Case 2: When the number of digits in multiplier is greater than the number of digits in multiplier, all the didits in multiplicand are 9.

Rule: We will solve the product in situation 2 using the rules situation 1.

Example: Find the product: 458×9999

Solution : Left side = 548 -1 = 457

Right side = 9999 – 457 = 9542

Hense, 458×9999 = 4579542

Example: 122 × 99999

Solution : Left side = 122 – 1 = 121

Right side = 99999 – 121 = 99878

Hense, 122×99999 = 12199787

Case 3: hen the number of digits in multiplier is less than the number of digits in multiplier, all the digits in multiplicand are 9

Rule: 1. Write as many zeros as there are number of 9 in the multiplier, at the end of the multiplicand.

2. Sulbtract the given multiplicand from the number obtained in Step 1.

Example: Find the product: 1584 × 99

Solution : There are 4 digits in multipicand and whereas there are two 9's in the muliplier.

1. Hense, writing two zeros at the end of 1584, we get 158400.
2. Subtracting the given multiplicand 1584 from 158400, we get

$$
\begin{array}{cccccccc}
 & & & & (9) & (9) & (10) \\
 & 1 & 5 & 8 & 4 & 0 & 0 \\
- & & & 1 & 5 & 8 & 4 \\
 & 1 & 5 & 6 & 9 & 1 & 6 \\
\hline
 & 1 & 5 & 6 & 8 & 1 & 6
\end{array}
$$

Thus, 1584 × 99 = 156816

158400 – 1584 = 156816

Thus, 1584 × 99 = 156816

Example: Find the product: 45678 × 999

Solution :

1. there are three nine's in multiplier, writing three zeros at the end of multiplicand

45678000

2. subtracting 45678 from 45678000

$$
\begin{array}{ccccccccc}
 & 4 & 5 & 6 & 7 & 8 & 0 & 0 & 0 \\
- & & & & 4 & 5 & 6 & 7 & 8 \\
\hline
 & 4 & 5 & 6 & 3 & 2 & 3 & 2 & 2
\end{array}
$$

45678000 - 45678 = 45632322

Do and learn:

(1) 234×999 (2) 234×9999 (3) 234×99

Multiplication by 11

Rule:

1 write zero on both sides of the number which has to be multiplied by 11

2 add two successive digits starting from right to left,if there is carry over, add it to the previous digit.

Example: Find the product:

134×11

Solution : according to the above rule

0 [1 3 4] 0

0 + 1/ 1 + 3 / 3 + 4 / 4 + 0

1 / 4 / 7 / 4

Hence, , $134 \times 11 = 1474$

Do and learn:

(1) 234×11 (2) 131×11

EXERCISE 3.1

1. using purnapurnabhyas and addition using subtraction rule solve the following

   (1) 82 + 18 + 96 + 24 + 17 + 13

   (2) 324 + 126 + 249 + 311

   (3) 145 + 25 + 248 + 122

2. find the subtract using Nikhilam Navatha Charam Dashata rule -

   (1) 7 0 0 0 0 − 1 3 4 6 (2) 5 4 3 4 − 2 1 7 6

   (3) 9 0 0 0 − 1 3 4 5 (4) 4 9 3 1 − 1 2 7 4

3. find the product using Vloknam Sutra:-

   (1) 7 8 × 1 0 (2) 3 5 × 1 0 0 (3) 4 3 2 × 1 0 0

4. using eknunena purvena rule find the product -

   (1) 81 × 99 (2) 78 × 99 (3) 23 × 999

   (4) 34 × 9999 (5) 134 × 9 (6) 345 × 99

5. find the product –

(1) 345 × 11 (2) 341 × 11 (3) 345 × 11

Fractions:

Dear students, Yor have the previous knowledge about fractions. We can simplify the consept of fractions using Vedic mathametics.

Observe the following fraction

When denominators are same

Example: Write the like fractions $\frac{5}{7}, \frac{3}{7}, \frac{9}{7}, \frac{1}{7}$ in their ascending order

Solution : The numerators of the given fraction are different but the denominators are the same we can write these fractions in their ascending order.

$$\frac{1}{7} < \frac{3}{7} < \frac{5}{7} < \frac{9}{7}$$

In like fractions (frations with same denominator), the fraction with greater numerator is greater.

When numerators are same:

Example: Write the fractions $\frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}$ in their ascending order.

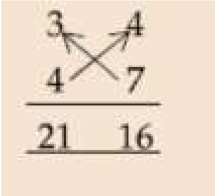
Solution : In the given fractions the numerator are same but their dinominators are different. 5 is the gratest dinominator, hence $\frac{1}{5}$ is the smallest fraction and the largest fraction is $\frac{1}{2}$. On writing in ascending order

$$\frac{1}{5} < \frac{1}{4} < \frac{1}{3} < \frac{1}{2}$$

Meaning, if the numerators in unlike fractions are same, the fraction with grater denominator smaller

Example: Find the larger fraction among $\frac{3}{4}$ and $\frac{4}{7}$

Solution : 1 write the digits without considering the fraction sign



2. cross multipling

$3 \times 7 = 21$ and $4 \times 4 = 16$

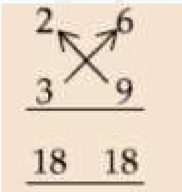
3. the side on which the product is more, the fraction on that side is greater.

4. since, 21 > 16, $\frac{3}{4} > \frac{4}{7}$

Example:

compare the fractions: $\frac{2}{3}$ and $\frac{6}{9}$.

Solution :



1 cross multipling $9 \times 2 = 18$ and $6 \times 3 = 18$

2 as a product are equal, the fractions are equal.

3 hence the fractions are equivalent fractions.

4 18 = 18, thus, $\frac{2}{3} = \frac{6}{9}$

Addition of Fraction:

- **If the denominarators are same**

Example: Find the sum of $\frac{1}{7}$ and $\frac{2}{7}$.

Solution : $\frac{1}{7}+\frac{2}{7}=\frac{1+2}{7}=\frac{3}{7}=\left(\frac{\text{sum of numerators}}{\text{denominator}}\right)$

Thus, sum of like fractions = $\left(\frac{\text{sum of numerators}}{\text{denominator}}\right)$

- **If the denominator are not same**

Example: Find the sum of $\frac{2}{3}$ and $\frac{4}{7}$

Solution : cross multiplied 2×7 and 3×4

product of denominators $3\times 7=15$

Solution: $\frac{2}{3}+\frac{4}{7}$

$$=\frac{2\times7+3\times4}{3\times7}$$

$$=\frac{14+12}{21}=\frac{26}{21}=1\frac{5}{21}$$

Example: $\frac{1}{2}+\frac{2}{3}+\frac{4}{5}$ Find the sum

Solution: $\frac{1}{2}+\frac{2}{3}+\frac{4}{5}$

$$=\frac{1\times3\times5+2\times2\times5+4\times2\times3}{2\times3\times5}$$

The cross multiplication in the sum

$1\times3\times5, 2\times2\times5$ and $4\times2\times3$

and

product of denominators: $2\times3\times5=30$

$$=\frac{15+20+24}{30}=\frac{59}{30}=1\frac{29}{30}$$

- If the denominators of fractions are not same and

Example: Add: $\frac{1}{4}+\frac{1}{10}$

Solution :

$$\frac{1\times10+1\times4}{4\times10}$$

$$=\frac{10+4}{40}=\frac{14}{40}$$

$$= \frac{14 \div 2}{40 \div 2}$$

$=$ (dividing both numerator and denominator by 2)

$$= \frac{7}{20}$$

Addition of mixed fractions using velokanam rule and cross multiplication method

We can easily find the sum of mixed fractions using velokanam rule and cross multiplication method.

$= 1\frac{3}{4} + 2\frac{1}{3}$ by splitting the mixed fraction using velokanam rule

$1\frac{3}{4} = 1 + \frac{3}{4}$ and $2\frac{1}{3} = 2 + \frac{1}{3}$

$$2 + 1 = 3$$

$\frac{3}{4} + \frac{1}{3}$ (by cross multiplication method)

$$= \frac{3 \times 3 + 1 \times 4}{4 \times 3}$$

$$= \frac{9 + 4}{12}$$

$$= \frac{13}{12} = \frac{12 + 1}{12}$$

$$= \frac{12}{12} + \frac{1}{12} = 1 + \frac{1}{12}$$

Hence, $1\frac{3}{4} + 2\frac{1}{3} = 5 + 1 + \frac{1}{12} = 6\frac{1}{12}$.

Subtraction of fractions

Subtraction of fractions is moreor less similar to the addition of fractions.

In addition we use (+) sign and in subtraction (–).

When the denominators of the fractions are same

Example: Find the drfference: $\frac{3}{7} - \frac{1}{7}$

Solution : $\frac{3}{7} - \frac{1}{7} = \frac{3-1}{7} = \frac{2}{7}$

Subtraction of unlike fractions (remove the phrase is incorrect)

Example: Subtract: $\frac{4}{5} - \frac{2}{3}$

Solution : $\frac{4\times3-5\times2}{5\times3} = \frac{12-10}{15} = \frac{2}{15}$

Example: Simplify: $\frac{1}{2} + \frac{1}{3} - \frac{1}{5}$

Solution : Solving by the method of addition of fractions

$$\frac{1\times3\times5+1\times2\times5-1\times2\times3}{2\times3\times 5}$$

$$= \frac{15+10-6}{30} = \frac{25-6}{30}$$

$$= \frac{19}{30}$$

Subtration of mixed fractions:

Using the method of addition of fractions by velokanam rule and cross multiplication method, we can subtract mixed fractions.

Example: Solve: $4\frac{3}{4} - 3\frac{2}{5}$

Solution : $\left(4 + \frac{3}{4}\right) - \left(3 + \frac{2}{5}\right)$

$$= (4-3) + \left(\frac{3}{4} - \frac{2}{5}\right)$$

$$= 1 + \frac{3\times5-4\times2}{4\times5} = 1\frac{15-8}{20} = 1\frac{7}{20}$$

Multiplication of Fraction:

We can multiply two fractions very easily. To find the numerator and the denominator of the product, we multiply both numerators and denominators respactively.

Example: Find the product of $\frac{1}{2}$ and $\frac{3}{7}$.

Solution: $\frac{1}{2} \times \frac{3}{7} \quad = \quad \frac{1 \times 3}{2 \times 7} \quad = \quad \frac{3}{14}$

Multiplication of two Mixed Fractions (Ekadhikena Purvena Sutra):

If the sum of fractional parts of the two given mixed fractions is 1, the whole parts of the fractions are same, we can write the product of two mixed fractions will be

The whole part is the product of a whole part of the mixed fractions and its one more (Ekadhiken).

The fractional part of the product is, the product of fractional parts of the mixed fractions.

Example: Solve $7\frac{1}{4} \times 7\frac{3}{4}$

Solution :

1. The fractional parts of the mixed fractions are $\frac{1}{4}$ and $\frac{3}{4}$. The sum of fractional parts is $\frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$
2. The remaining whole part of the fractions is the same (7).
3. Whole part of the product is whole part $\times$ one more than the whole part $= 7 \times (7+1)$ $= 56$
4. The fractional part of the product is $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$.

Hence, = $\quad 7 \times (7 + 1) \ / \ \frac{1}{4} \times \frac{3}{4}$

$$= \quad 7 \times (7 + 1) + \frac{1}{4} \times \frac{3}{4}$$

$$= \quad 7 \times 8 + \frac{3}{16}$$

$$= \quad 56 + \frac{3}{16} \quad = \quad 56\ \frac{3}{16}$$

Example: Multiply: $15\frac{4}{7} \times 15\frac{3}{7}$

Solution : $15\frac{4}{7} \times 15\frac{3}{7}$

$$= \quad 15 \times (15+1)/\frac{4}{7} \times \frac{3}{7}$$

$$= \quad 15 \times 16/\frac{12}{49}$$

$$= \quad 240\frac{12}{49}$$

Product of two fractions (by Viloknam Sutra)

Example: Solve by Vedik Method: $5\frac{1}{2} \times 6$

Solution :

$\left(5+\frac{1}{2}\right) \times 6$ (Vilokanam Sutra)

$= \quad 5 \times 6 + \frac{1}{2} \times 6$ (The result of the expression in brakets)

$= \quad 30 + 3$ (Half of six is 3)

$= \quad 33$

Veryfication:

$$= \quad 5\frac{1}{2} \times 6$$

$$= \quad \frac{11}{2} \times 6 \qquad \left(5\frac{1}{2} = \frac{11}{2}\right)$$

$$= \quad 11 \times \frac{6}{2}$$

$= \quad 11 \times 3$ (Half of six is 3)

$$= \quad 33$$

Example: Find the product of $7\frac{1}{2} \times 8\frac{1}{2}$

Solution :

$\left(7+\frac{1}{2}\right) \times \left(8+\frac{1}{2}\right)$ (*Vilokanam Sutra*)

$$= \quad 7 \times 8 + 7 \times \frac{1}{2} + 8 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \quad 56 + 3\frac{1}{2} + 4 + \frac{1}{4}$$

$$= \quad 56 + 3 + 4 + \frac{1}{2} + \frac{1}{4}$$

$$= \quad 63\ \frac{6}{8} \quad \text{or} \quad 63\ \frac{3}{4} \qquad\qquad \left(\frac{6 \div 2}{8 \div 2} = \frac{3}{4}\right)$$

By other rule:

$$7 \times 8 + \frac{1}{2} \times \frac{1}{2} + (7 + 8) \times \frac{1}{2} \ \text{(Pease check the sign)}$$

$$= 56\frac{1}{4} + 15 \times \frac{1}{2}$$

$$= 56 + 7 + \left(\frac{1}{4} + \frac{1}{2}\right)$$

$$= 63\frac{3}{4}$$

Division - 'Nikilam Navatashcharam Dashatah' formula (when the divisor is less than the base)

Method -

1. Take the divisor's nearest base (to the power of 10) and write its complement in the divisor column beneath the original divisor. **Complement = base - divisor**. The complement is easily obtained using the "Nikilam Navatashcharamam Dashatha" method, which is explained in the differences chapter. Below it, write the divisor's complement.
2. Draw a line to divide the rightmost digit or group of digits of a dividend into pieces equal to the number of digits of the divisor. This group is known as the remainder group, while the group on the left is known as the quotient group.
3. The number of digits kept in the remainder column will be equal to the number of zeros in the base.
4. Move the divisor's first digit to the second column. This will give you the quotient's first digit. Divide the quotient by the complement and enter the result in the dividend column next to the first digit of

the dividend5. To get the second digit of the quotient, simply write the sum of the digits of the second column.

6. Continue until you have numbers in the remainder column. If the remainder is greater than the divisor, repeat the process in the remainder group until the digit in the remainder column is less than the divisor.

Example : Divide 22 by 8

Plough :

$$
\begin{array}{lc|c|c}
\text{divisor :} & 8 & 2 & 2 \\
\text{Complement :} & 2 & \downarrow & 4 \\
\hline
 & & 2 & 6
\end{array}
$$

Here, the divisor is closer to base = 10. Complement = 10 – 8 = 2

Hint :

- Since there is a zero in the base, one digit will be separated from the line in the remainder column. In the above example, the rightmost digit 2 is separated by a line.
- Move down the first digit (2) of the dividend. This is the first digit of the quotient.
- Multiply the quotient's first digit by its complement and enter the result in the dividend column, to the right of the dividend's first digit.
- Write the sum of the remainder column's two digits. Our division is complete because the remainder is less than the divisor 6.
- quotient = 2, remainder = 6

Example: Divide 10025 by 88 -

Solution: Base = 100

Complement = 100 – 8 8 =1 2

		column **1**	column **2**	column **3** (***R***)
divisor	:	8 8	1 0 0	2 5
Complement	:	1 2	1 2	– –
			1	2 –
				3 6
			1 1 3	8 1

quotient = 113 and remainder = 81

Hint:

1. Here divisor = 88

   Nearest base (multiple of 10) = 100

   Complement = Base – Divisor

   = 100 – 88 =12

2. Arrange the digits in columns, separating the quotient and remainder as shown above. Because the base contains two zeros, the remainder column will contain the two digits to the right of the dividend.

3. In column 2, move 1 down (the dividend's first digit). This will be the quotient's first digit.

4. Multiply the first digit of the quotient by its complement and place it in the dividend column, to the right of the first digit of the dividend:

   12 × 1 = 12 is placed under 0.

Column 1	Column 2 (Q)	Column 3 (R)
Divisor: 8 8	1 0 0	2 5
Complement: 1 2	1 2	- -
	1	2 -
		3 6
	1 1	

5. Move the sum of the digits of the circle down. This will give you the second digit of the quotient, 0 + 1 = 1

6. Multiply the second digit of the quotient by its complement and place it in the dividend column to the right of the second digit of the dividend. 12 × 1 = 12 is placed under the second zero.

Column 1	Column 2 (Q)	Column 3 (R)
Divisor: 8 8	1 0 0	2 5
Complement: 1 2	1 2	- -
	1	2 -
	1 1 3	

7. Move the second digit of the circle down to get the quotient's third digit. quotient third digit 0 + 2 + 1 = 3

8. Multiply the complement of 12 by the third quotient (3) and write the result on the right side of the remainder's fourth digit12 × 3 = 36 is placed below 2 in **column 3 .**

Column 1	Column2 (Q)	Column 3 (R)
Divisor: 8 8	1 0 0	2 5
Complement:	1 2	- -
1 2	1	2 -
		3 6
	1 1 3	8 1

9. To calculate the remainder, add the digits in column 3. Repeat the preceding steps until the numbers in the remainder column are less than the original divisor.

   Here sum of digits of **column 3 = 81 and 81 < 88 (Divisor),**

   Thus, quotient = 113 and remainder = 81

Exercise: 3.2

1. Fill in the blanks using the signs (>,= and <) to compare the following fractions.

   (i) $\square$ $\frac{3}{5}$ $\quad \frac{4}{9}$ (ii) $\square$ $\frac{3}{5}$ $\quad \frac{6}{5}$ (ii) $\square$ $\frac{4}{7}$ $\quad \frac{5}{3}$

2. Write the following fractions in their asending order.

   (i) $\frac{3}{8}, \frac{5}{8}, \frac{7}{8}$ (ii) $\frac{3}{5}, \frac{2}{5}, \frac{4}{5}$

3. Write the following fractions in their desending order.

   (i) $\frac{4}{5}, \frac{1}{5}, \frac{2}{5}$ (ii) $\frac{4}{6}, \frac{4}{7}, \frac{4}{8}$

4. Find the sum (using Vilokanam rule and cross multiplication mathod)

   (i) $\frac{1}{5} + \frac{2}{5}$ (ii) $\frac{7}{15} + \frac{3}{15}$

   (iii) $\frac{4}{3} + \frac{2}{5}$ (iv) $\frac{2}{3} + \frac{1}{4}$

5. Find the difference (using Vilokanam rule and cross multiplication mathod)

   (i) $\frac{9}{10} - \frac{3}{10}$ (ii) $\frac{7}{5} - \frac{4}{5}$ (iii) $3\frac{1}{2} - 1\frac{3}{4}$

   (iv) $2\frac{3}{6} - 2\frac{1}{6}$ (v) $\frac{5}{9} - \frac{4}{9}$ (vi) $4\frac{1}{2} - 5\frac{3}{4}$

6. Find the product using Ekadhikena Purvena mathod

   (i) $\frac{1}{8} \times \frac{3}{5}$ (ii) $5\frac{1}{2} \times 5\frac{1}{2}$ (iii) $3\frac{2}{4} \times 3\frac{1}{4}$

   (iv) $8\frac{2}{5} \times 8\frac{1}{5}$ (v) $4\frac{1}{4} \times 4\frac{1}{4}$ (vi) $2\frac{1}{5} \times 3$

7. Divide:

   (i) $11011 \div 89$ (ii) $10025 \div 88$

   (iii) $22 \div 8$ (iv) $110 \div 8$

We learnt:

1. Using Purnapurnabhayam method, to add or subract two numbers by writing them close to the nearest to 10.
2. Using sanglan sutra, to add or subtract by writing the numbers as diviation with respect to multiples of 10.
3. Subtration using nikhilam navta charmam dashtaha rule.
4. Multiplication: Vilokanam sutra (10, 100, 1000), by 5 and oral mathod.
5. Product of a number and a number consisting of 9's by Eknunena Purvena mathod.
6. Addition and subtration of fractions (Vilokanam sutra and cross multiplication mathod)
7. Product of mixed fractions using by Eknunena Purvena mathod.
8. Division by Nikhilam Navatha Charmam Dashatha method.

Chapter – 4

DATA HANDLING

Dear students, In our daily lives, you must have heard about different types of data in television, radio and newspapers etc., in a cricket match we observe a graphs depicting runs scored by different players. These data give us some information about the event. In this chapter we will learn about organisation of data, methods of evaluation of mean, median and mode.

Let us understand about data using an example-

The following list is the number of marks scored by 15 students of a school, in a monthly test in which the maximum marks is 20

12, 10, 11, 12, 8, 9, 12, 8, 8, 7, 9, 7, 8, 10, 11

The above list gives us the information about the marks scored by the students in a monthly test. **Data is a collection of numbers which provides information about a particular event or experiment.**

❖ Do and learn-

Prepare a list containing the ages of all the students in your class

----

Are all the entries numbers?

Organisation of Data:

In a annual function of school, it was dicided to distridute sweets to the students of their favourite choice. The students were asked about the choice of their favourite sweet. The following list gives the information about the favourite choice of sweet of all the students, in which L stands for Laddu, V stands for Kaju katli and R stand for Rasagolla -

R, R, L, V, R, R, V, L, R, L, R, V, L, R, V, V, V, V, V

Students the organised the above data as follows

Radhkrishna organised the data as

L – 4, V – 8, R – 7. This information can be tabulated as

Name of the favourite sweet	Number of students
Rasagolla R	✓ ✓ ✓ ✓ ✓ ✓ ✓
Kajukatli V	✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓
Laddu L	✓ ✓ ✓ ✓

Vikas prepared the following table.

Name of the sweet	Tally Mark	Numbers of students
Rasagolla R	卌 | |	7
Kajukatli V	卌 | | |	8
Laddu L	| | | |	4

In the above, the collection of data is represented in three different ways, out of which the representation of data by Vikas is correct. In this representation, Vikas has used tally marks in groups of 5 which is easier to count. A group of 5 tally marks, the fifth line is drawn such that it crosses all the four vartical lines. Like 卌 each vartical line is called tally mark in the tally mark represention 卌 ||| observe that there is a group of 5 and three separate tally mark, which stands for the number of intries of the particular data is 5+3=8. The representation 𝍸 𝍸 stands for 5 + 5 = 10

Example: Aman tossed a dice for 20 times and listed the outcomes as shown below.

1, 3, 4, 5, 6, 6, 4, 3, 5, 2, 6, 6, 5, 5, 1, 2, 3, 3, 6, 4

Aman wanted to obtain the following information from the collection of data.

1. the number which the came out for maximum number of times.
2. the number that came out for minimum number of times.
3. the number of out comes which are even number.

Aman tabulated the out comes using tally marks which is show bellow

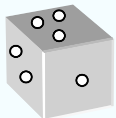
Out come	Tally marks	Frequency
1	| |	2
2	| |	2
3	| | | |	4
4	| | |	3
5	| | | |	4
6	𝍸	5

From the above table can you answer the questions asked?

Yes, the tally marks in the above table makes easier to answer the questions.

Dice -

In ancient India the game of dice was a very famous game. A dice in this game usually made of either wood or plastic, having grooves on their ractanglur faces. You must have used the dice while playing snake and ladder.

Dice - 

MEASURES OF CENTRAL TENDENCIES:

You must be aware of the " average". We hear the word average in the folloing

Situations very often.

Example:

1. in a cricket match, the average runs scored in an over is 4.
2. on an average, Rahul spendes 4 hour in a day studying Veda.

From the above statement, can we conclude that Rahul spandes 4 hour every day on studying Veda? There is no specific answer for the question. What do the statements convey? The word " average suggests that Rahul spands approximately four hours every day at time mores or at times less. Average is the central point of observations and it is one of the measures of central tendency. It is the value between the maximum and minimum values of the observation. Hence, we say that average is the measur of central tendencey

It is necessary to understand the behaviour of observations can be done using the central or average of the observations. The measures of central tendence is of three types namely arithmetic mean, median and mode.

ARITHMETIC MEAN:

The representative value that is very widely use in common practic is the mean value of the obsurevations. We can define arithmetic mean as

$$\text{Mean} = \frac{\text{sum of all the observations}}{\text{Total number of observations}}$$

Example: Madhav studies for 2, 5, 6 and 3 hours in four consecutive days. Find the average (mean) number of hours that he studies in a day.

Solution: $\text{Mean} = \frac{\text{sum of the hours studied by Madhav}}{\text{number of days he studied}}$

$$\text{Mean} = \frac{2+5+6+3}{4}$$

$$\text{Mean} = \frac{16}{4} = 4 \text{ hours}$$

Thus, we can say that he studies for 4 hours every day.

Example: Find the mean of 5 and 13.

Solution : We know that

$$\text{Mean} = \frac{\text{sum of all the observations}}{\text{Total number of observations}}$$

Hence,

$$\text{Mean} = \frac{5+13}{2} = \frac{18}{2} = 9$$

❖ **Discuss:**

1. Is mean the greatest value in thr obsurevations?
2. Is it smaller than all the values?

From the above discussion, you can observe that the mean value always lies between maximum and minimum observations. The mean of two values always lie between them.

Let us observe the following example and understand.

Example: Find the arithmetic mean of 1 and 3.

Solution :

$$\text{Mean} = \frac{\text{sum of all the observations}}{\text{Total number of observations}}$$

Hence,

$$\text{Mean} = \frac{1+3}{2} = \frac{4}{2} = 2$$

Hence, the mean of 1 and 3 is 2.

Do and Learn:

1. find the mean of first three odd numbers.

RANGE:

The difference between the largest and smallest observations gives us an idea of the spread of the observations. It can be found by subtracting the smallest observation from the largest observation. We call this result, the range of data or observations.

Range = largest observation – smallest observation

Example: A cricket player scored the following runs in 6 innings -

60, 43, 57, 15, 25, 40

1.. What are the highest and lowest runs scored by the player in 6 innings?

2. What is the range of runs scored by the player in the innings by him?

3. Find the average number of runs scored by the player per innings.

Solution :

1. Arranging the runs in ascending order,we get

   15, 25, 40, 43, 57, 60

   We can easily say by looking at the above numbers that

   highest runs scored : 60

   lowest runs scored : 15

2. **Range of runs = (highest runs) – (lowest runs)**

   =60 – 15 = 45

3. Average of runs scored by a player in each innings -

$$\text{Average} = \frac{\text{Sum of runs scored in total innings}}{\text{Total number of innings played}}$$

$$= \frac{15 + 25 + 40 + 43 + 57 + 60}{6}$$

$$= \frac{240}{6}$$

= 40

Let's try –

Find out the average of time (in hours) you have devoted for study of Vedas in a week.

Exercise 4.1

1. Twenty students participated in an essay competition on the topic Swachha Bharat Swasth Bharat, obtained the following marks out of 10 -

   5, 6, 7, 9, 10, 6, 8, 5, 6, 6, 7, 8, 8, 5, 6, 8, 9, 9, 5, 6

   Arrange these obtained marks using tally marks and give answer.

   i) Number of students who got 7 or 6 marks.

   ii) Number of students who got 5 marks.

   iii) Number of students who got marks 8 or more than 8

2. Find the mean (average) of the numbers 1, 3 and 5.

3. Find the arithmetic mean of the first five even natural numbers 2, 4, 6, 8, 10.

4. The net income of a fruit seller for a week is Rs.300, Rs.200, Rs.350, Rs.450, Rs.500, Rs.400 and Rs.600 respectively. Find the average (mean) income of the fruit seller.

5. Find the average and range of the first five odd natural numbers.

6. the following list is the heights (in CM) of 10 students of a ved patashala.

   125, 135, 151, 146, 144, 139, 148, 152, 140, 150

   i. Find the maximum height of the student.

   ii. Find the minimum height of the student.

   iii. Find the range of data.

   iv. Find the mean of heights.

   v. Find the number of students whose height is more than the mean height.

7. The number of students who were admitted in five consecutive years, in rashtriya adarsh ved vedyalayas is:

   1550, 1650, 1700, 2013,1528.

   Find the mean number of students admitted in vedyalaya

8. Find the range and mean of first five odd natural numbers.

9. Find the mean and range of the numbers 1, 6, 3, 5, 4, 6, 7, 8, 9 and 1.

10. The cost of 6 books in a library are Rs. 46, 40, 50, 58, 62 and 90. Find the range of the cost of the books.

11. The number of trees planted in the premises of the ashram is as follows, on the basis of this information, answer the following questions -

Mango	✮ ✮ ✮ ✮
Neem	✮ ✮ ✮ ✮ ✮ ✮ ✮ ✮ ✮
Bananas	✮ ✮ ✮ ✮ ✮ ✮ ✮
Peeapal	✮ ✮ ✮ ✮ ✮ ✮ ✮ ✮ ✮ ✮

   1) The number of mango trees?

   2) The difference between number of peepal and banana trees

   3) Is the number of neem trees more than 10?

12. The age (in years) of students of ved patashala is as follows:

   11, 13, 14, 15, 12, 15, 18, 14, 13, 19, 20, 14, 13, 14, 15, 15,

   14, 13, 14, 15, 15, 11, 12, 14, 12, 12, 15, 12, 14, 13, 12, 20,

   Prepare a table using tally marks and answer the following questions.

1 Find the number of students whose age is more than 14 years

3 Find the number of students and their age who are more in number

3 Find the number of students whose age is less than 13.

13. Find the value of x, if the mean of 5, 8, 3, 3, x and 4 is 8.

14. A student bought 5 books from a vedic sammelan on vedic vadya and moral studies whose costs are: Rs. 25, Rs.55, Rs.40,Rs. 60 and Rs.45. Find the mean and range of the cost of books.

❖ **Important point**

1. The mean of two numbers always lies between them.
2. The mean lies between the largest and the smallest observations (data).

MODE

Another type of representative value is the mode. The term that occurs most frequently in a set of data is called the mode. The term which has the highest frequency (number). That term is called the mode of the data.

Let us understand by **example** -

Example: Find the mode of the following data -

4, 5, 5, 2, 4, 7, 4, 4, 4, 3,

Solution : Arranging the data in ascending order -

2, 3, 4, 4, 4, 4, 4, 5, 5, 7

It is clear from the above observations that the number 4 has appeared for maximum number of times. Hence the mode will be 4.

Do and learn

Find the mode of the following numbers -

1, 1, 3, 3, 4, 5, 6, 6, 6, 6, 6, 6, 6, 7, 8

Answer – Mode will be …………….

Organisation of large and unclassified data -

If the number of data (observations) is more, then it is not so easy to count them by writing them in ascending order. In such a situation, we tabulate the data using tally marks.

observe the following **example**.

Example :

The number of family member of 30 students of Ved Bhushan(II year) in veda patashala is as followes

5, 5, 6, 4, 4, 3, 5, 4, 4, 7, 6, 5, 4, 3, 4,

5, 4, 6, 7, 4, 4, 4, 6, 5, 3, 7, 4, 5, 3, 4

Find the mode of the above data.

Solution : Tabulating the given data

Number of family members	Tally marks	Frequency
3	ǀǀǀǀ	4
4	卌 卌 ǀǀ	12
5	卌 ǀǀ	7
6	ǀǀǀǀ	4
7	ǀǀǀ	3
	Total	**30**

From the above table, we can easily conclude that the mode of data is 4, because the maximum number of families have 4 members. (The highest frequency is 12 which corresponds to 4 members in a family)

❖ **Discuss -**

Can a set of numbers have two modes ?

Median (median) -

If the data is arranged in ascending or descending order, the value of the middle term is called median.

Example: Find the median of the following data having odd number of terms -

24, 12, 27, 38, 40, 13, 16

Solution : Arranging the data in ascending order, we get

12, 13, 16, 24, 27, 38, 40

Here, n = 7 (odd number)

Median = $\frac{n+1}{2}$value of term

$= \frac{7+1}{2}$value of term

= 4 (fourth) term value

By arranging the above data, 4 (fourth) term is 24, so median = 24.

Example: The values of a variable are 78, 56, 22, 34, 45, 52, 39, 68, 69, 84 when the given number of entries is even. Find their median.

Solution : On writing in ascending order 22, 34, 39, 45, 52, 56, 68, 69, 78, 84.

Here number of entries is = 10 (even number)

Median $= \frac{1}{2}$[value of $\frac{n}{2}$th term + (value of $\left(\frac{n}{2}+1\right)$) th term]

$= \frac{1}{2}$ [value of $\frac{10}{2}$th term + (value of $\left(\frac{10}{2}+1\right)$) th term]

$= \frac{1}{2}$[value of 5th term + value of (5+1) th term]

$= \frac{1}{2}$ [value of 5th term + value of 6th term]

$= \frac{1}{2}$ [52+56]

$= \frac{1}{2}$ [108]

= 54

Do and learn:

Find the median of the runs scored by a cricket player in 5 matches.

16, 60, 45, 75, 40

Bar graph

The collected information is first arranged in the form of a frequency distribution table and then the combined information is represented in the form of graphs called as bar graphs or histograms.

From the bar graphs we can conclude various information about the data. The tallest bar in a bar graph is the mode of the data represented. In other words, the data can be represented in the form of bar graphs.

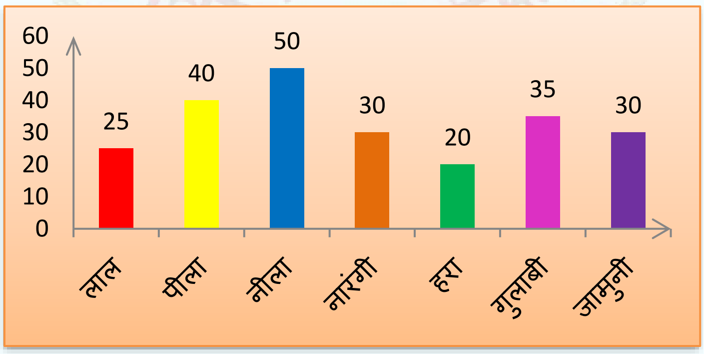
Every bar in a bar graph has a same width and the distance between any two successive bars is the same. The height of a bar represent the frequency of a category of the data.

These bars can be drawn horizontally or vertically. This representation of presenting data is called bar graph or bar diagram.

Reading bar graph -

In a Veda School, 230 students were asked to name their favorite colour which colour should be painted on the walls of the school. For this, the names of the colours have been shown on the ' x' axis and the number of students who like the colour on the ' y' axis. Also 1 unit equals 10 students.

custom colour	red	Yellow	blue	orange	green	pink	purple
No. of students	25	40	50	30	20	35	30



Favourite colour

Answer the following questions -

1. Which bar graph is this? (Horizontal/vertical)
2. Which is the colour that is liked the least?
3. Which is the colour that is liked by most of the students?

   Or on finding the mode of the above data, what is a colour obtained?
4. Find the number of students of the colour that is liked by most of the students.
5. Find the number of students of the colour, that is liked by least number of students.
6. What are the colors which are liked by the same number of students?

Solution :

1. We can see that this bar graph is a vertical bar graph.
2. On studying this bar graph, the least liked colour is green.
3. The most liked colour by the students is blue.

   Or the highest frequency (50) is blue.

   Hence the mode is blue in colour.
4. The number of students, who like a colour for the most number of times, is 50.
5. The number of students who like a colour for the least number of times, is 50...
6. Violet and Orange are the colours which are liked by the equal number of students.

Do and Learn

The number of students studying for Vedas in rashtray Adarsh Ved Vidyalaya, Ujjain is mentioned below. Represent it in the form of a bar graph.

Veda	Rigveda	Jurveda	Samveda	Atharvaveda
Number of students	40	80	30	50

Exercise 4.2

1. Following are the marks obtained by 10 students of Mathematics (out of 20) in an examination.

   15, 16, 17, 17, 18, 17, 19, 17, 20, 17

   Find the mode and median of this data.

2. Find the mode and median of the following numbers.

   (a) 10, 12, 13, 13, 12, 13, 13, 14, 15, 13

   (b) 12, 13, 24, 25, 25, 3, 7

   (c) 18,18, 14, 15, 18, 18, 24, 25

   (d) 13, 24, 25, 25, 3, 25, 11, 12, 3, 12, 3, 25, 4

3. Find the mode of the following data.

   7, 6, 5, 5, 4, 4, 5, 5, 3, 5, 5, 5, 3

4. Find the median of the following data.

   (a) 5, 6, 1, 2, 3, 4, 7

   (b) 2, 4, 11, 12, 3, 15, 4, 9

(c) 6, 1, 2, 3, 12, 3, 4, 11, 12, 12, 0

(d) 6, 1, 22, 4, 11, 12, 3, 12, 3, 15, 4

5. The following data is the runs scored by a team of players of a school in a cricket match: 20, 15, 50, 58, 90, 75, 10, 15, 18, 10, 55

Find the mean, mode and median of the data.

6. The heights (in cm) of 16 students of a class is given below:

138, 142, 135, 137, 145, 150, 132, 143,

143, 140, 136, 138, 143, 138, 147, 140

Find the mode and the median of the data.

7. The numbers of students of Ved Bhushan 1st to 5th year are given below. Represent these data by bar graph -

Vedas	1st	2nd	3rd	4th	5th
Number of students	20	15	30	10	15

8. Consider the following data obtained from a survey conducted in a Ved Pathshala.

Game	Basketball	Swimming	Cricket	Kho-Kho
Number of students	30	25	40	20

1. By choosing a proper scale, draw a bar graph.
2. Which is the most liked sport?
3. The number of students of the sport which least preferred.
4. How many students like swimming ?
5. Which game is in the mode of the data ?

9. Given below are the number of patients who came positive during the corona virus test, in Ujjain.

Checked Dates (May 2020)	13.5.2020	14.5.2020	15.5.2020	16.5.2020	17.5.2020
Number of Patients	15	12	10	13	33

1. Draw a bar graph between the date tested and the number of positive patients by choosing a proper scales.
2. On which date did the least number of patients come ?

10. The following list is a result of a survey conducted during Akhil Bharatiya Veda Sammelan:

Competition	Ved Mantra antakshari	Ved Shloka	Vedik Q & A	Shubhashitani Shloka
Audience	1330	1525	1440	1520
Participants	550	1000	800	850

1. Draw a double bar graph using proper scale.
2. which is the favourite Competition?
3. Which is most preferred watching or participating in a competition
4. Which is the competition in which most number of students participated?

11. Write T for true or F for false

1. Mean is the least entry of the data set.
2. Mode is always one of the data points of the given data set.
3. The median of 3, 1, and 2 is 2.
4. The median is one of the central tendencies.
5. Median is the least entry of the data set.

❖ **We learned -**

1. Collecting and organizing data helps us to draw conclusions.
2. It is necessary to tabulate (pictograph) the collected data, so that it can be understood easily and conclude sertain statements.
3. The reperntetive values of a data set reprents the vehaviour of the data. We called these values as mesures of central tendencies. Mean, mode and median are the measures of central tendencies.
4. The mean is obtained by adding the data and dividing it by the number of data.

   Mean $= \frac{\text{Sum of all data}}{\text{Number of data}}$
5. The term that occurs most frequently in a set of data is called the mode. It can be one or more than one. In other words, the term which has maximum frequency is called mode.
6. If the data is arranged in ascending or descending order, then the value of the middle term is called median.

- If n is odd then median = $\frac{n+1}{2}$value of term
- If n is even, then median = (value of $\frac{n}{2}$ th term + value of $\frac{n}{2+1}$ th term)th / 2

7. Range: - It is found by subtracting the smallest data point from the largest data point of the data set. It is called the range or spread of the data.

   Spread or Range = Largest data point – Smallest data point

Miscellaneous Questionnaire Example:

Example: Find the range of the data - 1, 3, 4, 5, 6, 7, 10, 6, 7

Solution : **Range = Largest data point – Smallest data point**

= 10 – 1

= 9

Hence, the range of the above data set is 9.

Example: Find the mean, range and mode of the following data.

3, 4, 5, 10, 3, 5

Solution : 1. Mean $= \frac{\text{sum of all data}}{\text{number of data}}$

$= \frac{3+4+5+10+3+5}{6}$

$= \frac{30}{6} \quad = 5$

2. Range = Largest data point – Smallest data point

=10 – 3

=7

3. To find the mode -

First of all, writing the data in ascending order -

3, 3, 4, 5, 5, 10

So we can tell by looking that the mode will be 3 and 5, because both have the same frequency.

Example: Find the median of the following data -

Figures 1, 3, 4, 5, 10, 3, 4

Solution : For median - on arranging in ascending order

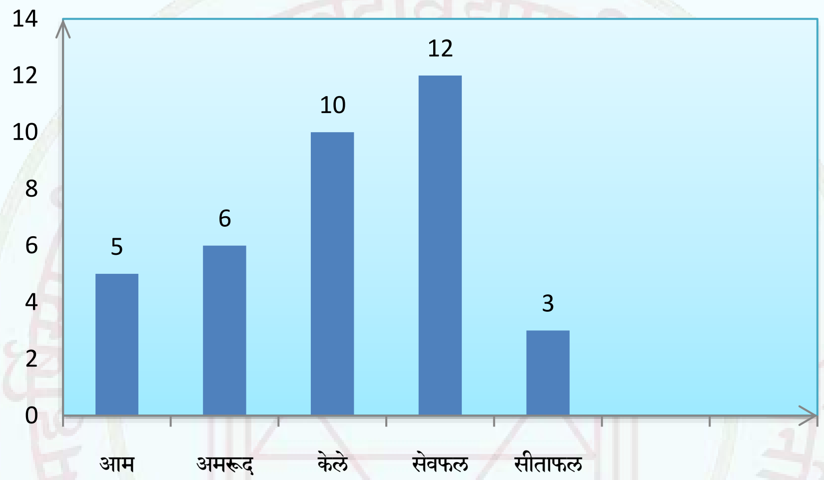
1, 3, 3, 4, 4, 5, 10

We know that the median is the middle value of all the data.

Hence the median is 4.

4. Some fruits are in the basket (mango, guava, banana, nutmeg, custard apple). Students were asked to pick the fruit of their choice. Draw a bar diagram repersenting the following data which is aboute the favourate choice of fruits.

Fruit	Mango	Guava	Banana	Apple	Custard Apple
Number of students	5	6	10	12	3



Chapter 5

Simple Equation

Dear Vedic students, Let us start learning a new chapter in Mathematics, called Simple Equations. We will learn about forming equations using mathematical expressions.

Let us understand the simple equation through a mind game. Try to answer the questions asked in this game. Let's start the game.

The following questions were discussed in the class between the teacher and students.

1 What is the number that should be added to get 18?

Ashish - That number is 16, to which, when 2 is added to get 18.

Guruji - How did you find?

Ashish - Unknown number + 2 = 18,

16 + 2 = 18, hence, the unknown number is 16.

Guruji - Very nice! Perfect answer.

Guruji : What is the number, on multiplying by10, we get 60?

Hemlata - That number will be 6, in the following manner.

Unknown number × 10 = 60

6 × 10 = 60

Guruji - Absolutely correct answer.

3. What is the number to which, multiplying by 10 and adding 5, gives 65 ?

All students think logically. Aaradhya explains based on the previous question.

Aaradhya - the number is 6.

Unknown number $\times$ 10 + 5 = 65

$6 \times 10 + 5 = 65$

$60 + 5 = 65$

Guruji - Very good! You have answered all the questions correctly.

Variable Quantity:

The quantities that vary randomly are called variables or unknown quantities

यावत्तावत्कालको नीलकोऽन्यो वर्ण :पीतो लोहितश्चैतदाद्या:।

अव्यक्तानां कल्पिता मानसंज्ञा-स्तत्संख्यानं कर्तुमाचार्यवर्यै:।

(Algebra, Athavyakt-Sangvidham Nirupayati, 7)

Meaning, to calculate the unknown or variable, we use their Yavat-Tavat, Kalak, Nilak, Pitak and Lohit etc. so that all the variables can be expressed separately.

Express the unknown quantity or unknown number or variable quantity by a letter x. You can replace x with other letters like a,b,c,……… x,y,z. You can use any of the etc.

For example, find the number such that when 5 is added,we get 8. Then unknown amount + 5 = 8

We can denote the unknown quantity by any of the letters a,b,c,……… x,y,z. If we denote by y then we get

$$y+5=8$$

Shubham - I have denoted the unknown quantity with P. Is it correct?

$P+5=8$

Sheetal - Shubham you have written correctly. Unknown or variable quantity can be represented by any letter.

Guruji - what is the value of y (unknown quantity) ?

Shubham - Value of y is 3.

Because $3 + 5 = 8$ both sides will be equal.

Only when $y = 3$.

Guruji - Very good! Correct.

You have guessed the value of an unknown number during the mind game. In which you got different values of the unknown quantities. Hence, the value of variable quantity or unknown quantity keeps on changing.

Constant Quantity:

A quantity whose value does not change, is called a constant quantity.

Constants are represented by any number.

Example: In 2x, 2 is a constant and x is a variable.

We can also write 2x in the following form. $2x = 2 \times x$

Here, substituting the value of x as 1, we get $2x = 2\times1= 2$.

Substituting the value of x as 2, we get $2x = 2\times2= 4$.

Substituting the value of x as 3, we get $2x = 2\times 3=6$.

From the above we can conclude that x is a variable whose value keeps on changing. In the above expression, for every value of x, it is always multiplied with 2. Hence, here 2 is a constant.

Note: When there is no sign (+ or –) between variable and constant quantities, the sign between them is multiplication (×).

You must have understood about variable and constant quantities. A variable can have different values.it means, that its value is not fixed. Often the variables are expressed with the English alphabet x, y, z … etc. We form algebraic expressions using variables. These expressions are formed by

Do and learn

Distinguish the variable and constant quantities in the following - $x, 2, 3, y, P, Q, 1, 8, 4, 5, R, S, t, V, 1$

Variable quantities = ……………………………......

Constant quantities = ……………………………......

Performing operations on the variables such as addition, subtraction, multiplication and division. To form the expression (3x+5), we first multiplied x by 3 and then added 5 to the product.

The value of an expression depends on the chosen value of the variable. If on keeping the value of x as 1

x=1 then, 3x+5 = 3×1+5 = 3+5 = 8

If on keeping the value of x as 2

x=2 then, 3x+5 = 3×2+5 = 6+5 = 11

The conditional statement 3x+5= 8 is satisfied only when x = 1.

When x=2, the value of the expression (3x+5) is 3x+5=11, which is not equal to 8.

OR The **Solution** of the equation 3x+5=11 is not 1.

Therefore, in the equation 3x+5=11, when x= 2, the equation is satisfied. x=2 is called the **Solution** of the equation.

Let us now, learn how to form an equation.

The following sholka is found in beejaganitam written by bhaskracharya.

यावत्तावत्कल्प्यमव्यक्तराशेर्मानं तस्मिन् कुर्वतोद्दिष्टमेव ।

तुल्यौ पक्षौ साधनीयौ प्रयत्नात् त्यक्त्वा क्षिप्त्वा वाऽपि संगुण्य भक्त्वा ॥

(बीजगणितम्,एकवर्णसमीकरणम्, पृ. 263)

Meaning, unknown quantities are found considering as mathematical quantities using yavat - tavat principles.

What is the equation?

In an equation, there are two sides separated by an equals sign. The expression on the left of the equal sign is called Left Hand Side (LHS) and the expression on the right side of the equal sign is called Right Hand Side (RHS). Always the expressions on both sides of an equation are equal.

Example:

$$\underbrace{3x + 5}_{L.H.S.} \;=\; \underbrace{11}_{R.H.S.}$$

equivalence or equal sign

Example:

$$\underbrace{8x + 2}_{L.H.S.} = \underbrace{4x + 5}_{R.H.S.}$$

The expression on the left side of the equality sign is 8x + 2 and the expression on its right side is 4x + 5.

In other words - an equation is a conditional statement having two sides separated by and equal sign which should by true for certain value/s of the variable.

On adding, subtracting, multiplying or dividing by any number other than zero, the equality of the expression is not disturbed.

Remember: in an equation there must be a variable at least on one of its sides.

Do and learn

In the following, select expression and equation separately -

3x+2, 4x-2=5, 4x=1, 8+5x, 4x-1=0, 5x+4,

8x-2=1, 4x+2

	expression		equation
1.	3x+2	1.	4x-2=5
2.		2.	
3.		3.	

Think:-

How can we recognize the equation and an expression?

Writing the statements in the form of equation -

Let us understand using an **example**.

Example: Adding 4 to a number gives 20.

Solution : Let the consider the unknown number as x. Adding x and 4 gives x+4. According to the question the sum is 20.

Hence, the required equation is x+4=20.

Example: subtract 5 from 6 times a number, you get 7.

Solution : Now let this number be y. Multiplying y by 6 gives 6y.

On subtracting 5 from 6y gives (6y-5).

Given that the result is 7.

Hence, the required equation is 6y – 5 = 7.

Example: Adding 7 to one fourth of P gives 8.

Solution : One fourth of P = $\frac{P}{4}$.

According to the question, adding 7 to one fourth ($\frac{P}{4}$) of P is ($\frac{P}{4}$+7).

The sum is equal to 8.

Hence, the required equation is P/4 + 7 = 8.

Example: If 4 is subtracted from 3 times a number, we get 5.

Solution : Let the number be Z. Multiplying Z by 3 gives 3Z.

Subtracting 4 from 3Z will give (3Z-4). This result is equal to 5.

Hence, the required equation is 3Z-4=5.

Writing equations in the form of statements:

(1) x-4=5 (2) 5p=25

(3) 3x+7=11 (4) m/5=4

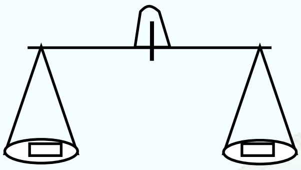
Solution : 1. Subtracting 4 from x gives 5.

2. Five times a number p is 25.

3. Adding 7 to three times a number x gives 11.

4. Dividing a number m by 5 gives 4.

Solving an equation- An equation can be considered as a weighing scale or balance. Performing a mathematical operation (+,-) on an equation should be understood as adding or subtracting equal weights to or from both the pans of the balance respectively.



left side **right side**

(A balanced equation is like a scale with equal weights on both ends.)

The following verse is found to beejaganitam to ensure certain points before solving an equation.

एकाव्यक्तं शोधयेद् अन्यपक्षाद् रूपाण्यन्यस्य इतरस्माच्च पक्षात् ।

शेषाव्यक्तेन उद्धरेद् रूपशेषं व्यक्तं मानं जायते ऽव्यक्ता राशेः।।

अव्यक्तानां द्वयादिकानामपीह,यावत्तावद् द्वयादिनिघ्नं हृतं वा।

युक्तोनं वा कल्पयेद् आत्मबुद्ध्या मानं क्वापि व्यक्तमेव विदित्वा।

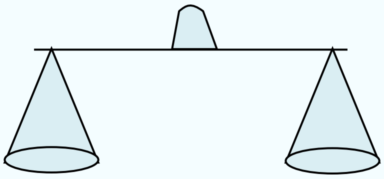
(बीजगणितम्,एकवर्णसमीकरणम्, पृ. 263)

From the above verse, it is clear that validity of an equation is not disturbed till.

1. Add or subtract the same number to or from both sides of the equation.
2. Multiply or divide by the same non-zero number on both sides of the equation.

Let us consider the following equation.

Example: Find the value of x in the equation x+5=10.



Subtracting 5 from both sides of the equation -

New Left Side: $x + 5 - 5 = x$

And

New Right Side: 10 – 5 = 5

5 is subtracted from both sides because subtracting 5 leaves the left side with x. And subtracting the same number from both sides of the equation does not change is equality,

Hence,

new left side = new right side

x = 5

Thus, in the equation x+5=10, the value of x is 5.

Example : Solve the equation x - 3=10.

Solution : Given : x - 3=10

Left side = x - 3 and Right side = 10

(on adding 3 to both sides of the equation)

x – 3 + 3 = 10 + 3

x = 13

Hence, the Solution of the equation is x=13.

Example : Solve : 3P=15

Solution : $3P=15$

Now dividing both sides by 3

$$\frac{3P}{3} = \frac{15}{3}$$

Hence, P=5 which is the Solution of this equation.

Solution of an equation is the value of variables such as x, y, z … etc. for which the equation is true.

Let us learn to find the value of a variable by transposition method. In this method, we move all the variable to one of the sides of the equation and move all the numbers (constants) to the other side.

While moving a number from the left side to the right side or a number from the right side to the left side, the signs of the number are changed.

left side	:	right side
+		–
–		+
×		÷
÷		×

Example: Find the value of x -

$x - 12 = 18$

Solution : Left side = Right side

$x - 12 = 18$

$x = 18 + 12$

(Sign is changed when 12 is transposed from left side to right side)

$x = 30$

Hence, the value of x is 30.

Example:

Solve: $3x+1=13$

Solution : Left side = $3x+1$

right side = 13

$3x+1=13$ { On moving 1 from left side to right side }

$3x = 13 - 1$ {change of sign }

$3x=12$

dividing both sides by 3

$$\frac{3x}{3} = \frac{12}{3}$$

$x = 4$

Hence the value of x is 4.

Example: Find the value of x -

$x + 2 = 18$

Solution : Left side = Right side

$X + 2 = 18$

$x = 18 - 2$

(Sign is changed when 2 is transposed from left to right side)

$x = 16$

Hence, the value of x is 16.

Exercise 5.1

1. Define the terms variable and constant with suitable examples.

2. Write the following statements in the form of equations -

   (1) The sum of x and 4 is 6.

   (2) If 12 is subtracted from a number, we get 7.

   (3) When the number p divided by 5 gives 8.

   (4) 10 times of a is 70.

   (5) Adding 11 to three times x gives 32.

   (6) One fourth of m is 15.

3. Convert the following equations into simple statements -

   (1) x-4=12 (2) 3p=18 (3) 3x+4=1

   (4) $\frac{p}{5}$-3=6 (5) m+3=15 (6) m+13=25

4. Solve the following equations -

   (1) $10m = 100$ (2) $3x = 12$ (3) $2p + 6 = 18$

   (4) $3t + 12 = t + 18$ (5) $3x + 1 = 19$ (6) $x - 12 = 8$

   (7) $\frac{x}{4} - 3 = 56$ (8) $\frac{x}{5} + 3 = 85$ (9) $4x + 1 = x + 17$

   (10) $5y + 1 = y - 25$ (11) $\frac{3x}{5} - 3 = \frac{x}{5} - 3$ (12) $\frac{2x}{5} - 5 = 7$

5. Match the following-

(1)	If x+1=3 then	x=4
(2)	If x+4=9 then	x=2
(3)	If x+3=7 then	x=14
(4)	If x-4=10 then	x=5
(5)	If x-2=8 then	x=10

We learned

1. An equation is a conditional statement which has two sides, constants and variables on one or both sides, is always true for certain values of the variable.
2. The value of the variable for which the equation is satisfied, is called the **Solution** of that equation.
3. In an equation, transposing of a term is moving it from one side to the other. The sign of the terms on transposing change.
4. Interchanging the expression on left side and the right side of an equation does not change the equation.
5. In an equation, we can add, subtract, multiply or divide a number on both the sides simultaneously.
6. The **Solution** of the equation is found by following steps. More than one mathematical operation has to be done on both the sides, so that we get only variable on one of the two sides, to solve the equation.

Chapter 6

Line and angle

Dear Students! You have studied the basic concepts of geometry as an introductory course in your previous class. In which you have studied about line, ray, line segment, point, angle (acute angle, right angle, obtuse angle), parallel and intersecting lines. In this chapter, we will revise the concepts studied in our previous class and learn about the angles made by a transversal on lines.

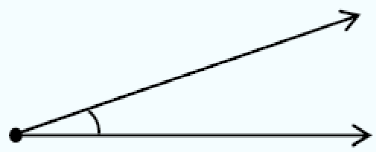
Revision: Recall that a line segment has two endpoints. If we extend these two end points infinitely in their respective directions, we get a line. Thus we can say that a line does not have any end points. Remember that a ray has an end point.

For example

Ray	O	P	$\overrightarrow{OP}$
Line segment	A	B	$\overline{AB}$
Line	M	N	$\overleftrightarrow{MN}$

Angle -

When two rays start from a point, an angle is formed between them.

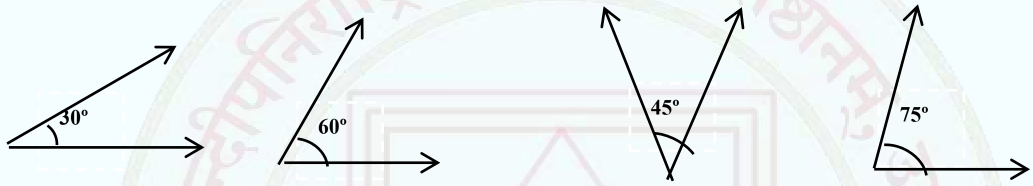


You have also studied about the classification of angles in your previous class. You will recall that angles are classified as

Acute angles (angles greater than 0° and less than 90°), Right angles (angles of 90°), Obtuse angles (angles greater than 90° and less than 180°) and Straight angles (angles of 180°). Now, we will learn about some more angles.

Complementary angle :

When the sum of the measures of two angles is 90°, these angles are called Complementary angles of each other.



(i) (ii) (i) (ii)

Are these two angles complementary angles ? Yes, are these two angles complementary angles ? - no

When two angles are complementary, the angles are called complementary angles of each other.

For example: the complementary angle of 30° will be 60° and the complementary angle of 60° will be 30°.

Do and learn

the complementary angle of 45° is ________.

Think –

(i) What is the complementary angle of a right angle ?

(ii) Can two angles be complementary to each other ?

(ii) Can two acute angles be complementary angles of each other?

Complementary angles

When the sum of two angles is 180°, these angles are called supplementary angles of each other. Which are the supplementary angles in the pair of angles given below?

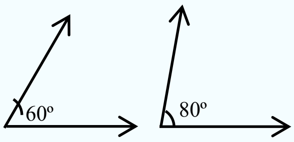


figure 1

$(60^0 + 80^0 \neq 180^0)$

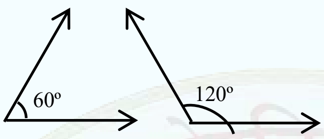


figure 2

$(60^0 + 120^0 = 180^0($

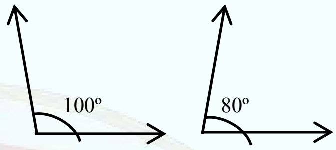


figure 3

$(80^0 + 100^0 = 180^0)$

We observe that in the above paires of angles, except in figure (i) the sum of the measure of angles in figure (ii) and (iii) is 180°. Hence, in the figure (ii) and (iii) such pairs of angles are called supplementary angles.

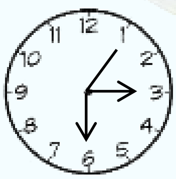
❖ **Think:**

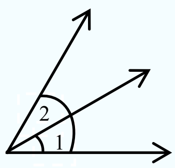
(i) Can two obtuse angles be supplement of each other ?

(ii) Can two right angles be supplementary angles of each other ?

(iii) Can two acute angles be supplementary angles of each other ?

Adjacent angle –





In the above figures you can observe that there are two angles are next to each other with a common arm. Such pairs of angles are called adjacent angles. Adjacent angles have a common vertex and a common arm and both angles are not on the same side of the common side but on opposite sides(Please check).

❖ **Think:**

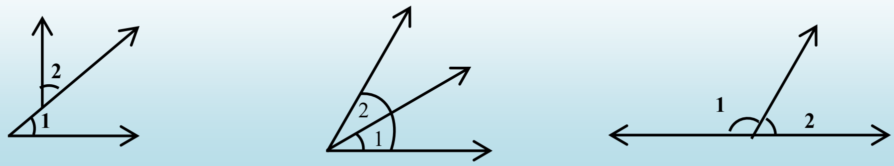
(i) Can the two adjacent angles be complementary to each other ?

(ii) Can the two adjacent angles be supplementary to each other ?

(iii) Can two acute or obtuse angles be adjacent angles ?

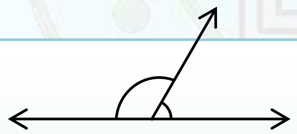
Do and learn

Which of the following angles are adjacent angles, why ?



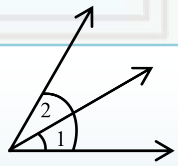
linear pair

A linear pair is a pair of adjacent angles whose non-common sides are rays in opposite directions



Is $\angle 1$ and $\angle 2$ a linear pair ? – Yes

$\angle 1 + \angle 2 = 180°$



Is $\angle 1$ and $\angle 2$ a linear pair ? – no

$\angle 1 + \angle 2 \neq 180°$

Think:

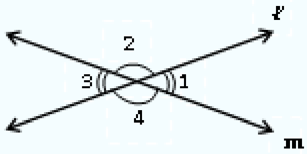
(i) Can a pair of two acute angles be a linear pair ?

(ii) Can two right angles form a linear pair ?

(iii) Can two abtuse angles form a linear pair ?

Vertically Opposite Angles (VOA):

When two lines intersect each other at a point (vertex), two pairs of angles which are opposite to each other, are called vertically opposite angles.



Vertically opposite angles are equal to each other.

Let us try to prove it using geometry.

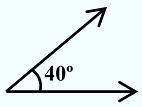
Let us consider two lines *l* and *m* which intersect each other at O forming four angles namely ∠1, ∠ 2, ∠ 3, ∠ 4, shown below.

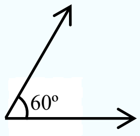
Observe that in the figure there are 4 angles. in which angle 1 is equal to angle 3 and angle 2 is equal to angle 4. These pairs of angles namely ∠1 and ∠ 3, ∠2 and ∠ 4 are called vertically opposite angles. Observe that the pairs of angles ∠1 and ∠2, ∠2 and ∠3, ∠3 and ∠4, ∠4 and ∠1 form linear pairs.

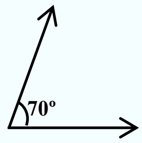
Think - when two lines intersect each other, how many angles are formed?

Exercise 6.1

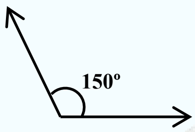
1. Find the complementary angle of each of the following angles ?

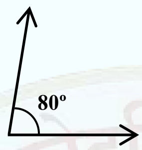
(1 

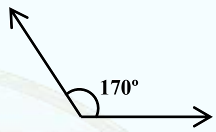
(2 

(3 

2. Find the supplementary angles of each of the following angles ?

1) 

2) 

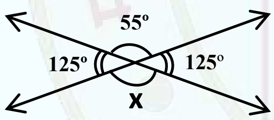
(3 

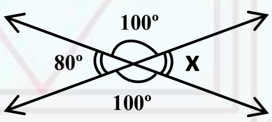
3. Classify the following pairs of angles in complementary and supplementary pairs.

(i) 64°, 26° (ii)65°, 115° (iii)45°, 45°

(iv) 10°, 80° (v) 115°, 65° (vi) 112°, 68°

4. Find the angle x^0in each of the following –





5. Find an angle such that it is equal its complementary angle.

6. Can two angle be supplementary angles, if both the angles

(a) Acute angles (b) Right angles (c) Obtuse angles

7. Fill in the following blanks :

(180°, 60°, 80°, 1, 90°, equal, linear pair, 180°)

1. If two angles are complementary angles, their sum is …………….

2. If two angles are supplementary,the sum of their measures is.........

3. Vertically opposite angles are ………….. to each other.

4. Complementary angle of 30° is...........

5. Supplementary angle of 100° is...........

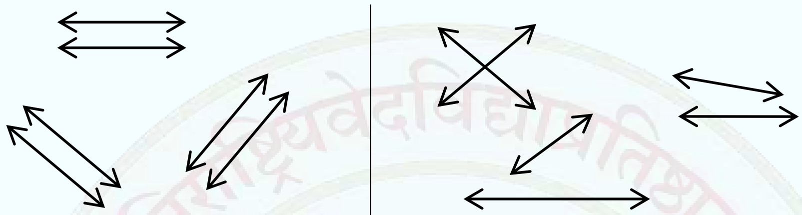
6. How many common vertices are there in adjacent angles ________?

7. The of the measure of angles forming a linear pair is …………….

8. If two adjacent angles are supplementary, they form a.........

Pairs of Lines:

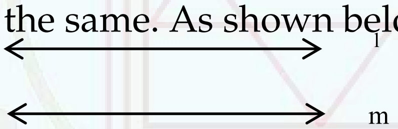
Look at the pairs of lines given below:



is a parallel line | There are not parallel lines.

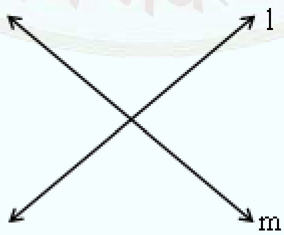
Parallel line:

In a plane, if two lines do not intersect each other are called parallel lines the perpendicular distance between the two parallel lines at every point, is the same. As shown below:



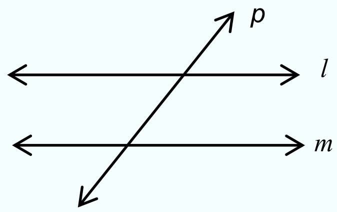
Intersecting line

Two lines that intersect each other are called intersecting lines, As shown below.



Transversal:

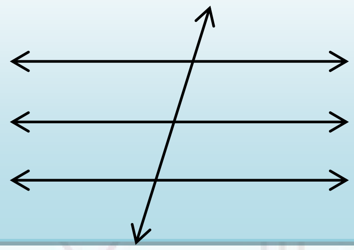
A line which intersects two or more lines at different points is called a transversal.



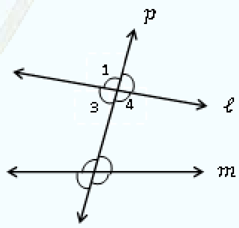
In the above figure, the transversal *P* intersects the pair of lines *l* and *m* at two distinct points.

Do and learn

If a transversal is drawn on three lines, how many intersecting points will be obtained.

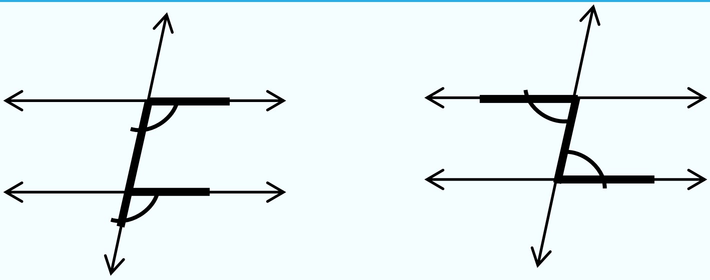


Angles formed by a transversal:



When the transversal *p* intersects the lines *l* and *m*, then 8 different angles are formed. Look at the 8 angles in the figure which have distinct names :

The angles ∠1, ∠ 2, ∠ 7 and ∠8 are formed above and below the lines (Externally), these are called exterior angles. Similarly, the angles formed inside (between the two lines) ∠3, ∠ 4, ∠ 5 and ∠6 are called interior angles. Corresponding angles form an F shape(the term "corresponding angles " is at to be taught.)



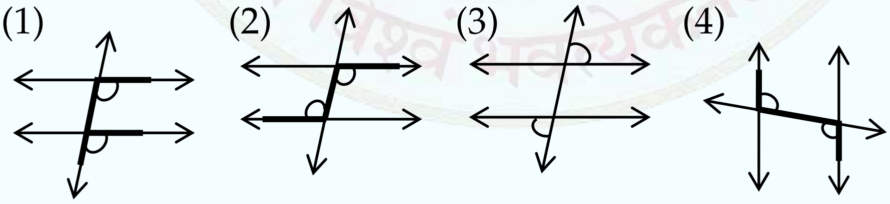
An F shape is formed by the corresponding angle. A Z shape is formed in alternate angles.

Types of Angels	Angles
Corresponding angles – pair	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$
Alternate interior angle pairs	$\angle 3$ and $\angle 6$, $\angle 5$ and $\angle 4$
Alternate exterior angle pairs	$\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$
The pair of interior angles on the same side of the transversal are	$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$.
The pairs of exterior angles on the same side of the transversal are	$\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$

❖ Do and Learn

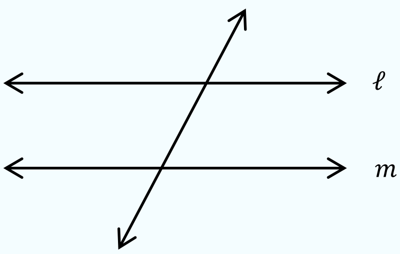
Identify the angle pair of each shape and write their names –

(1) (2) (3) (4)



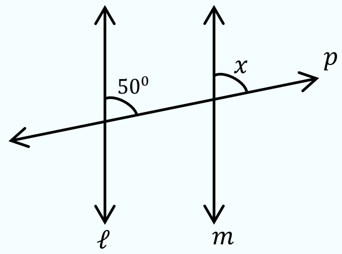
(.................) (.................) (.................) (.................)

Transversal on Parallel Lines :



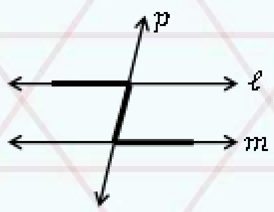
When a transversal p intersects two parallel lines l and m

1. Corresponding angles are equal to each other.



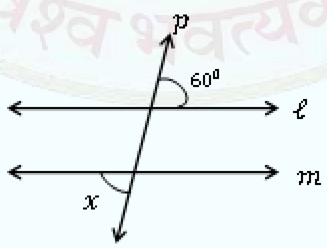
In the above figure, l and m are parallel lines ($l \parallel m$) and p is a transversal. Hence, on the intersection of the transversal with lines, the corresponding angles are equal. Thus $x=50^{o}$

2. Alternate interior angles are equal.



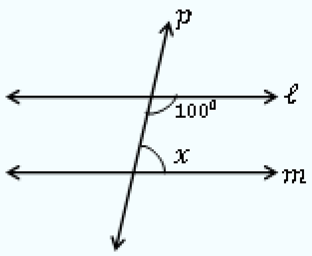
In the above, l and m are parallel lines ($l \parallel m$). P is a transversal. Hence $x=100^{o}$.

3. Alternate exterior angles are equal.



In the above, l and m are parallel lines ($l \parallel m$). P is a transversal. Hence $x=60^{o}$.

4. Interior angles on one side of a transversal are supplementary.



In the above, l and m are parallel lines (l ∥ m). P is a transversal.

Hence, every pair of interior angles is supplementary.

$100° + x = 180°$

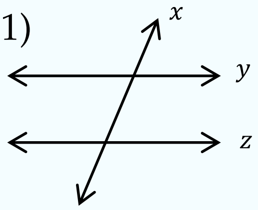
$x = 180° - 100°$

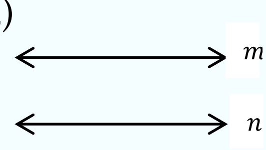
$x = 80°$

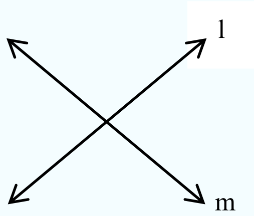
Hence $x = 80°$

Exercise 6.2

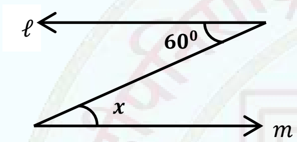
1. Write the names of parallel, intersecting and transversal lines in the following figure –

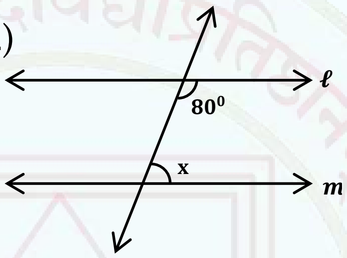
1) 

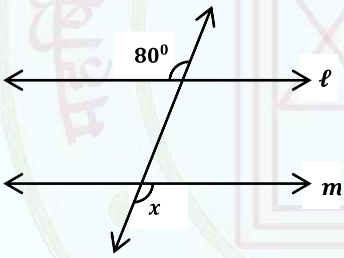
2) 

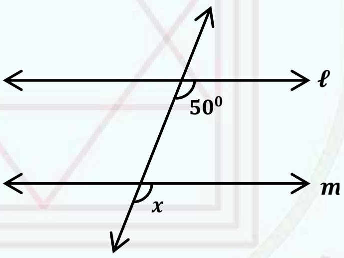
3) 

2. If *l* and *m* are parallel lines ($l \parallel m$), then find the value of x.

1) 

2) 

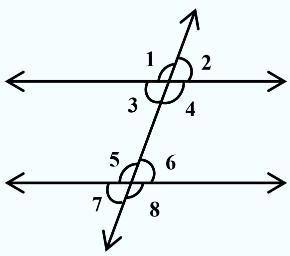
3) 

4) 

3 . Using the below given figure, answer the following questions.

(1) Name the interior angles

(2) Name the exterior angles



4. Draw a line PQ and a parallel line RS to it.

5. Fill in the following blanks:

1. If a transversal intersects to parallel lines, then the

corresponding angles are ________ (Equale / Not equale)

2. If a transversal intersects two parallel lines, then the alternate intirior angles are _________. (Equale / Not equale)

3. If a transvarsal p is drown on two parralal lines $l \parallel m$, the sum of the angles formed on the same side is _______ (180° / 90°)

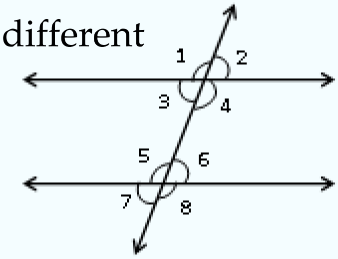
4. In alternate intirior angles, a .... shape is formed. (F/Z)

5. The corresponding angle forms a .... shape. (Z/F)

We learned -

1. When the sum of two angles is 90°, they are called complementary angles of each other.
2. If the sum of two angles is 180°, they are called supplementary angles of each other.
3. The angles having a common vertex and the common arm are called adjacent angles.
4. When two adjacent angles are supplementary, they form a linear pair.
5. When two lines intersect at a point (vertex), then the angle formed opposite to each other are called the vertically opposite angle.
6. A pair of vertically opposite angles are always equal.
7. When a transversal intersects two parallel lines. Then corresponding angles, alternate interior angles and alternate exterior angles are equal.

8. A line which intersects two or more lines at different points is called a transversal line. A line intersecting two lines makes an eight angle.



Angle formed by a transversal

Sl. No.	types of angles	number of pairs	angles
1	interior angle	-	∠3 and ∠4, ∠ 5 and ∠6
2	exterior angles	-	∠1 and ∠2, ∠ 7 and ∠8
3	vertically opposite angles	4 pairs	∠1 and ∠4, ∠ 2 and ∠3 ∠ 5 and ∠8, ∠ 6 and ∠7
4	Corresponding angles	4 pairs	∠1 and ∠5, ∠ 2 and ∠63 ∠ 3 and ∠7, ∠ 4 and ∠8
5	alternate interior angles	2pairs	∠3 and ∠6, ∠ 4 and ∠5
6	alternate exterior angles	2pairs	∠1 and ∠8, ∠ 2 and ∠7
7	2 pairs of interior angles on one side of the transversal	2pairs	∠3 and ∠5, ∠ 4 and ∠6

Chapter 7

Comparison of Quantities

Ratio -

The following verse is found in beeja ganitam written by bhashkaracharya about ratio and proportion.

एकः पदार्थस्तत् सजातीयद्वितीयपदार्थेन यद् गुणितः स एव सम्बन्धो निष्पत्तिर्वा।

यथा अ, व अनयोः सम्बन्धः $\frac{अ}{व}$, वा अ : व एवं लिख्यते ।

(बीजगणितम्, परिशिष्टम्पृ. 243)

The comparison of two similar quantities or numbers is called a ratio. The sign ' : ' is used to express the ratio, in a ratio, the quantities should be in the same units.

A ratio can be expressed in the form of a fraction.

Example: The length of Bajot (Platform) used in worship is 50 cm. and width is 80 cm., find the ratio of the length and breadth of the bajot.

Solution : Given: Length of bajot = 50 cm.

The width of the bajot = 80 cm.

Then, Ratio $= \frac{\text{Length of Bajot}}{\text{Breadth of Bajot}}$

$$= \frac{50 \text{ cm}}{80 \text{ cm}}$$

$$= \frac{5 \text{ cm}}{8 \text{ cm}}$$

Hence, the required ratio is 5 : 8.

Example: 2 kg of wheat and 500 grams of rice are used for making Shodash Matrika and Navagraha in the worship.Find the ratio of weights of wheat and rice.

Solution : Given

Quantity of wheat – 2 kgs

Quantity of rice – 500 grams

then,

$$\text{Ratio} = \frac{\text{quantity of wheat}}{\text{quantity of rice}}$$

We know that the units of both the quantities should be same. The unit of quantity of wheat is kg. converting it into grams

1 kg = 1000 grams

2 kg = 1000 × 2

= 2000 grams

Hence,

$$\text{Ratio} = \frac{2000\text{ grams}}{500\text{ grams}} = \frac{4}{1}$$

Thus, the required ratio will be 4 : 1.

Example: Convert the ratio into simplest form -

(1) $\frac{25}{45}$

$$= \frac{25 \div 5}{45 \div 5}$$

$$= \frac{5}{9}$$

(2) $\frac{32}{24}$

$$\text{or} = \frac{32 \div 2}{24 \div 2} = \frac{16}{12}$$

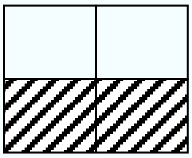
$$\text{or} = \frac{16 \div 2}{12 \div 2} = \frac{8}{6}$$

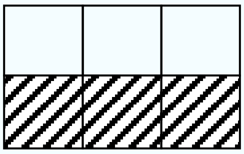
$$\text{or} = \frac{8 \div 2}{6 \div 2} = \frac{4}{3}$$

$$\text{or} = \frac{32 \div 8}{24 \div 8} = \frac{4}{3}$$

Equivalent ratio

Two ratios are said to be equivalent if their corresponding fractions are equivalent. Let us understand using figures –





$= \frac{2}{4}$ or $\frac{1}{2}$ $= \frac{3}{6}$ or $\frac{1}{2}$

the ratio of figure (1) and (2)

$$\frac{2}{4} = \frac{3}{6}$$

$\frac{1}{2} = \frac{1}{2}$ (Corresponding ratios are equivalent.)

Example: Find three equivalent ratios of 4 : 3 or 4/3.

Solution : Three equivalent ratios of $\frac{4}{3}$

$$\frac{4\times 2}{3\times 2} = \frac{8}{6}$$

$$\frac{4\times 3}{3\times 3} = \frac{12}{9}$$

$$\frac{4\times 4}{3\times 4} = \frac{16}{12}$$

Then, $\frac{4}{3} = \frac{8}{6} = \frac{12}{9} = \frac{16}{12}$

Hence the equivalent ratios are 8 : 6, 12: 9 and 16 : 12.

Example: Find two equivalent ratios of 5 : 7.

Solution : Two find to equivalent ratios, expressin the ginen ratio in the form of a fraction we get $\frac{5}{7}$-

$$\frac{\mathbf{5\times 2}}{\mathbf{7\times 2}} = \frac{\mathbf{10}}{\mathbf{14}}$$

$$\frac{\mathbf{5\times 3}}{\mathbf{7\times 3}} = \frac{\mathbf{15}}{\mathbf{21}}$$

Proportion

In the Indian knowledge system, the following verse is found in the context of proportion in Aryabhatiyam and Beejganitam.

त्रैराशिकफलराशिं तमथेच्छाराशिना हतं कृत्वा।

लब्धं प्रमाणभाजितं तस्मादिच्छाफलमिदं स्यात्॥

(आर्यभट्टीयम्,गणितापाद : 26)

Meaning, multiplying the final amount with the amount given and dividing the result by the amount, the Requisition result is obtained.

$$\text{Requisition}(\textit{ichaaphala}) = \frac{(\textbf{fruit}\ (\textbf{\textit{phala}}) \times \textbf{Requisition}(\textbf{\textit{ichaa}})\ )}{\textbf{Argument}(\ \textbf{\textit{pramana}})}$$

The rule of proportion was known to the Indians in Vedang jyotish period before Aryabhatta. Later Acharyas knew the rure even for more than three quantities. From India, this rule reached Europe and Arabia resulting in a lot of propaganda, it spread across the world and it was praised.

यदि चत्वारो राशयः सम्बन्धिनो भवेयुस्तदा आद्यन्त्योर्घातः

द्वितीयतृतीयराश्योर्घाततुल्यो भवेत् ।

कल्प्यन्ते राशयः अ, व, क, ड तदा अः व = कः ड

अर्थात् $\frac{\text{अ}}{\text{व}} = \frac{\text{क}}{\text{ड}}$ **पक्षौ व ड अनेन गुण्यते तदा अ ड = व क॥**

(बीजगणितम्, परिशिष्टम् पृ. 245)

Meaning when two rasios are equale, the quantities are said to be in proportion.

Consider the following example.

Ram bought 6 hand – kerchives for rs. 60 and Shyam bought 10 hand-kerchives for rs. 100.

four quantities will be said to be in a proportion when the ratio of the first quantity and the second quantity is equal to the ratio of the third and fourth quantities. The proportion is denoted by the symbol '::' or by '='.

Example: If the cost of 6 flower garlands is Rs.60. Find the of 10 such garlands ?

Solution : By law of proportion -

Ratio of number of garlands :: Ratio of costs of garlands

$$\frac{6}{10} = \frac{60}{x}$$

$$\frac{6}{10} = \frac{60}{x}$$

$$x = \frac{60\times10}{6}$$

$$x = \frac{600}{6}$$

$$x = 100$$

Thus, the cost of 10 flower garlands is Rs.100.

Solution by unitary method –

Cost of 6 garlands = 60

Cost of 1 garland = $\frac{60}{6}$ = Rs.10.

Hence, the cost of 10 garlands is 10×10 = Rs.100.

Exercise 7.1

1. Find the ratio –

   1. 3kg and 100 g 2.12cm and 36 cm 3. 340cm and 4m

   4. 6km and 400m 5. 80 paise and Rs.4 6. 3 days and 40 hours

2. The length of a room of Gurukul is 14 meters and the width is 20 meters. Find the ratio of the length and breadth of the room.

3. Write in the simplest form –

   (i) $\frac{12}{16}$ (ii) $\frac{45}{20}$ (iii) $\frac{32}{18}$ (iv) $\frac{48}{36}$

4. Find two equivalent ratios of the following ratios –

   (i) $\frac{5}{3}$ (ii) $2 : 3$ (iii) $\frac{3}{8}$ (iv) $\frac{4}{7}$

5. A shopkeeper sells a set of 3 panch-patra for Rs.150, find the cost two sets of panch-patra.

6. In an office out of 400 employees 25 are women. Find the ratio of

   (i) Number of men and number of women.

   (ii) Number of women and number of men.

   (iii) Total number of emploees and number of men.

7. Fill in the following blanks.

   $\frac{5}{3} = \frac{25}{....} = \frac{.....}{24} = \frac{....}{30}$ (are they equivalent ratios ?)

8. Cost of a dozen books is Rs. 180 and the cost of 8 pencils is Rs. 56. Find the ratio between the costs of a book and the pencil.

9. Divide 20 apples between Vishal and shital in the ratio of 3:2.

10. Out of 150 students, 70 like paying cricket, 25 like to play basket ball and the remaining like to play kho-kho. Find the ratio between:

(i) Number of students who like to play cricket and the number of students.

(ii) Total number of students and the number of students who like to play kho-kho.

(iii) Number of students who like to play basket ball and the number of students who like to play criket.

11. Find the number that has to be subtracted from 15 and 19, such that the ratio between the results is 1 : 2.

12. Find the value of x if 4: x :: 8:16.

13. Find the total amount, on dividing it among three person in the ratio 3: 4: 7 the second person gets Rs. 150.

14. Find the 4 ratio in 4, 5 and 2

15. Find the third ratio in 4, 8 and 3.

16. Find the number of runs scored in a over by each, if Aardhya scored 35 runs in 5 overs and Animesh scored 63 runs in 7 overs.

17. The weight of 64 books is 8 kg., find the weight 40 such books.

Percent :

The word percentage is made up of two words 'per' and 'cent' or ‘per hundred. Percentage is represented by the symbol (%).Per means - 'for every'.

percentage of a number means divinding it into 100 equal parts.

Example :

1. Out of total 100 trees in an ashram, 20 trees bear fruits, then the percentage of trees bearing fruits is 20. 20 percent of trees bear

fruit means 20 trees out of every hundred trees bear bruit.Where the total number of trees is 100.

2. Suppose a student scored 40 marks out of 100, it means that student scored 40 percent (%) marks.

Percent sign

Percentage is expressed by writting a sign (%) after the number.

Example: 40 percent to 40%

2 percent to 2%

Percentage as a fraction

Percentage can be represented as a fraction in which the denominator is always 100.

$$a\% = \frac{a}{100}$$

$$40\% = \frac{40}{100}$$

$$13\% = \frac{13}{100}$$ etc.

Converting fraction into a percentage:

Rule :

1. Multiply the given fraction by 100.
2. Simplify the product.
3. At the end of the quotient write percentage (%) symbol.

fraction $\frac{a}{b}$ corresponding percentage form: $\frac{a}{b}\times 100$

Example: $\frac{3}{4}$ into percentage.

Solution : Given Fraction : $\frac{3}{4}$

To convert into percentage, multiply by 100

$$\frac{3}{4}\times 100$$

$$= \frac{300}{4}$$

$$= 75\% \qquad \text{or} \qquad \frac{3}{4} \times 100 = \frac{3}{4} \times 25 \times 4 = 75\%$$

Example: Out of 50 students in class, 20 are boys. Find the percentage of boys in the class.

Soluton:

In the class out of 50 students, 30 are boys.

Hence, the percentage of boys in the class is $\frac{20}{50} \times 100 = 40\%$

Thus, 40% of class are boys. Can we calculate the percentage of girls in the class ?

Converting Percentage into Fraction:

Rule: Remove the percentage sign and divide by 100 or multiply by $\frac{1}{100}$.

Hence, simplify.

For Example:

$$a\% = \frac{a}{100}$$

Example: Convert 3% into a fraction.

Solution : $3\% = \frac{3}{100}$

Now since numerator and denominator do not have any common factor. Therefore

$$3\% = \frac{3}{100}$$

Example: Convert 10% into a fraction.

Solution : $10\% = \frac{10}{100}$

$$= \frac{10}{10 \times 10} = \frac{1}{10}$$

Hence, the fraction of 10% is $\frac{1}{10}$.

Converting a decimal percentage into a simple fraction -

1. First divide the given number by 100 by after removing the % sign.
2. To remove decimal point in the given number, divide the numbers by powers of 10, like10, 100, 1000 etc. as required.
3. Simplify the fraction to get the answer.

Example: Convert 0.4% into a fraction.

Solution : Given - 0.4% = $\frac{0.4}{100}$

Since there is one digit on the right side of the decimal point in the numerator. So multiplying by 1/10 to remove the decimal point.

$$= \frac{4}{100} \times \frac{1}{10}$$

$$= \frac{4 \times 1}{100 \times 10} = \frac{1}{250}$$

Converting percentage into a decimal:

Decimal is a way of writing numbers in exact form Using decimal point. For example, 3.5 means 3 whole parts and one half (0.5) part. Let us follow the steps given below to convert the following percentages to decimals:

Step 1: Remove the percent sign (%).

Step 2: Place the decimal point by mooving two digits from right to left.

For example: Convert 45% into decimal form.

Solution : 45% can be written as 45 without the percent sign. Now, this is a whole number. So consider the decimal point on the right end. 45 is the same as 45.0. Now, shifting the decimal point two places to the left, we get 0.45. Therefore, 45% = 0.45

let's consider as another

example 3.5% we have to shift the decimal point two places to the left.

Then 3.5% = 0.035

Find out the percentage of a Number :

Example: Find 10% of 100

Solution : 10% of 100

According to the question $100 \times 10\%$.

Because, 'of ' stands for multiplication.

$$100 \times \frac{10}{100}$$

Hence, 10% of 100 is 10

Example:

Find 4% of 300

Solution :

4% of 300

$$= 300 \times \frac{4}{100}$$

$$= 3 \times 4 = 12$$

Example: Find 40% of 20

Solution : 40% of 20

$$= \frac{20 \times 40}{100}$$

$$= 2 \times 4$$

$$= 8$$

Hence, 40% of 20 is 8.

Example: Find 40% of 150.

Solution : 40% of 150

$$= \frac{150 \times 40}{100}$$

$$= 15 \times 4$$

$$= 60$$

Hence, 40% of 150 is 60.

Example: A trader offers a discount of 10% on the coas of a chair, if the cost of the chair is Rs 1000, how much will the customer have to pay to buy a chair ?

Solution : Given: a trader gives a discount of 10% on the cost of a chair is Rs.1000.

Means, 10% of 1000 is the discount amount

Hence, the discount amount =

$$= \frac{1000 \times 10}{100} = 100$$

Then, 1000 – 100 = 900

Thus, the customer has to pay Rs.900 to buy a chair.

Do and learn

(i) 10% of 30 (ii) 20% of 40

Note :- We can understand that two zeros are removed from one (%) percentage. That and can also solve immediately.

Example: In a Gurukul, out of 30 students in class, 50% of the students like to play cricket.What is the number of students who like to play cricket ?

Solution : Students who like to play cricket = 50% of 30

$$= 30\times \frac{50}{100}$$

$$= 3\times5$$

$$= 15$$

Hence 15 students like to play cricket.

Example: Vishal got 30 marks out of 50 in Veda's exam, find the percentage of marks got by him ?

Solution : Given that obtained marks is - 30 out of 50.

Percentage of marks score =

$$= \frac{\text{Marks obtained}}{\text{Full Marks}} \times 100$$

$$= \frac{30}{50} \times 100$$

$$= 60$$

So Vishal got 60% marks.

Do and learn :

Utkarsh scored 15 out of 20 marks in a maths monthley test. Fint the percentage of marks got by him.

Exercise 7.2

1. Convert the given percentage to a simple fraction.

   (1) 13% (2) 15% (3) 19% (4) 75%

2. Convert the given decimal fractions into percentages.

   (1) 0.19% (2) 0.1237% (3) 23.7% (4) 253.7%

3. Convert the given fractions into percentages ?

   (1) $\frac{2}{5}$ (2) $\frac{7}{2}$ (3) $\frac{11}{50}$ (4) $\frac{5}{20}$

4. Solve:

   (1) 12% of 300 (2) 70% of 50 (3) 50 % of 40

   (4) 8% of 600 (5) 5% of 900 (6) 50% of 150

5. Out of 50 members of a group, 20 donated blood on Blood Donation Day. What percentage of the members donated blood ?

6. Out of 200 students of the school, 40 students study in Ved Bhushan's second year class. What percentage of students study in second year ?

7. An employee earns Rs.5000 per month. If he saves 10% of his income, then how much money does he save every month ?

8. Out of 200 students of Rashtriya Adarsh Ved Vidyalaya, 15% students study Samaveda and 20% students study Shukla Yajurveda in Madhyandin branch, how many students study Samaveda and Shukla Yajurveda.

9. Out of 150 students of a Veda school, 20% students study in Shuklayajurveda Madhyandin branch and 15% students study in Kanva branch, how many students study in Madhyandin and

Kanva branch.

10. In town out of 15000 voters, 75% of the voted. Find the number of voters who voted.

11. In a village, out 1500 people 850 are literate. Find the number of illterates.

12. Out 2000 thousand in an orchard, 12% is mango trees, 18% is lemon trees and rest orange trees. Find the number of orange trees in the orchard.

13. Out of 1500 students in veda vidayalaya, 15% rigveda, 45% yajurveda and 20% samaveda and rest are atharvaveda students. Find the number of students who study each veda.

14. In a gaushala, there is an increase of 15% of cows every year. If the present number of cows is 800, find the number of cows in the previous and the next year.

Buying and Selling

Cost price (buying):

The cost at which a commodity is bought is called its cost price. In short it is represented as C.P.

Selling Price (Selling)

The cost at which a commodity is sold called its selling price. In short it is represented as S.P.

Let us understand with an example –

for example: A shopkeeper sells a garment to vedik students at Rs.130 each, which he bought at Rs. 100 each. we can understand.

Cost price = Rs.100. And selling price = Rs.130.

Guruji - In this transaction, find whether the shopkeeper got profit or loss ?

Utsav – He earned profit. Because (Bought at less price Rs. 100 and sold at higher price Rs. 130).

Guruji -Very good! Find the amount that he earned as profit.

Lakshmi- He earned Rs.30 as profit.

Guruji - Very good! Correct.

Let us understand the concept of profit and loss using the above example.

Profit :

When an article is bought (purchased) at a lower price and sold (sold) at a higher price. ie

selling price > cost price

profit = selling price – cost price

Profit = SP – CP

Loss-

Contrary to the profit, when an article is bought (purchased) at a higher price and sold (sold) at a lower price. ie

selling price < cost price

loss = cost price – selling price

loss = CP – SP

Percentage of Profit and Loss

Profit and loss can be calculated as a percentage. Remember that the profit-loss percentage is always calculated on the cost price (buying price).

Profit percentage = $\frac{\text{profit}}{\text{cost price}} \times 100$

Loss percent = $\frac{\text{Loss}}{\text{cost price}} \times 100$

Example: If the shopkeeper sells a chair for Rs.300. Wich costs him Rs. 250. How much profit did he earned ?

Solution : Selling Price > Cost Price

300 > 250

Hence,

Profit = Selling Price – Cost Price

= 300 – 250

= Rs.50

Then, profit percentage = $\frac{\text{profit}}{\text{cost price}} \times 100$

$= \frac{50}{250} \times 100$

$= \frac{5000}{250}$

$= \frac{5000 \div 250}{250 \div 250}$

$= \frac{20}{1} = 20\%$

Example: If the shopkeeper bought a toy car for Rs.100 and sells it for Rs. 80. Find the percentage of loss.

Solution : Selling Price <Cost Price

80 < 100

Then, loss = cost price – selling price

= 100 – 80

= 20

Now, loss percent = $\frac{\text{Loss}}{\text{cost price}} \times 100$

$$= \frac{20}{100} \times 100$$
$$= 20\%$$

Example: An article is bought at Rs.50 and profit of 10%. Find the selling price.

Solution : Given: cost price = Rs.50 and % of profit = 10.

hence, selling price = cost price + 10% of cost price

$$= 50 + \left(50 \times \frac{10}{100}\right)$$
$$= 50 + 5$$
$$= 55$$

Thus, selling price Rs.55.

Example: Umashankar sold a cooler to his friend Ajit for Rs.3500 at a loss of 10 %. Find the cost at which Umashankar bought the coolar.

Solution : Given selling price = Rs.3500

$$3500 = x - \left(\frac{10}{100} \times x\right)$$
$$3500 = \frac{100x - 10x}{100}$$
$$3500 = \frac{90x}{100}$$
$$3500 \times \frac{10}{9} = x$$
$$x = \frac{35000}{9}$$
$$x = 3888.89$$

EXERCISE 7.3

1. Find the loss or profit in the following transactions between the shopkeeper and customer. Find the percentage loss or percentage profit in each case.

   (a). A gaumukhi was bought for Rs.50 and sold for Rs.40.

   (b). A cotton shawl (Dushala) was bought Rs.200 ans sold at Rs 220.

   (c). A shirt was bought at Rs.250 and sold at 300.

2. Rahul bought a book for Rs.200 and sold at a profit of 12% to ram. Find the selling price of the book.

3. The cost of a bag of potatos is Rs.500,.which is sold at a profit of 3%. Find the selling price of the bag.

4. Raman bought a vehicle at Rs. 25000 and sold it for Rs.27000. Find the percentage of profit.

5. Anant sold a toy (Musicle instrument) at Rs.540 and earned a profit of 20%. Find the cost price of toy.

6. In Ujjain, in tree plantation programme out of the trees planted 10% withered. If the present number of trees is 1800, find the total number of trees planted.

7. Find the profit percentage, if Radha sold a sewing machine at Rs. 5100 which costed her Rs. 4800.

8. The population of a city increased from 1500 to 1650. Find the percentage increase of population of the city.

We learned -

1. In our daily life we often have to make comparisons between two

quantities. These quantities can be heights, weights, salaries, marks etc.

2. Two ratios can be compared by converting them into fractions with the same denominator. If the two fractions are equal, the corresponding ratios are called equivalent ratios

3. Percentage is also a method of comparison. Percent means 'per hundred'. Means to divide into 100 equal parts.

$$3\% = \frac{3}{100}$$

Decimal to Percent - 0.4%

$$= \frac{0.4}{10} \times 100 \quad = 40\%$$

Convert fraction to decimal - $\frac{a}{b}\% \quad = \frac{a}{b\times100}$

$$= \frac{3}{4}\% \quad = \frac{3}{4\times100} = \frac{3}{400}$$

$$\text{or } \frac{3}{400} \quad = \frac{3\times25}{4\times25\times100} \quad = \frac{75}{10000} \quad = 0.0075$$

Number in percentage - 4% of 300

$$= 300 \times \frac{4}{100} \quad = \quad 3 \times 4 \quad = 12$$

price - Cost price = buy

selling price = selling

Profit - Selling Price >Cost Price

Profit = Selling Price – Cost Price

Loss – Selling Price <Cost Price

loss = cost price – selling price

profit percentage = $\frac{\text{profit}}{\text{cost price}}\times100$

Loss percent = $\frac{\text{Loss}}{\text{cost price}}\times100$

Chapter 8

Rational Number

Dear Students! We have learned the numbers to counting the objects in daily life, the numbers used in counting (calculation) are called natural numbers. 1, 2, 3, 4, 5........ On adding 0 to the natural numbers, we get the whole numbers. On inclusion of negative natural numbers with whole number, 0, 1, 2, 3, 4........, we get...... – 3, –2, –1, 0, +1, +2, +3...... are called integers we have studied the number system in detail till integers.

natural numbers	whole numbers	integers
1, 3, 4, 9, 8, 5, 2, 7	0, 1, 2, 7, 6, 8, 70, 100	–3 –2 –1 –6 0 12 5 7

In the previous class, you were introduced about Fractions. The fractions are written in the form of $\frac{\text{Numerator}}{\text{Denominator}}$.

In this chapter we will extend our studies to rational number and its behaviour

Need for Rational Numbers:

In our daily life, we use integers to represent various situations, such as a profit of Rs. 50 is represented as +50 and a loss of Rs. 50 as -50. Similarly a point at 500m above the sea level is represent as ½ km. at the same time a point which is at 500m below the sea level can be shown as 1/2 km below sea level. This is also written as -1/2 km.

We can understand that -1/2 is neither an integer nor a fraction. To understand such numbers, we are required to extend the number system.

Rational Number –

Rational number are formed using the concept of ratios. We know that 4:5 can be written as a fraction 4/5 in which 4 and 5 are integers. Similarly, for any two integers p and q where $q \neq 0$, the ratio p : q can be written of the form of $\frac{p}{q}$. Such form of numbers are called rational numbers. Hence, a rational number is in form of $\frac{p}{q}$, where p, q are integers and $q \neq 0$. The following verse is found in beejaganitam about rational numbers.

सच्छेदकराशौ भाज्ये भाजके वा ऋणत्वं चेत् फलमृणमेव,

द्वयोर्ऋणत्वे फलं धनमेव । यथा -

$$\frac{-\text{अ}}{\text{व}} = \frac{\text{अ}}{\text{व}} \quad , \frac{\text{अ}}{-\text{व}} = -\frac{\text{अ}}{\text{व}} \quad \textbf{तथा} \quad \frac{-\text{अ}}{-\text{व}} = \frac{\text{अ}}{\text{व}}$$

(Bijaganitam, Sachchedkarashiganitam p. 238)

In the above verse, the form of rational numbers is explained. The rational number is in the form of numerator and non - zero denominator, where both are integers.

Example: $\frac{5}{7}, \frac{-3}{5}, \frac{11}{15}, \frac{-8}{-16}$ etc

Rational numbers = { $\frac{p}{q}$ / where q ≠ 0 and p, q ∈ integer (I) }

Numerator and Denominator of a Rational Number:

A rational number is in the form of $\frac{p}{q}$ where $q \neq 0$, in this, p is numerator and q is the denominator. In the rational number $\frac{2}{3}$ 2 is the numerator and 3 is the denominator. Similarly, in $-\frac{5}{7}$, -5 is the numerator and 7 is the denominator.

After understanding the concept of rational numbers, the following points were discussed.

Guruji : Are all natural numbers, rational numbers ?

Pratyush: Yes, all the natural numbers are rational numbers. 1, 2, 3, 4 .... are natural numbers. Since all natural numbers can be written in the form of $\frac{p}{q}$.

Example: 2 as $\frac{2}{1}$, 3 as $\frac{3}{1}$

Hence, all the natural numbers are rational numbers.

Guruji : Are all the integers, rational numbers?

Aradhya: Like 0, –3, 1, 3, –4, 5........ etc are integers.

Since – 3 can be written as $\frac{-3}{1}$, 4 as $\frac{4}{1}$, –5 as $\frac{-5}{1}$,which is in the form of $\frac{p}{q}$.

Hence, all integers are rational numbers. Zero which is a whole number is also a rational number. Because 0 can be expressed as $\frac{0}{1}$.

Guruji : Are all fractions rational numbers ?

Utsav : Yes, all fractions are rational numbers, because fractions are in the form of $\frac{p}{q}$.

where $q \neq 0$.

Guruji : Is $\frac{4}{0}$ a rational number ?

Lakshmi: No, $\frac{4}{0}$ is not a rational number. Because the denominator of a rational number is not zero and the denominator of $\frac{4}{0}$ is 0.

Guruji : Very good! You answered all the questions correctly.

Equivalent rational number -

The numerator and denominator of a rational number can be converted into the desired numerator or denominator by multiplying or dividing by the same non – zero number.

Let us consider the rational number $\frac{(-2)}{3}$.

$$\frac{(-2)}{3} = \frac{(-2)\times 2}{3\times 2} = \frac{(-4)}{6}$$

$$\frac{(-2)}{3} = \frac{(-2)\times 3}{3\times 3} = \frac{(-6)}{9}$$

$$\frac{(-2)}{3} = \frac{(-2)\times(-4)}{3\times(-4)} = \frac{8}{(-12)} \text{ or } \frac{(-8)}{12}$$

Thus $\frac{(-2)}{3} = \frac{(-4)}{6} = \frac{(-6)}{9} = \frac{(-8)}{12}$ are equivalent rational number.

Let us consider another rational number

$$\frac{(-30)}{25} \frac{(-30)\div 5}{25\div 5} = \frac{(-6)}{5}$$

$$\frac{(-30)\div(-5)}{25\div(-5)} = \frac{6}{(-5)} \text{ or } \frac{(-6)}{5}$$

Hence, $\frac{-30}{25}$ and $\frac{-6}{5}$ are equivalent rational numbers.

Such fractions which are equal in value are called equivalent fractions.

Positive and negative rational numbers

Positive rational numbers -

Rational numbers in which both numerator and denominator are positive or both are negative (the minus sign of both gets cancelled), are called positive rational numbers.

For example:

1. In $\frac{2}{3}, \frac{11}{13}, \frac{2}{5}$ both numerator and denominator are positive.

2. In $\frac{-2}{-3}, \frac{-11}{-15}, \frac{-7}{-5}$ both numerator and denominator are negative.

$\frac{-2}{-3} = \frac{2}{3}$ is a positive rational number. Thus $\frac{-11}{-15}, \frac{-7}{-5}$ are also positive rational numbers.

In these, both the numerator and the denominator are negative hence, both the negative signs get cancelled and the number changes to a positive sign. $\frac{-2}{-3} = \frac{2}{3}$ is a positive rational number.

Important point -

Zero is neither a positive rational number nor a negative rational number

Do and Learn:

1. write three positive and four negative rational number
2. is (- 7) a rational number?
3. write rational numbers which are not fractions.
4. write a rational number which cannot be expressed as a ratio

Rational numbers in standard form

A rational number whose denominator is a positive integer and its numerator and denominator do not have any common factor other than 1 (the numerator and the denominator cannot be divided by any number other than absence of 1), is called the standard form of that rational number.

Example : $\frac{2}{3}, \frac{3}{4}, \frac{11}{13}$

Think –

- the rational number ¾ is expressed in a standard form ?

  Yes, because its denominator is positive, and numerator and denominator have no common factor other than 1.

- Is 4/6 a rational number expressed in standard form?

  No, because the common factors of 4 and 6 is 2. Expressing this in standard form, $\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

  So the standard form of $\frac{4}{6}$ is $\frac{2}{3}$.

Example : Convert $\frac{(-12)}{18}$ to standard form -

Solution : $\frac{(-12)}{18}$

HCF of 12 and 18 = 6.

Or the common factors of 12, 18 are (1, 2, 3, 6).

on dividing numerator and denominator by 6

$$= \frac{(-12) \div 6}{18 \div 6} = \frac{(-2)}{3}$$

the standard form of $\frac{(-12)}{18}$ is $\frac{(-2)}{3}$.

Do and learn

Convert the following to standard form.

1. $\frac{35}{25}$ 2. $\frac{(-15)}{25}$ 3. $\frac{(-20)}{24}$

Comparison of rational numbers (=, > and <) –

Vedic Method (UrdhwatiryaBhyam)

Example: Compare the rational numbers $\frac{3}{5}$ and $\frac{6}{7}$?

Solution : The given rational number is

$$\frac{3}{5} \text{ and } \frac{6}{7}$$

(On multiplying with the formula Urdhvatirya↑Bhyam)

$$\frac{3}{5} \times \frac{6}{7}$$

or $\quad 3 \times 7$ and 6×5

Or $\quad 21$ and 30

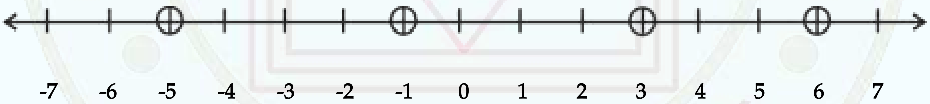
It is clear that $21 < 30$

Hence, $\frac{3}{5} < \frac{6}{7}$

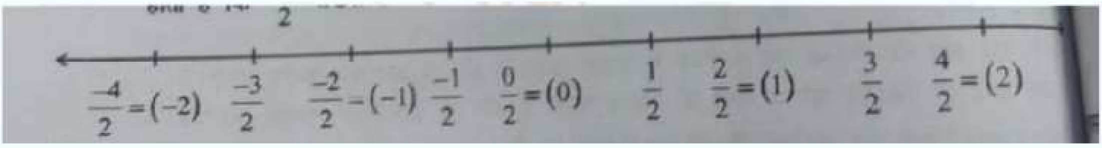
Rational Numbers on a Number Line:

We know that, all the integer can be located on a number line as follows.

In the given below number line, the integers -5, -1, 3 and 6 are located. Similarly, all the rational numbers can be located on a number line.



Let us located $\frac{-3}{2}$ can be rewritten as $-\left(1\frac{1}{2}\right)$, which is a negative rational number. Hence, we move towards left of 0 on a number line to locate. The rational $\frac{-3}{2}$ is located at the midpoint of -1 and -2.



In the above number line, other than $\frac{-3}{2}$ some more rational numbers are located. Following the same method located the rational numbers $\frac{-6}{2}$ and $\frac{7}{2}$ on a number line.

Operations on Rational Number:

Using the method of addition, subtraction, multiplication, and division of whole numbers and fractions, let us understand the operations on rational numbers.

Addition: When the denominator of rational numbers is the same, the result of the sum of the rational numbers will be having the same denominator and numerator is the sum of all the numerators of the rational numbers.

Example: $\frac{-15}{3}+\frac{5}{3}=\frac{(-15+5)}{3}=\frac{-10}{3}$

How to add rational numbers with different denominators?

To add rational numbers with different denominators, just like fractions, find the LCM of the denominators, write equivalent rational numbers such that their denominator is the LCM found and add the rational numbers.

Example: Find the sum of $\frac{-7}{5}$ and $\frac{-2}{3}$.

Solution : LCM (5, 3) = 15

Hence, $\frac{-7}{5}=\frac{-21}{15}$ and $\frac{-2}{3}=\frac{-10}{15}$

Thus, $\frac{-7}{5}+\left(\frac{-2}{3}\right)=\frac{-21}{15}+\left(\frac{-10}{15}\right)=\frac{-31}{15}$

Additive Inverse:

A number when added to the given number gives 0 is called its additive inverse. In general, the additive inverse of a is (- a) and additive inverse of $-a$ is a. in other words, $a+(-a)=0$.

in the same way, additive inverse of $\frac{-5}{9}$ is $\frac{5}{9}$. Because,

$$\frac{-5}{9} + \left(\frac{5}{9}\right) = \frac{(-5+5)}{9} = \frac{0}{9} = 0$$

Hence, the additive inverse of $\frac{-5}{9}$ is $\frac{5}{9}$.

Do and Learn: Find the additive inverses of the following:

(i) $\frac{5}{9}$ (ii) $\frac{-11}{19}$ (iii) $\frac{55}{209}$ (iv) $\frac{-14}{21}$ (v) $\frac{500}{19}$

Subtraction of Rational Numbers: To subtract a rational number from the other, we add the additive inverse of the second rational number (subtrahend) and the first rational number (minuend).

Example: Ram adopted the method discussed to obtain the difference between $\frac{3}{7}$ and $\frac{4}{5}$, which is given below.

$$\frac{3}{7} - \frac{4}{5}$$

$$= \frac{3\times5}{7\times5} - \frac{4\times7}{7\times5}$$

$$= \frac{15}{35} - \frac{28}{35}$$

$$= \frac{15-28}{35} = \frac{13}{35}$$

Exercise 8.1

1. Classify the following into positive and negative rational numbers.
$\left(\frac{-2}{3}, \frac{5}{7}, \frac{-4}{-5}, \frac{2}{7}, \frac{-2}{-1}, \frac{-3}{4}, \frac{-3}{-7}, \frac{+8}{+3}\right)$

2. Write three equivalent rational numbers of the following rational numbers.

1) $\frac{5}{3}$ 2) $\frac{-2}{7}$ 3) $\frac{-4}{-7}$ 4) $\frac{12}{5}$

3. Convert into their standard forms -

1) $\frac{-14}{12}$ 2) $\frac{36}{30}$ 3) $\frac{-25}{45}$ 4) $\frac{96}{48}$

4. Fill in the blanks by choosing the correct sign from the comparison signs (=, > and <) -

1) $\frac{3}{5} \square \frac{-4}{7}$ 2) $\frac{-3}{7} \square \frac{4}{5}$ 3) $\frac{4}{3} \square \frac{3}{7}$

4) $\frac{-5}{-4} \square \frac{-2}{-3}$ 5) $\frac{5}{7} \square \frac{7}{9}$ 6) $\frac{9}{1} \square \frac{1}{4}$

7) $\frac{4}{6} \square \frac{2}{3}$ 8) $\frac{5}{9} \square \frac{4}{6}$

5. Which is the largest rational number in each of the following groups -

1) $\frac{-2}{3}, \frac{5}{5}$ 2) $\frac{-5}{6}, \frac{-3}{4}$

3) $\frac{-3}{1}, \frac{4}{3}$ 4) $\frac{2}{3}, \frac{1}{-4}$

6. Find the sum of the rational numbers

1) $\frac{-2}{3} + \frac{5}{5}$ 2) $\frac{-5}{6} + \frac{-3}{4}$ 3) $\frac{-3}{1} + \frac{4}{3}$

4) $\frac{2}{3} + \frac{1}{-4}$ 5) $\frac{7}{4} + \frac{5}{5}$ 6) $\frac{9}{8} + \frac{-7}{4}$

7. Find the difference between the rational numbers:

1) $\frac{-2}{3} - \frac{5}{5}$ 2) $\frac{-5}{6} - \frac{-3}{4}$ 3) $\frac{-3}{1} - \frac{4}{3}$

4) $\frac{2}{3} - \frac{1}{-4}$ 5) $\frac{7}{4} - \frac{6}{5}$ 6) $\frac{9}{8} - \frac{-7}{4}$

8. Locate the following rational numbers on a number line:

(1) $\frac{5}{6}$ (2) $\frac{-8}{3}$ (3) $\frac{7}{5}$ (4) $\frac{4}{3}$ (5) $\frac{5}{4}$

9. Write (T) for True / (F) for False -

1. Zero is neither a positive nor a negative rational number.
2. The standard form of $\frac{-4}{6}$ is $\frac{2}{3}$.
3. $\frac{-4}{-7}$ is a negative rational number.
4. Among the numbers $\frac{2}{5}$ and $\frac{3}{5}$, the larger rational number is $\frac{3}{5}$.
5. Rational numbers which are equal to each other are said to be equivalent to each other.
6. $\frac{-4}{7}$ is located on right of 0 on a number line.
7. $\frac{4}{7}$ is located on right of 0 on a number line.

We learned -

1. A rational number is written in the form $\frac{p}{q}$. where p and q are integers and $q \neq 0$.

   Like - $\frac{7}{8}, \frac{3}{4}, \frac{0}{4}, \frac{5}{4}$

2. All natural numbers, whole numbers and integers are rational numbers. As -

   $4 = \frac{4}{1}, 6 = \frac{6}{1}, -7 = \frac{-7}{1}$

3. If the numerator or denominator of a rational number is multiplied or divided by the same non-zero integer, we get a rational number. Which is called the equivalent rational number of the given rational number.

Example: $\frac{3}{2} = \frac{3 \times 5}{2 \times 5} = \frac{15}{10}$

Hence, $\frac{15}{10}$ is the equivalent form of the number $\frac{3}{2}$.

$\frac{45}{25} = \frac{45 \div 5}{25 \div 5} = \frac{9}{5}$

Classification of Rational Numbers	
Positive Rational Number	**Negative Rational Number**
When both the numerator and the denominator are positive or negative integers, that number is called a positive rational number. For example: $\frac{3}{8}, \frac{5}{3}, \frac{2}{5}$	When one of the numerator and the denominator is negative integer, they are called negative rational numbers. For example: $\frac{-3}{4}, \frac{-4}{5}, \frac{6}{-7}$

5. The number ' 0' is neither a positive rational number nor a negative rational number.

6. A rational number is said to be in standard form when its denominator is a positive integer, the numerator and denominator have no common factors other than 1 or are divisible by any number other than 1.

For example: $\frac{-1}{3}$, $\frac{5}{7}$, $1\frac{-2}{3}$ etc. are standard forms.

7. Comparison of rational number is done easily by doing cross multiplication (Urdhwatirya꣡Bhyam - Vedic method). As -

$\frac{-4}{5}$ and $\frac{6}{7}$ on cross multiplication for comparison $(-4) \times 7$ and 6×5

It is clear that $(-28) < 30$

So $\frac{(-4)}{5} < \frac{6}{7}$

8. To add or subtract rational numbers, we find the equivalent rational numbers such that the denominator is the LCM of all the denominators and evaluate.

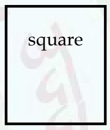
9. On a number line, positive rational number and negative rational number are located on the right and left of 0 respectively.

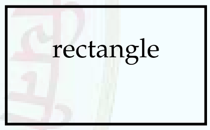
Chapter 9

Perimeter and Area

Dear students! We learned to find the perimeter and area of figures, square and rectangle under the chapter on perimeter and area in the previous class. The distance around a closed figure its perimeter.Whereas the part or region of the plane enclosed by a closed figure is called its area. In this chapter, we will study about the perimeter and area of some other planar figures.

Revison:

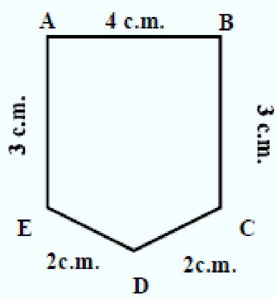




- Perimeter of square = 4 × one side
- Area of square = side × side
- Perimeter of the rectangle = 2 × (length + breadth)
- Area of rectangle = length × breadth

Perimeter - The sum of the lengths of all the sides of a closed figure is called its perimeter.

Example: Find the perimeter of the figure given below –



Solution : Perimeter = Sum of the distances around

= AB + BC + CD + DE + EA

= 4 cm + 3 cm + 2 cm + 2 cm + 3 cm

=14 cm

In Sanskrit litrature, the method of finding the area of a quadrilateral with equal sides has been explained in the following verse of Aryabhatiyam.

वर्गस्समचतुरस्त्र: फलञ्च सदृशद्वयस्य संवर्ग: ॥

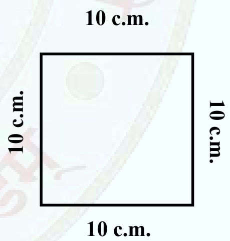
(आर्यभट्टीयम्,गणितापाद, 3 अ)

Meaning, such a quadrilateral whose four sides are equal is called a *samacaturashra* (square) of equal sides and the area is found by multiplying two equal sides. It is classified as $(\text{square})^2$.

Example: If the side of a square field is 10 centimeter, find its perimeter and area.

Solution : Given

One side of a square field = 10 centimete



Then, Perimeter of square = 4 × side

= 4 × 10 meter

= 40 meter

And, Area of square = side × side

= 10 meter × 10 meter

= 100 (m × m)

$= 100\ (\text{m.})^2$

Hence, the perimeter of the square field is 40 meters and the area is $100\ (\text{m.})^2$.

Example: Find the perimeter of a square class whose length is 5 m.

Solution : Given : length of the square room = 5 m

Then, perimeter of square = 4 × side (length)

$= 4 \times 5 \text{ m} = 20 \text{ m}$

Example: The length of a rectangular blackboard is 4 meters and the width is 3 meters, find the perimeter and area of the blackboard.

Solution : Given : Length of rectangular black board = 4 m, breadth = 3 m

Perimeter = 2 × (Length + Breadth)

$= 2 (4 \text{ m} + 3 \text{ m})$

$= 2 (7 \text{ m})$

$= 14 \text{ m}$

And, **area = length × breadth**

$= 4 \text{ m} \times 3 \text{ m}$

$= 12 (\text{m} \times \text{m})$

$= 12 (\text{m})^2$

Hence, the perimeter of the rectangular blackboard is 14 m and the area is $12 (\text{m})^2$.

Think –

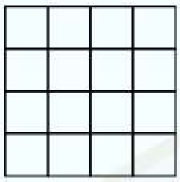


figure 1

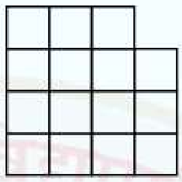


figure 2

1. Are the perimeters of the above figure 1 and figure 2 equal or not ?
2. Are the areas of the above figure 1 and figure 2 equal or not ?

Do and learn

In the following situations identify, when do we find the area or perimeter.

1. If the carpet is to be spread in the class ....
2. If wire fencing is to be done around the Gurukul ....
3. If a lace is to be stitched around edges of a silk dupatta ....

Example: Area of a rectangular sheet is 100 $(cm)^{2}$, if the width of the sheet is 20 cm.What will be its length ?

Solution : Given

Area of rectangular sheet = 100 $(cm)^2$

Breadth = 20 cm, length = to be determined.

We know that –

Area of rectangular sheet = length × breadth

$100 = \text{length} \times 20$

or $\text{length} = \frac{100}{20}$

$= 5$

Hence the length of the rectangular sheet is 5 cm.

Example: Pawan bought a plot whose length is 10 m. and width 20 m. Find how much land (area) Pawan bought ?

Solution : Given, area of rectangle = length × breadth

$= 10 \text{ m} \times 20 \text{ m}$

$= 200 \text{ (m.)}^2$

Pavan bought 200 square meter of land.

Exercise 9.1

1. Find the perimeter and area of a rectangle whose length and breadth are as follows -

   1. Length = 10 cm, breadth = 30 cm.
   2. Length = 20 cm, breadth = 50 cm.
   3. Length = 40 meters, Width = 30 meters

2. Find the perimeter and area of the square whose side is as follows-

   1.10 cm 2. 30 cm 3.40 m 4. 15 m

   5. 15cm 6. 20 m 7. 65 m 8. 16m

3. If the area of a rectangle is 500 $(\text{meters})^2$and the length is 50 meters, what will be the width ?

4. If the area of a rectangular field is1000 $(\text{meters})^2$and the width of the field is 200 meters, what will be the length of the field ?

5. What will be the area of the rectangular land whose length is 15 m and the width is 20 meters ?

7. In a square field of side 60 m, a pathway is constructed in side its periphery of width 2 m.

   (i) Find the area of the pathway.

   (2) In the cost of constructing the pathway is Rs. 250 per square m, find the total cost involved.

8. A pathway of width 2.5m is constructed, boardaring out side the rectangular field of length 55m and breadth 40m. Find the area of the pathway.

9. Match the following

1)	Area of square	=	$2 \times$ (length + breadth)
2)	Area of rectangle	=	$4 \times$ one side
3)	Perimeter of square	=	side $\times$ side
4)	Perimeter of rectangle	=	length $\times$ breadth

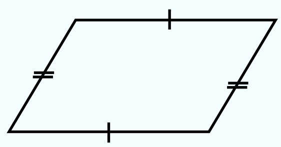
10. **Fill in the blanks :**

(12m, 16m, 25cm^2, 12cm^2)

1. If the length of a rectangle is 3 cm, and width 4 cm. then the area will be ....
2. If one side of a square plate is 4 m., the perimeter of the plate will be ....
3. If the length of a rectangle is 4 m. and width 2 m. the perimeter of the rectangle will be..........
4. If one side of a square is 5 cm, its area will be..........
5. Area of the square = ...
6. Area of the rectangle = ...
7. Perimeter of the square = ...
8. Perimeter of the rectangle = ...

Area of parallelogram:

Apart from square and rectangle, we also observe many other shapes in the surrounding environment. How will you find the area of a plot which is in the shape of a parallelogram?



A parallelogram is a quadrilateral whose opposite sides are equal and parallel. A parallelogram can be converted into a rectangle of equal area.

Draw a perpendicular from a vertex of the parallelogram to its opposite side. Cut out the triangle so formed and place it along the other side of the parallelogram

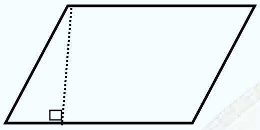


figure **1**

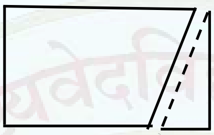


figure **2**

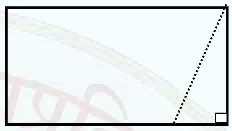


figure **3**

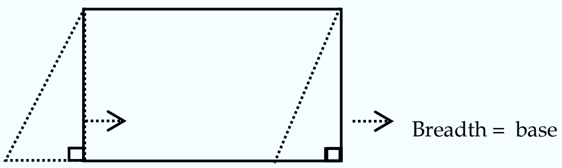
Guruji - What shape do you get in figure 3 ?

Vishal - We get the shape of a rectangle in figure 3.

Guruji - absolutely right! Is the area of the rectangle equal to the area of the parallelogram ?

Pratyush - Yes, because of the change in figure 1, the rectangle obtained in figure 3 also encloses the same (equal) area.

Guruji - Very good! Yes, we have seen that the length of the rectangle so formed is equal to the height of the parallelogram and the breadth of the rectangle is equal to the base of the parallelogram.



Length = Height

now,

Area of parallelogram = Area of rectangle

base × height = length × breadth

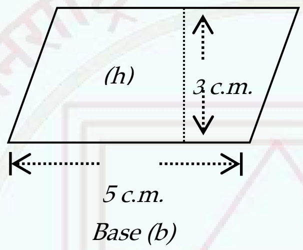
But the breadth (b) and length (l) of the rectangle are the base (*b*) and height (*h*) of the parallelogram respectively.

Thus, **area of parallelogram = area of rectangle**

base × height = length × breadth

= b × h

Example: Find the area of the figure given below.



Solution : We know that the above given figure is a parallelogram, in which the length of the base = 5 cm. And the corresponding height of the parallel sides = 3 cm.

Then, **area of parallelogram = base × height**

= 5 cm × 3 cm

= 15 $(cm)^2$

Therefore, the area of the above figure is 15 $(cm)^2$.

Example: One side of a parallelogram is 5 cm and the corresponding height is 6 cm. Find the area of the parallelogram.

Solution : Given

The length of the base of the parallelogram (b) = 5 cm.

Height (h) = 6 cm

Then, area of parallelogram = base × height

$$= 5 \text{ cm} \times 6 \text{ cm}$$

$$= 30 \text{ cm}^2$$

Example: The area of a parallelogram is $30(\text{m})^2$ whose one side is 10m, find the correponding height.

Solution : Given

Area of parallelogram = $30(\text{m})^2$and one side = 10 m

we know,

Area of parallelogram = bese × height

$30(\text{m})^2 = 10 \text{ m} \times \text{height}$

or height = $\frac{30(\text{m})^2}{10 \text{ m}}$

or height = 3 m

Area of triangle:

The description of the area of a triangle has been found in the below given mantra of Atharvaveda.

यो अक्रन्दयत् सलिलं महित्वा योनिं कृत्वा त्रिभुजं श्यान:।

वत्स कामदुघो विराज: स गुहा चक्रे तन्व: पराचै॥

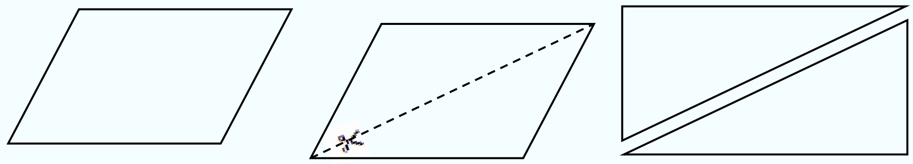
(अथर्ववेद : 8/9/2)

Meaning the space contained by the triangles is dependent on its base and height

This means that the area under the triangle remains constantAnd the basis of line mathematics is the property of area, that is, it is based on area only

To find the cost of planting grass in a triangular field, we need to find the area of the field. How to find the area of a triangular field ?

diagonal of a parallelogram divides the parallelogram into two triangles



You can understand by looking at the above figures that on drawing the diagonal of a parallelogram, two congruent triangles of equal area are obtained.

The sum of the areas of the two triangles is equal to the area of the parallelogram. The base and altitude height of a triangle are equal to the base and altitude of the parallelogram respectively.

Area of triangle $= \frac{1}{2}$(Area of parallelogram)

Area of triangle $= \frac{1}{2}(\text{base} \times \text{height})$

or area of triangle $= \frac{1}{2}(b \times h)$

The formula for the area of a triangle has been explained in the following verses of Aryabhatiyam and Lilavati Mathematics.

त्रिभुजस्य फलशरीरं समदलकोटीभुजार्धसंवर्गः।

(आर्यभट्टीयम्,गणितपाद : 6)

From the above verse, the area of a triangle is equal to half of the product of (base) and the perpendicular.

लम्बगुणं भूम्यर्धं स्पष्टं त्रिभुजे फलं भवति।।

(लीलावती गणित, 209)

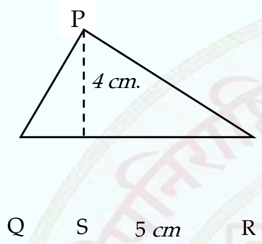
Means multiplying the perpendicular in the triangle by the (base) and halving it, (area) of the triangle is obtained.

On the basis of the interpretation of the verses of Aryabhatiyam and Lilavati Mathematics mentioned above, the formula of area of a triangle can be written in the following form.

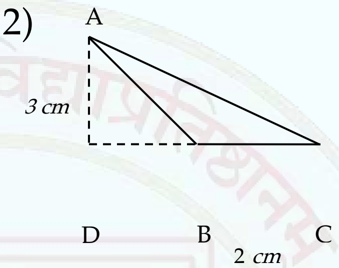
That is, **Area of triangle = $\frac{1}{2}$(base × height)**

Example: Find the area of the following triangles:

1)



2)



Solution : 1) Area of the triangle $= \frac{1}{2}(QR \times PS)$

$= \frac{1}{2}$ (5 cm × 4 cm)

$= (\frac{20}{2})$ cm 2

$= 10\ cm^2$

2) Area of the triangle $= \frac{1}{2}(BC \times AD)$

$= \frac{1}{2}$ (2 cm × 3 cm)

$= (\frac{6}{2})\ cm^2$

$= 3cm^2$

Example: Find the area of a triangle whose base is 6 cm and height is 4cm.

Solution : Given - Base of the triangle = 6 cm.

Height = 4 cm

We know that the **area of the triangle** $= \frac{1}{2}$ **(Base × Height)**

$$= \frac{1}{2}(6 \text{ cm} \times 4 \text{ cm})$$

$$= (\frac{24}{2}) \text{ cm}^2 = 12(\text{cm})^2$$

Example: Find the area of a triangular park whose base is 60 m and height is 20 m.

Solution : Given - Base of the triangle = 60 m

height = 20 m

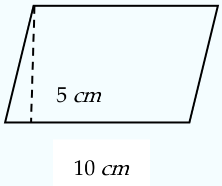
We know that the **area of the triangle** $= \frac{1}{2}$**(Base × Height)**

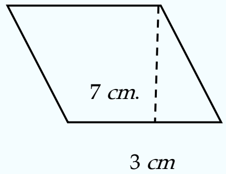
$$= \frac{1}{2}(60 \text{ m} \times 20 \text{ m})$$

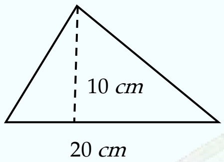
$$= 600(\text{m})^2$$

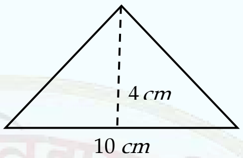
Exercise 9.2

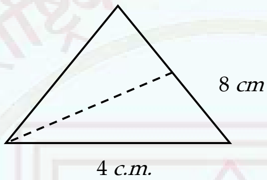
1. Find the area of parallelogram and triangle :

1) 

2) 

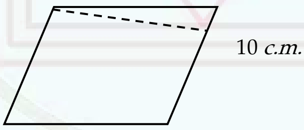
3) 

4) 

5) 

2. The base of a parallelogram is 7 cm, and the corresponding height of the parallelograme 8 cm. find the area of the parallelogram.

3. The area of a parallelogram is $30(\text{cm})^2$and the base is 10 cm, find the height.



4. Find the area of a parallelogram whose base and height are 5 m and 7 m respectively.

5. The base and height of a triangle are respectively 7 cm. and 2 cm. Find the area of the triangle.

6. Find the area of a triangle whose base is 10 m and height is 5 m.

7. In the following figure ΔABC, base BC = 8 cm. and Height AD = 4 cm. Find the area of ΔABC.

8. If height of the triangle is twice its base and base = 8cm, find the

area of the triangle.

9. Find the base and height and base of the ΔABC which are in ratio 3:2, if its area is 216 cm^2.

10. Find the base and height and base of the parallelogram which are in ratio 5:2, if its area is 640 m^2.

11. Complete the following table:

Sl.	Base	Height	Area Of Parallelogram
1)	10cm	……..	300cm^2
2)	……..	75cm	150cm^2
3)	15cm	……….	30cm^2

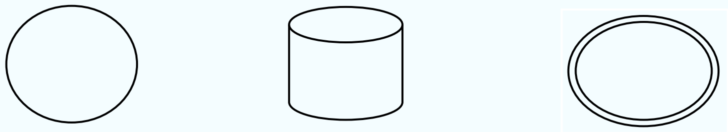
9. Fill in the blanks.

Base × Height, $\frac{1}{2}$(Base × Height), 50 $(cm)^2$,100 cm^2, two, half

1) Area of the triangle = ……………
2) The diagonal of a parallelogram divides it into................. triangles.
3) The area of a triangle is ………….. of the area of the parallelogram.
4) Area of parallelogram = ………………………
5) If base = 5 cm and height = 10 cm. The area of the parallelogram will be …………….
6) If base = 10 cm and height = 20 cm. the area of the triangle will be …………….

Circle :

You must have seen many circular shaped pictures in daily life :



Bangle WellRunning Track in the Playground

Apart from the above mentioned shape, if any shape is circular in shape and closed, it is called a circular shape.

Do and learn :-

Identify and mark (□) tick mark the circle in the following?

....

So we can say Circle is a closed planar figure which is a two-dimensional (2D) shape.

Definition of circle:

The locus of all the points that are equidistant from a fixed point is called a circle. This fixed point is called the center of the circle. Hence circle is a closed planar figure.

Circular

Circumference of circles:

Utsav cuts individual cards in a curved shape. He wants to decorate these cards with a border around them. What length of the decorating lace is required for each card.

Can you measure the perimeter (circumference) of the above figure with the help of a scale ?

The following mantra taken frome rigveda describes that the angle around the centre of the circle is 360 digree.

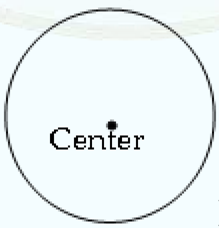
द्वादश प्रधश्चक्रमेकं त्रीणि नभ्यानि क उ तच्चिकेत् ।

तस्मिन्त्साकं त्रिशता न् शंकवोऽर्पिता षष्टिर्न चलाचलास:।।

(ऋग्वेद : 1/164/48)

That is, there is a wheel surrounded by twelve (twelve) rays, with three divisions (nabhi) known only to a scholar (mathematician). It is attached to 360 moving nails.

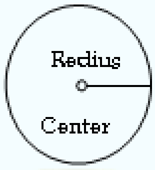
The center of a circle is bounded by a curved line which is closed around the point. The length of the curved line is called the circumference of the circle. In other words, the distance around the circular path is called the circumference of the circle and is denoted by 'C'.



Perimeter

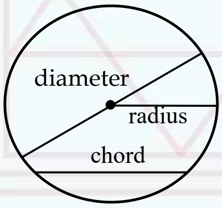
Radius of circle

The distance from the center of the circle to any point on the circumference of the circle is called the radius of the circle. The radius of the circle is denoted by ' r'.



Diameter of circle

The line segment joining any two points on the circumference and passing through the center of the circle is called the diameter of the circle. The diameter is denoted by 'D'. The diameter of a circle is twice its radius. The line segment joining any two points on a circle is called a chord of the circle. The diameter of a circle is the longest chord of the circle.



The ratio of the circumference and diameter of a circle is a constant ratio. It is denoted by π (Pi). Its value is $\frac{22}{7}$ or 3.14 approximately.

Regarding the value of π the following verse is found in Aryabhattiyam.

चतुरधिक शतमष्टगुणं द्वाषष्ठिस्तथा सहस्राणाम् ।

अयुत द्वय विष्कम्भस्यासन्ने वृत्त परिणाहः ॥

(आर्यभट्टीयम् गणितपाद : 10)

Meaning, the length of the adjacent circumference of two ayutas (20,000) finite diameter is 62,832. In 499 AD, Aryabhatta has given the value of π Pi more subtle way.

therefore, $\pi = \frac{62,832}{20,000} = 3.1416$

Formula: $\frac{\text{circumference (C)}}{\text{diameter (D)}} = \pi$

circumference = $\pi \times$ Diameter

or $C = \pi D$

or $= \pi \times 2r$

Thus, Circumference of the circle = $2 \pi r$

Example: If the radius of the circle is 5 cm, find the circumference of the circle.

Solution : Given: radius of the circle = 5 cm.

Then,we know that

circumference of the circle = $2\pi r$

$= 2 \pi \times 5$ cm

$= 2\times5\times \pi$cm

$= 10\pi$cm

Thus the circumference of the circle is 10πcm.

Example: If the radius of the circle is 14 cm, find the circumference of the circle.

[where $\pi = \frac{22}{7}$]

Solution : Given:

Radius of the circle (r)=14 cm.

Then, we know that,

Circumference of circle = $2 \pi r$

$= 2\times \pi \times 14$ cm(substituting o $\pi = \frac{22}{7}$)

$$= 2\times \frac{22}{7}\times 14 = 88 \text{ cm}$$

Hence, circumference of the circle = 88 cm.

Example: If the diameter of the circle is 30 cm, what will be the circumference of the circle ?

Solution : We know that Radius of circle (r) = $\frac{\text{Diameter (D)}}{2}$

$$= \frac{30 \text{ cm.}}{2}$$

$$= 15 \text{ cm}$$

Then, **circumference of the circle = 2 π r**

$$= 2\times \pi \times 15 \text{ cm}$$

$$= 30\pi \text{cm}$$

In another way, circumference of a circle = π × diameter

$$= \pi \times 30 \text{ cm}$$

$$= 30\ \pi \text{cm}$$

Area of circle

The area enclosed by the circle is called the area of the circle. It is denoted by 'A'. In Lilavati Mathematics, the following verse is found to find the area of the circle.

वृत्तक्षेत्रे परिधिगुणितव्यास पाद: फलं तत् ।

(लीलावती गणित : 41 अ,पृ.81)

Meaning, multiplying the circumference by the diameter and dividing it by four gives the area of the circle.

$$\text{Area of circle} = \frac{\text{circumference} \times \text{diameter}}{4}$$

$$= \frac{\pi \times \text{diameter} \times \text{diameter}}{4}$$

$$= \frac{\pi \times 2 \text{ radius} \times 2 \text{ radius}}{4}$$

Simplifying the above expression

Or Area of circle = $\pi \times (\text{radius})^2$

or area of circle = πr^2

Example: If the radius of the circle is 5 cm, find the area of the circle.

Solution : Given : radius of the circle = 5 cm.

we know that

Area of circle = πr^2

$= \pi\,(5\text{cm})^2$

$= 25\pi\text{cm}^2$

Substituting $\pi = 3.14$

$= 3.14 \times 25 = 78.5\ \text{cm}^2$

Example: Find the area of the circle of radius 3 cm.

Solution : Given: radius 3 cm.

Area of circle = πr^2

$= \pi\,(3\text{cm})^2$

$= \pi \times 9\ \text{cm}^2$

$= 9\pi\text{cm}^2$

Try:

1. If the radius of the circle is 3 cm., the diameter of the circle will be ....
2. Circumference of a circle of radius 6 cm is......... πcm.
3. The diameter is the .... of the radius of the circle.
4. If the diameter of the circle is 100 cm, the radius of the circle is ....

5. If the radius of the circle is 18 cm, the diameter is......... cm..

6. If the diameter of the circle is 13 cm, the radius of the circle will be

   …………….

7. Diameter of the circle =........................

8. Circumference of the circle =........................

9. Area of the circle =........................

10. Radius of the circle =........................

EXERCISE 9.3

1. Find the circumference of the circles of the following radii –

   (where $\pi = \frac{22}{7}$)

   1) 14 cm 2) 21 m. 3) 28 cm 4) 16 m.

2. Find the area of the circles of the following radii -

   1) 4 cm 2) 12 cm 3) 35 cm 4) 49 m.

3. If the radius of a circular flower garden is 6 m, find the area of the garden.

4. If the diameter of a circular sheet is 6 cm., find the perimeter and area of the sheet.

5. The radius of a circular pipe is 5 cm., what will be the required length of tape wrapping around the pipe ?

6. The circumference of a circle is 31.4 cm, find the radius and its area.

7. Find the distance travelled by Ram if the wheel of the vehicle of radius 42 m. Makes 3 revolutions.

8. Anuradha divides a circular sheet of radius 14 cm into two equal halves. Find the perimeter each half.

9. The diameter of a circular park is 15.6 m, find the area of the park.

10. From a circular sheet of radius 5 cm, a circle of radius 4 cm is cut and removed. Find the area of the left part of the sheet.

11. If the circumference of a circle is 31.4 cm, find its radius and area.

❖ **We learned :**

1. Perimeter is the distance around a closed figure while area is refers to the space enclosed by the closed figure.

2. Formulas for finding the perimeter and area of a square and a rectangle, learnt in the previous class:

    1. Perimeter of square = 4 × one side

    2. Area of square = side × side

    3. Perimeter of the rectangle = 2 × (length + breadth)

    4. Area of rectangle = length × breadth

3. Area of a parallelogram = base × height

    = b × h

4. Area of a triangle = $\frac{1}{2}$ base × height

5. Area of a triangle = $\frac{1}{2}$ (Area of the parallelogram)

6. The distance around a circular region is called its circumference.

    Circumference of the circle = 2 π r (Where r = radius.)

    Circumference of circle = π × diameter (diameter = 2×r)

7. The space enclosed by the circle is called the area of the circle.

    Area of circle = πr^2

    Here r is the radius of the circle.

Chapter 10

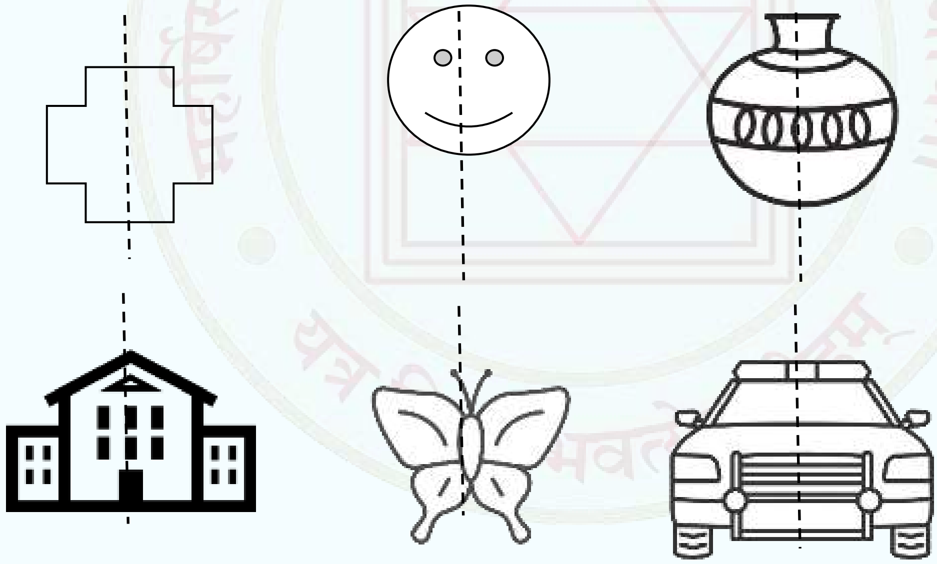
Symmetry

We often observe many pictures and objects around us in our daily lives, in which there are many geometrical shapes are found. Many a times, you must have seen objects which can be divided or cut into two equal halves coinciding with each other on supper-imposing one on the other. Such objects are called symmetrical objects.

In this chapter, we will learn about symmetry in detail.

Line symmetry

Look at the following pictures carefully –



The objects which are symmetrical about a line, can be folded about that line of symmetry such that both the parts overleap each other. Such objects are said to be having the line of symmetry.

On looking at the picture given above, Find out the answer to the following questions by discussing with your friends.

1. Into how many parts does the line drawn in the pictures divides the picture?
2. Are the divided parts equal to each other?
3. Can more such lines be drawn in the pictures? Try drawing the lines.

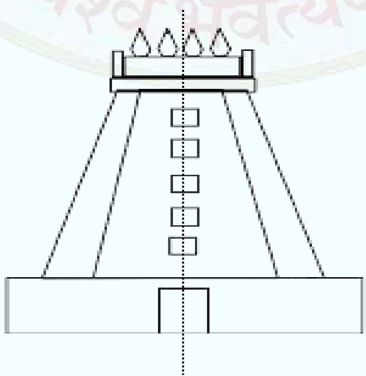
You will notice that the dotted line divides the picture into exactly two equal parts. Such lines are called lines of symmetry. And such figures are called symmetrical figures.

Do and learn

Draw the lines of symmetry on the following figures –

M E A

The line of symmetry divides the figure into equal parts. It can also be vertical, horizontal or oblique. When folded along the line of symmetry, the parts overlap each other exactly.

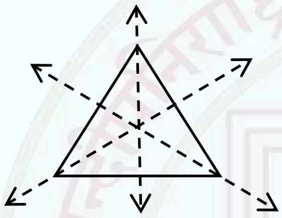
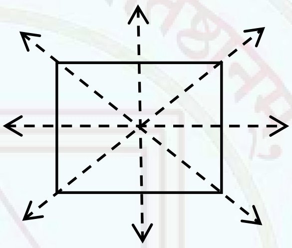


Generally, symmetrical shapes look more beautiful than asymmetrical shapes.

The picture of the temple shown above is more beautiful, because of its symmetry, where us asymmetrical shape pictures are not so beautiful.

More than one line of symmetry

Many pictures (figures) may have more than one line of symmetry. How many lines of symmetry can be drawn in the figures given below?

1)  2) 

It is clear from the above examples that a figure can have more than one line of symmetry. Recall the properties of equilateral triangles.

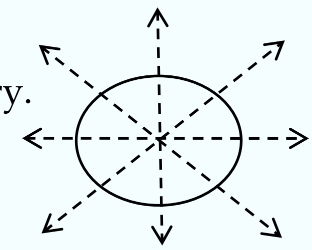
- All the sides are equal
- All the internal angles are equal

The interesting fact is that there are three lines of symmetry in an equilateral triangle and four lines of symmetry in a square.

Find the number of lines of symmetry in a regular pentagon and a regular hexagon

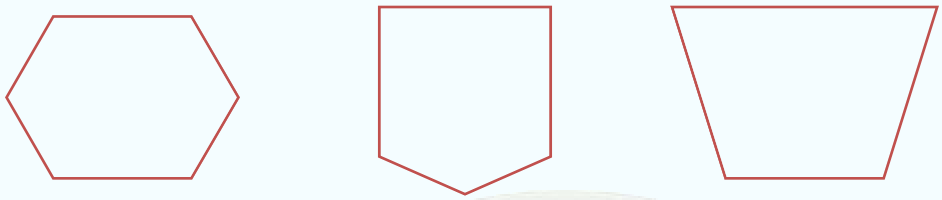
As the sides of a regular polygon increase, the number of lines of symmetry of the polygon also increases.

A circle has an infinite number of lines of symmetry.

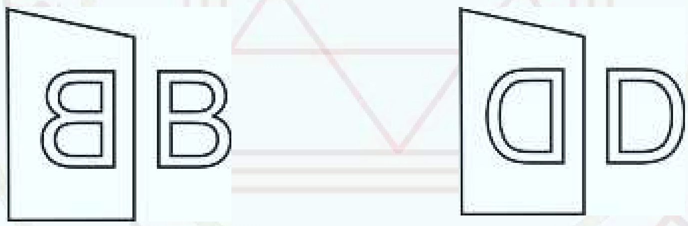


Try -

How many lines of symmetry can be drawn in the following figures?



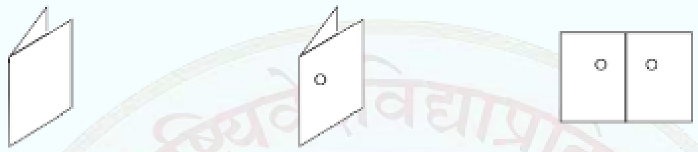
Reflection symmetry : Using a plane mirror, look at different objects and there reflections, individually. Keep a plane mirror on the figure such that half a fit is in front of the mirror. We can see that one half is in front of the mirror and the other half its reflection in mirror the combination of both gives us the complete picture this is called reflections symmetry. Look at the images formed in the mirror.



In the above pictures the mirror image is half of the picture. The edge of the mirror is the axis of symmetry thus the concept of line symmetry is closely related to the reflection symmetry. The edge of the mirror helps us to find a line of symmetry. The mirror reflection of B and D is shown in the following figures. In the mirror reflection of the above figure, there is a lateral inversion or right to left change of orientations and vice-versa.

Activity:-

Play the game of making symmetrical figures by cutting holes on a paper – Take a sheet of paper and fold it as shown below. Further draw a circle and cut it. Now, open the folded paper. The crease formed on folding is the line of symmetry and the holes on the two parts are symmetrical.

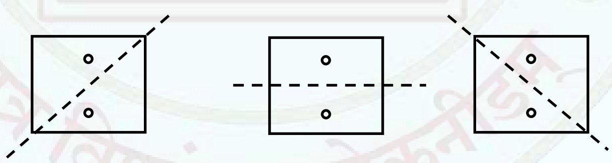


Fold a paper in half and make a hole | There are two holes | along the fold mark.

The turning mark is a line (or axis) of symmetry.

Try –

Find the line of symmetry by looking at a hole in a paper (choose the correct option)



....

Rotational symmetry

When the hands of a clock, blades of a ceiling fan or spokes of the wheel of a cycle move they make rotation. In certain objects, rotation is possible in both the directions. In a clock, the rotation occurs about a point and the direction in which it rotates is called clock wise

direction. The direction opposite to it is called anti-clock wise direction. The wheel of a cycle can be rotated in both the direction. You three more such object which can move in both directions. The point about which an object rotates is called centre of rotation.

Find the centre of rotation in a clock.

The angle turned during rotation is called the angle of rotation. We know that there is a rotation of 360º in one complete revolution.

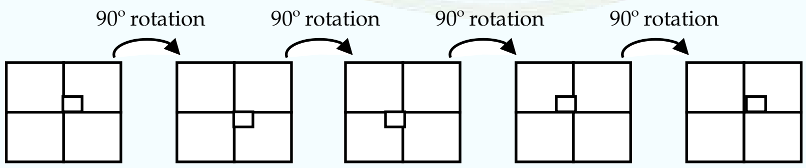
What are the measures of the angles of rotation in a half and one-fourth of a revolution?

Half or half a turn means a rotation of 180º and a quarter turn means a rotation of 90º. The rotation can be counterclockwise or clockwise. If an object after rotation, in terms of position, appears the same as before, we say that it has rotational symmetry.

The number of times an object appears the same position as before in one full rotation (360º) is called the order of that rotational symmetry.

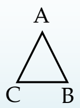
Let us understand the order of rotation of the square which is given below

Rotation of square

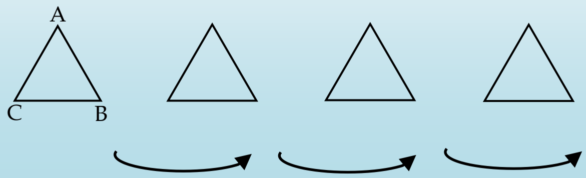


The square returns to its initial state four times in its full rotation. Hence its order of rotation is 4 and at every 90° it returns to its previous state. Hence the angle of rotation of the square is 90°.

Do and learn



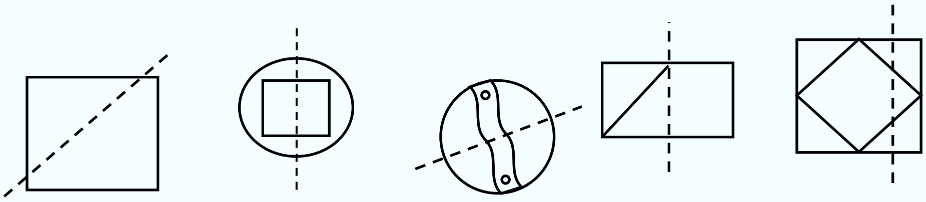
State the order of rotation of the triangle in the equilateral triangle in the clockwise direction.



Form the above answer we conclude that the equilateral triangle on rotating in clockwise direction, resembles the same for three times. Hence, its order of rotation is three. An equilateral triangle retains its position for every rotation of 120°. Hence, the angle of rotation of the equilateral triangle is 120°.

EXERCISE 10.1

1. State whether the dotted line shown in the figure below is the line of symmetry of the figure or not.

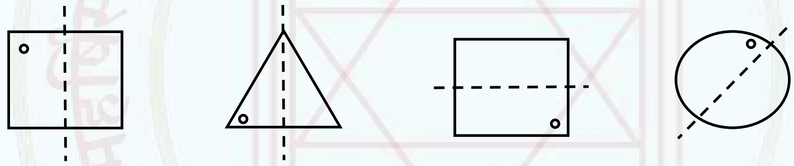


(a) (*b*) (c) (*d*) (*e*)

2. Draw the line of symmetry of the figures given below -

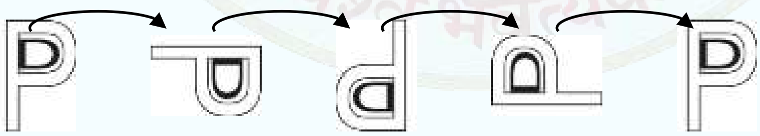
(a) (b) (c)

3. Given below are the lines of symmetry. Find the other holes.

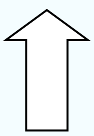


(a) (b) (c) (d)

4. Draw the direction of rotation, angle of rotation and order of rotation.



5. State the order of rotation of the following figure.



6. Write the names of two figures which has line symmetry and more than1 rotational symmetry.

7. Write the names of quadrilaterals which has line symmetry and more than1 rotational symmetry.

We learned

1. A figure has linear symmetry when a line can be drawn along which the two parts of the figure coincide with each other when the figure is folded.
2. Every regular polygon has as many lines of symmetry as the number of sides of the regular polygon.

Regular polygon	Regular pentagon	Square	Triangle
Number of lines of symmetry	5	4	3

3. In mirror reflection, there is a change of orientations from left to right.
4. In rotation, an object is rotated about a fixed point. The fixed point is called the center of rotation. The angle at which the object rotates is called the angle of rotation.
5. If the position of an object after rotation remains the same as before, we say that it has rotational symmetry.
6. The number of times an object (figure) appears in the same position as before in a complete rotation (360°) is called the order of that rotational symmetry.

Introduction and contribution of Indian mathematicians

❖ Bhaskaracharya (1114-1185):

Bhaskaracharya was born in 1114 AD in a Desashtha Rigvedi Brahmin family of Bijapur, Karnataka. His father Maheshwar (Maheshwar Upadhyay) was a mathematician, astronomer and astrologer who taught mathematics to Bhaskaracharya ji. From childhood, Bhaskar was very interested in mathematics and astronomy. After receiving his initial education, he continued in this work. Later, he also became the president of the institute of Astronomy, Jantar-Mantar of Ujjain. This Jantar Mantar was the biggest observatory of that time.

He used to live in the Sahayadri region (present day Jalgaon district, Maharashtra). He also taught, the knowledge received from his father, to his son Lok Samudra about mathematics and astronomy. Lok Samudra's son established a school in 1207 where the principles of his grandfather Bhaskara, were taught. Bhaskar composed Siddhant Shiromani when he was 36 years old. In this Siddhant Shiromani, he has presented the basic knowledge and formulas of many branches of mathematics. He composed Karan Kutuhal when he was 69 years old. Siddhant Shiromani, the main work of Bhaskaracharya, has mainly four parts. All these parts are branches of mathematics.

1. Lilavati Ganitam	2. Beejaganitam (Algebra)
3. Grahaganitam	4. Goladhyaya

❖ **Bibhuti Bhushan Dutta (1888-1958):**

He was a historian of Indian mathematics. He was educated at the University of Calcutta, from where he obtained a master's degree in various branches of mathematics, and he receved a sudent research fellowship. In days of youth it self, he was inclined towards sanyasa. In 1920, he was named Swami Vishnu Tirtha Maharaj. In 1921, for his dissertation work on in the field of hydrodynamics, he was awarded the D.S. C. Got the degree. After this, Under the influence of prof Ganesh Prasad, his interest turned towards the history of mathematics.

1926–35, Datta published more than fifty research papers, in which he collected and studied various aspects of Indian mathematics. During this time a large number of Pandu scripts were designed which formed the basis of books like History of Hindu Mathematics. In 1929, Datta resigned from the post of Professorship of University of Calcutta, mentioning this in a letter to Professor Karpinski in 1934 that he resigned in order to aspire to the life of a Vedanti i.e. the 'Eternal Self' living in Brahman. In 1931, he briefly returned to the University, to honour the whishes of Prof. Ganesh prashad, he Gave several lectures which were published as 'Science of Shulva' in (1932). He retired from the university in 1933 and took sanyasa in 1938 under the name of Swami Vidyaranya. He spent the last years of his life in Pushkar where he wrote several books on Indian philosophy in Bengali. Before leaving the university in 1929, he gave the manuscript of the book History of Hindu Mathematics: A Source Book to his junior colleague Awadhesh

Narayan Singh, who published it in two parts, Arithmetic and Algebra, in 1935 and 1938, respectively.

❖ **Malur Rangachari (1861-1916):**

He was a science graduate from the Christian College, Madras, who later became professor of Sanskrit at the Presidency College. He translated the works of Mahavira's Ganita Saar Sangrah with English translation and articles (1912). This was the first detailed account of Indian mathematical work since Cole Brooke's translation of Brahmagupta and Bhaskara in 1817.

Appendix

Names of numbers in the Indian number system

UNIT	1
TEN	10
HUNDREDS	100
THOUSAND	1,000
TEN THOUSAND	10,000
LAKH	1,00,000
TEN LAKH	10,00,000
CRORE	1,00,00,000
TEN CRORE	10,00,00,000
BILLION	1,00,00,00,000
TEN BILLION	10,00,00,00,000
TRILLION	1,00,00,00,00,000
TEN TRILLION	10,00,00,00,00,000
NIL	1,00,00,00,00,00,000
TEN NIL	10,00,00,00,00,00,000
PADMA	1,00,00,00,00,00,00,000
DUS PADMA	10,00,00,00,00,00,00,000
SHANKH	1,00,00,00,00,00,00,00,000
TEN SHANKH	10,00,00,00,00,00,00,00,000

आदर्श प्रश्नपत्र/ Model Que. Paper /गणित /

वेदभूषण द्वितीय-वर्ष / Vedabhushan Second Year/

कक्षा 7वीं/ प्रथमा - II / Class 7th / Prathama - II

विषय – गणित

पूर्णांक – 100 समय – 3 घण्टे

प्रश्न - 01. सही विकल्प के सामने (✓) चिह्न लगाइए – $10 \times 2 = 20$

Question - 01 Put a (✓) mark in front of the correct option -

1. पूरक कोण के सन्दर्भ में कौन-सा कथन सत्य नहीं है-

   Which of the following statements is not true with respect to complementary angles?

   (अ) 50 अंश के पूरक कोण का मान 30 अंश है।
   The value of the complementary angle of 50 degree is 30 degree.

   (आ) दो पूरक कोण का योग 90 अंश होता है ।
   The sum of two complementary angles is 90 degree.

   (इ) 80 अंश के पूरक कोण का मान 10 अंश है।
   The value of the complementary angle of 80 degree is 10 degree.

   (ई) 30 अंश एवं 60 अंश दोनों पूरक कोण के युग्म है।
   Both 30 degree and 60 degree is a pair of complementary angles.

2. मान ज्ञात करें - 40 का 20% = ...

   Find the value of - 20% of 40

   (अ) 10 + 20 + 50 - 10 (आ) 100 - 80 + 20

   (इ) 200 – 40 (ई) 20 + 20 + 50 - 10

3. निम्नलिखित में कौन-सा कथन सही है –

Which of the following statement is correct -?

(अ) 5 का गुणात्मक प्रतिलोम (–5) है।

The multiplicative inverse of 5 is (–5)

(आ) (-5) का गुणात्मक प्रतिलोम $\frac{1}{5}$ है।

The multiplicative inverse of (–5) is $\frac{1}{5}$.

(इ) 7 का गुणात्मक प्रतिलोम $\frac{1}{7}$ है।

The multiplicative inverse of 7 is $\frac{1}{7}$.

(ई) 7 का गुणात्मक प्रतिलोम $\frac{(-1)}{7}$ है।

The multiplicative inverse of 7 is $\frac{(-1)}{7}$.

4. $(-5) \times 0 \times (-1)$ का हल है –

$(-5) \times 0 \times (-1)$ is the solution -

(अ) $5 + 1$ (आ) $5 + 0$

(इ) 5×0 (ई) इनमें से कोई नहीं/ None of these

5. तुल्य भिन्न के सम्बन्ध में निम्न में से कौन-सा कथन गलत है –

Which of the following statements is incorrect regarding equivalent fractions -?

(अ) भिन्न $\frac{3}{5}$ की तुल्य भिन्न $\frac{5}{3}$ है।

The equivalent fraction of the fraction $\frac{5}{3}$ is $\frac{3}{5}$

(आ) $\frac{25}{15}$ की तुल्य भिन्न $\frac{5}{3}$ है।

The equivalent fraction of $\frac{25}{15}$ is $\frac{5}{3}$.

(इ) दो तुल्य भिन्न के क्रास गुणन परस्पर समान होते हैं।

The cross products of two equivalent fractions are equal to each other.

(ई) $\frac{30}{25} = \frac{6}{5}$

6. मापन के सम्बन्ध में निम्न में से कौन-सा सही हैं –

Which of the following can be written correctly regarding measurement?

(अ) 13 ग्राम = 13 कि.ग्रा.
13 grams = 13 kg.

(आ) 13 ग्राम = 0.13 कि.ग्रा.
13 grams = 0.13 kg.

(इ) 13 ग्राम = 0.013 कि.ग्रा
13 grams = 0.013 kg.

(ई) 13 ग्राम = 1.3 कि.ग्रा
13 grams = 1.3 kg.

7. निम्न में से सही नहीं है/Which of the following is not correct -

(अ) $2.4 + 0.2 = 2.6$

(आ) $2.4 \div 0.2 = 1.2$

(इ) $2.4 \times 0.2 = 0.48$

(ई) $2.4 - 0.2 = 2.2$

8. कथन (A) – दो पूरक कोण का योग 90° होता है ।

Assertion (A) - The sum of two complementary angles is.

कथन (R) – 30° का पूरक कोण का मान 60° है ।

Statement (R) - The value of the complementary angle of is.

(अ) A एवं R दोनों सही है । R, A की सही व्याख्या करता है ।
Both A and R are correct. R is the correct explanation of A.

(आ) A एवं R दोनों सही है । R, A की सही व्याख्या नही करता है ।
Both A and R are correct. R does not explain A correctly.

(इ) A सही है परन्तु R गलत है ।/A is correct but R is incorrect.

(ई) A गलत है परन्तु R सही है ।/A is wrong but R is correct.

9. निम्न में सही स्थति का चयन करें/Select the correct statement from the following -

(अ) लाभ = विक्रयमूल्य – क्रयमूल्य
Profit = Selling price – Cost price

(आ) लाभ = क्रयमूल्य – विक्रयमूल्य

Profit = Cost price – Selling price

(इ) लाभ = विक्रयमूल्य / क्रयमूल्य

Profit = Selling price / Cost price

(ई) लाभ = क्रयमूल्य / विक्रयमूल्य

Profit = Cost price / Selling price

10. निम्न कथनों को पढ़कर दी गये विकल्प में सही विकल्प का चयन करें -

Read the following statements and select the correct option from the given options -

(अ) किसी बन्द आकृति के चारों ओर की दूरी का योग परिमाप कहलाता है ।

The sum of the distances around a closed figure is called its perimeter.

(आ) किसी बन्द आकृति द्वारा घेरा गया स्थान, उस आकृति का क्षेत्रफल कहलाता है ।

The space enclosed by a closed figure is called the area of that figure.

(इ) दोनों (अ) एवं (आ) / Both (A) and (B)

(ई) इनमें से कोई नहीं / None of these

प्रश्न – 02. रिक्त स्थानों की पूर्ति कीजिए – $5 \times 2 = 10$

Question - 02. Fill in the blanks -

1. 345 मे 4 का एकाधिकेन पूर्वेण..................... होगा ।

Akhine Purven of 4 in 345 will be.....................

2. $\frac{2}{7}$ का व्युत्क्रम प्रतिलोम.............. होगा ।

The inverse inverse of $\frac{2}{7}$ will be.....................

3. संख्या 6×1000 का गुणनफल .... है ।

The product of the number 6×100 is.....................

4. समान्तर माध्य $= \dfrac{\text{.........................}}{\text{कुल आकडों की संख्या}}$ ।

Arithmetic mean $= \dfrac{\text{.............................}}{\text{Total number of figures}}$

5. प्रथम तीन अभाज्य संख्याओं का औसत............. होता है ।

The average of the first three prime numbers is....................

प्रश्न - 03. निम्नलिखित युग्मों के मिलान पर विचार कीजिए – **$5 \times 2 = 10$**

Question - 03. Consider matching the following pairs -

1.	13 का विचलन Deviation of 13	क.	माध्यिका Median
2.	4 cm. भुजा वाले वर्ग का परिमाप Perimeter of square with side 4cm.	ख.	आपस में समान होताहै। Equal to each other.
3.	मध्य मे आने वाले पद का मान Value of the middle term	ग.	16 cm.
4.	संगत कोण Corresponding Angle	घ.	180^{o}
5.	दो सम्पूरक कोण का योग Sum of two supplementary angles	ङ.	+3

उपर्युक्त युग्मों के आधार पर सही विकल्प का चयन कीजिए –

Select the correct option based on the above pairs -

(अ) (1) (ङ), (2) (ग), (3) (क), (4) (ख), (5) (घ)

(आ) (1) (घ), (2) (अ), (3) (ङ), (4) (ग), (5) (ख)

(इ) (1) (ङ), (2) (ग), (3) (घ), (4) (क), (5) (ख)

(ई) (1) (ङ), (2) (ख), (3) (क), (4) (ग), (5) (घ)

प्रश्न - 04. सत्य / असत्य कथन पर विचार कीजिए – $5 \times 1 = 5$

Question - 04. Consider the true / false statement -

1. दो संख्याओं का माध्य सदैव उनके बीच स्थित होता है ।

   The mean of two numbers always lies between them.

2. संख्या "0" न तो धनात्मक और ना ही ऋणात्मक परिमेय संख्या है ।

   The number "0" is neither positive nor negative rational number.

3. पूर्ण संख्या के समूह में 0 सम्मिलित होता है ।

   0 is included in the set of whole numbers.

4. 1005 का विचलन 1000 होता है ।

   The deviation of 1005 is 1000.

5. 28 का विनकुलन संख्या 3 होता है ।

   The Vinkulam number of 28 is 3.

उपर्युक्त कथनों को पढ़कर सही विकल्प का चयन कीजिए –

Read the above statements and choose the correct option -

(अ) (1) सत्य, (2) असत्य, (3) सत्य, (4) सत्य, (5) सत्य
(1) True, (2) False, (3) True, (4) True, (5) True

(आ) (1) असत्य, (2) असत्य, (3) सत्य, (4) असत्य, (5) सत्य
(1) False, (2) False, (3) True, (4) False, (5) True

(इ) (1) सत्य, (2) सत्य, (3) सत्य, (4) असत्य, (5) सत्य
(1) True, (2) True, (3) True, (4) False, (5) True

(ई) (1) असत्य, (2) सत्य, (3) असत्य, (4) असत्य, (5) सत्य
(1) False, (2) True, (3) False, (4) False, (5) True

प्रश्न - 05. अति लघूत्तरीय प्रश्न – $10 \times 2 = 20$

Question - 05. Very short answer type questions -

1. एक वर्गाकार कक्ष का परिमाप ज्ञात कीजिए जिसके कक्ष की लम्बाई 5 मीटर हो ।

   Find the perimeter of a square room whose length is 5 meters.

   ----

2. यदि 6 पुष्पमाला का मूल्य 60 रु. है तब ऐसी ही 10 पुष्पमाला का मूल्य क्या होगा ।

If the cost of 6 garlands is Rs. 60 then what will be the cost of 10 garlands?

3. 1 और 3 का समान्तर माध्य ज्ञात करें ।

Find the arithmetic mean of 1 and 3.

4. गुणफल ज्ञात करें – $(-1) \times (-5) \times (-1)$

Find the product - $(-1) \times (-5) \times (-1)$

----

5. सङ्कलन-व्यवकलन सूत्र का उपयोग कर योगफल ज्ञात करें –

Find the Sum using the sankalan-vyavakalan formula

75 + 33 + 89+ 18 + 28

----

6. 13% को साधारण भिन्न में बदलिये । / Convert 13% to a simple fraction.

----

7. 45° का पूरक कोण क्या होता है । What is the complementary angle of 45° ?

----

8. निम्न दिये कथन को समीकरण रूप मे लिखिए।

Write the following statements in equation form.

किसी संख्या में 4 को जोड़ने पर योगफल 16 प्राप्त होता है।

On adding 4 to a number, the sum is 16.

----

9. $3\bar{4}5\bar{3}$ विनकुलम संख्या को सामान्य संख्या मे लिखिए।

Write $3\bar{4}5\bar{3}$ Vinkulam number in general number.

----

10. हल करें/Solve : $(-3) \times (-2)$ [5 + 2]

प्रश्न - 06. लघूत्तरीय प्रश्न – **$5 \times 3 = 15$**

Question - 06. Short answer questions -

1. संख्या 5 एवं 13 का माध्य ज्ञात करें।

Find the mean of the numbers 5 and 13.

----

2. परिमेय संख्या की परिभाषा लिखिए ।

   Write the definition of rational number.

----

3. 5 : 7 के तीन तुल्य अनुपात ज्ञात करें ।

   Find three equivalent ratios of 5 : 7.

----

4. एक दुकानदार एक कुर्सी 250 रु. में क्रय कर ग्राहक को 300 रु. में विक्रय कर देता है, तब बताइये दुकानदार को कितनें रुपये का लाभ या हानि हुआ ।

A shopkeeper a chair purchase in Rs. 250. If he sells the customer by 300. Then how much profit or loss did the shopkeeper make.

----

5. यदि वृत्त का व्यास 14 सेमी. है तो वृत्त की त्रिज्या तथा क्षेत्रफल ज्ञात करें ।

If the diameter of the circle is 14 cm. Find the radius and area of the circle.

प्रश्न - 07. दीर्घ उत्तरीय प्रश्न – 4 × 5 = 20

Questions - 07. Long answer type questions -

1. योगे युतिः स्यात् क्षययोः स्वयोर्वा धनर्णयोरन्तरमेव योगः।

Yoge yutiḥ syāt kṣayayoḥ svāyorvā dhanarṇayorantarameva yogaḥ।

उक्त श्लोक की व्याख्या को समझाते हुए निम्न को हल करें –

Solve the following explaining the interpretation of the above shloka

+ 50 – 45 =

+ 45 + 56 =

– 30 + 40 =

----

2. शून्यर्णयोः खधनयोः खशून्ययोर्वा वधः शून्यम्।

Śūnyarṇayoḥ khadhanayoḥ khaśūnyayorvā vadhaḥ śūnyam।

उक्त श्लोक की व्याख्या को समझाते हुए निम्न को हल करें –

Solve the following explaining the interpretation of the above shloka

$$(+50) + 0 \quad = \quad (+50) + 0 \quad =$$

$$(-45) - 0 \quad = \quad \frac{(-45)}{0} \quad =$$

3. तिर्यक् छेदी रेखा द्वारा बनने वाले कोणों को स्पष्ट करें।

Explain the angle formed by the transversal.

4. π के मान से सम्बन्धित श्लोक की व्याख्या कीजिए।

Explain the verse related to the value of π.

Rashtriya Adarsh Veda Vidyalaya Run and Proposed by

MAHARSHI SANDIPANI RASHTRIYA VEDA VIDYA PRATISHTHAN, UJJAIN (M.P.)

(Ministry of Education, Government of India)

MAHARSHI SANDIPANI RASHTRIYA VEDA VIDYA PRATISHTHAN, UJJAIN (M.P.)

(Ministry of Education, Government of India)

Veda Vidya Marg, Chintaman, Jawasiya, Ujjian, Madhya Pradesh 456006

Phone : (0734) 2502266, 2502254, E-mail : msrvvpujn@gmail.com, website - www.msrvvp.ac.in