आज़ादी का अमृत महोत्सव

MATHEMATICS TEXTBOOK

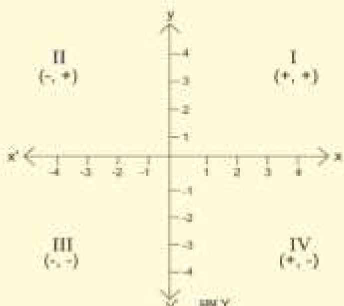
(On the basis of vedic Mathematics and Sanskrit Literature of Mathematics)

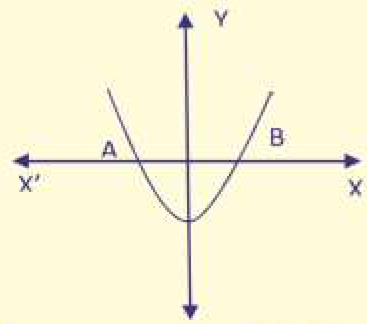
Veda Bhushan V Year / Purva Madhyama - II Year / Class X

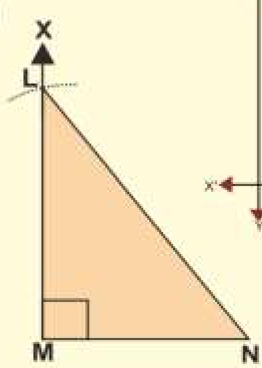
MAHARSHI SANDIPANI RASHTRIYA VEDA SANSKRIT SHIKSHA BOARD

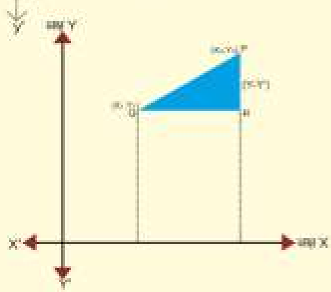
(Established and Recognized by the Ministry of Education, Government of India)

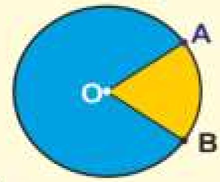
शून्यैषी निर्ऋते याजगन्धोत्तिष्ठाराते प्रपत मेह रंस्था: ।
वर्गाक्षराणि वर्गेऽवर्गेऽवर्गाक्षराणि कात् ङ्मौ यः।
खद्विनवके स्वरा नव वर्गेऽवर्गे नवान्त्यवर्गे वा ॥
नञावचश्च शून्यानि संख्या: कटपयादय: ।
मिश्रे तूपाऽन्त्यहल्संख्या न च चिन्त्यो हल: स्वरा: ॥
योगोऽन्तरं तेषु समानजात्योर्विभिन्नजात्योस्तु पृथक् स्थितिश्च।
भाज्याच्छेदः शुध्यति प्रच्युतः सन् स्वेषु स्वेषु स्थानकेषु क्रमेण।
यैर्यैर्वर्णैः संगुणो यैश्च रूपैर्भागाहारे लब्धयस्ताः स्युरत्र ॥
वर्गाहतरूपाणामव्यक्तार्धकृतिसंयुतानां यत् ।
पदमव्यक्तार्धोनं तद्वर्गविभक्तमव्यक्त: ॥
चतुराहतवर्गसमैरूपै: पक्षद्वयं गुणयेत् ।
अव्यक्तवर्गरूपैर्युक्तौ पक्षौ ततो मूलम् ॥
वृत्तव्यासस्य कृतेर्मूलं परिधिर्भवति दशगुणाया: ।
द्वादश प्रधयश्चक्रमेकं त्रीणि नभ्यानि क उ तच्चिकेत ।
तस्मिन्त्साकं त्रिशता न शंकवोऽर्पिता षष्टिर्न चलाचलास:।
आयाममायामगुणं विस्तारं विस्तरेण तु ।
समस्य वर्गमूलं यत्तत्कर्णं तद्विदो विदु: ॥

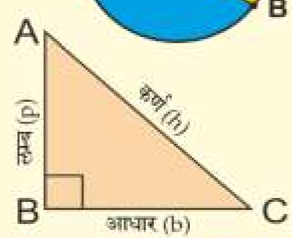












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dhvajāṅka: | vilokanam | ānurūpyeṇa | ūrdhvatiryagbhyām

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PREFACE

(In the light of NEP 2020)

The Ministry of Education (Department of Higher Education), Government of India established Rashtriya Veda Vidya Pratishthan in Delhi under the Chairmanship of Hon'ble Education Minister (then Minister of Human Resource Development) under the Societies Registration Act, 1860 (XXI of 1860) on 20th January, 1987. The Government of India notified the resolution in the Gazette of India vide no 6-3/85- SKT-IV dated 30-3-1987 for establishment of the Pratishthan for preservation, conservation, propagation and development of oral tradition of Vedic studies (Veda Samhita, Padapatha to Ghanapatha,Vedanga, Veda Bhashya etc), recitation and intonation of Vedas etc and interpretation of Vedas in scientific lines. In the year 1993 the name of the organization was changed to Maharshi Sandipani Rashtriya Veda Vidya Pratishthan (MSRVVP) and it was shifted to Ujjain, Madhya Pradesh.

The National Education Policy of 1986 and Revised Policy Formulations of 1992 and also Programme of Action (PoA) 1992 have mandated Rashtriya Veda Vidya Pratishthan for promoting Vedic education throughout the country. The importance of India's ancient fund of knowledge, oral tradition and employing traditional Guru's for oral education was also emphasized in the PoA.

In accordance with the aspirations of the nation, national consensus and policy in favour of establishing a Board for Veda and Sanskrit Education at national level, the General Body and the Governing Council of MSRVVP under the Chairmanship of Hon'ble Education Minister, Government of India, have set up "Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha

Board” (MSRVSSB) in tune with the mandate of the Pratishthan and its implementation strategies. The Board is necessary for the fulfillment of the objectives of MSRVVP as envisioned in the MoA and Rules. The Board has been approved by the Ministry of Education, Government of India and recognized by the Association of Indian Universities, New Delhi. The bye-laws of the Board have been vetted by Central Board of Secondary Education and curriculum structure have been concurred by the National Council of Educational Research and Training, New Delhi.

It may also be mentioned here that the committee “Vision and Roadmap for the Development of Sanskrit - Ten year perspective Plan”, under the Chairmanship of Shri N. Gopalaswamy, former CEC, constituted by the Ministry of Education Govt. of India in 2015 recommended for establishment of a Board of Examination for standardization, affiliation, examination, recognition, authentication of Veda Sanskrit education up to the secondary school level. The committee was of the opinion that the primary level of Vedic and Sanskrit studies should be inspiring, motivating and joyful. It is also desirable to include subjects of modern education into Vedic and Sanskrit Pathashalas in a balanced manner. The course content of these Pathashalas should be designed to suit to the needs of the contemporary society and also for finding solutions to modern problems by reinventing ancient knowledge.

With regard to Veda Pathashala-s it is felt that they need further standardization of recitation skills along with introduction of graded materials of Sanskrit and modern subjects so that the students can ultimately acquire the capabilities of studying Veda bhashya-s and mainstreaming of students is achieved for their further studies. Due emphasis may also be given for the study of Vikriti Patha of Vedas at an appropriate level. The members of the committee have also expressed their concern that the Vedic recitation studies are not uniformly spread all over India; therefore, due steps may be taken to improve the situation without in anyway interfering with regional variations of recitation styles and teaching method of Vedic recitation.

It was also felt that since Veda and Sanskrit are inseparable and complementary to each other and since the recognition and affiliation problems are same for all the Veda Pathashalas and Sanskrit Pathashalas throughout the country, a Board may be constituted for both together. The committee observed that the examinations conducted by the Board should have legally valid recognition enjoying parity with modern Board system of education. The committee observed that the Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain may be given the status of Board of Examinations with the name "Maharshi Sandipani Rashtriya Veda Sanskrita Vidya Parishat with headquarters in Ujjain which will continue all programs and activities which were being conducted hitherto in addition to being a Board of Examinations.

The promotion of Vedic education is for a comprehensive study of India's glorious knowledge tradition and encompasses multi-layered oral tradition of Vedic Studies (Veda Samhita, Padapatha to Ghanapatha,Vedanga, Veda Bhashy aetc), recitation and intonation, and Sanskrit knowledge system content. In view of the policy of mainstreaming of traditional students and on the basis of national consensus among the policy making bodies focusing on Vedic education, the scheme of study of Veda stretching up to seven years in Pratishthan also entails study of various other modern subjects such as Sanskrit, English, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, etc. as per the syllabus and availability of time. In view of NEP 2020, this scheme of study is with appropriate inputs of Vedic knowledge and drawing the parallels of modern knowledge in curriculum content focusing on Indian Knowledge System.

In Veda Pathashala-s, GSP Units and Gurukula-s of MSRVVP, affiliated to the Board transact the curriculum primarily based on oral tradition of a particular complete Veda Shakha with perfect intonation and memorization, with additional subsidiary modern subjects such as English, Sanskrit, Mathematics, Science, Social Science and SUPW. Gradually, the Veda Pathashala-s will also introduce other skill and vocational subjects as per their resources.

It is a well-known fact that there were 1131 shakha-s or recensions of Vedas; namely 21 in Rigveda, 101in Yajurveda, 1000 in Samaveda and 9 in Atharva Veda. In course of time, a large number of these shakhas became extinct and presently only 10 Shakhas, namely, one in Rigveda, 4 in Yajurveda, 3 in Samaveda and 2 in Atharvaveda are existing in recitation form on which Indian Knowledge System is founded now. Even in regard to these 10 Shakhas, there are very few representative Vedapathis who are continuing the oral Vedic tradition/ Veda recitation/Veda knowledge tradition in its pristine and complete form. Unless there is a full focus for Vedic learning as per oral tradition, the system will vanish in near future. These aspects of Oral Vedic studies are neither taught nor included in the syllabus of any modern system of school education, nor do the schools/Boards have the systemic expertise to incorporate and conduct them in the conventional modern schools.

The Vedic students who learn oral tradition/ recitation of Veda are there in their homes in remote villages, in serene and idyllic locations, in Veda Gurukulas, (GSP Units), in Veda Pathashala-s, in Vedic Ashrams etc. and their effort for Veda study stretches to around 1900 – 2100 hours per year; which is double the time of other conventional school Board's learning system. Vedic students have to have complete Veda by-heart and recite verbatim with intonation *(udatta, anudatta, swaritaetc),*on the strength of memory and guru parampara, without looking at any book/pothi. Because of unique ways of chanting the Veda mantras, unbroken oral transmission of Vedas and its practices, this has received the recognition in the UNESCO-World Oral Heritage in the list of Intangible Cultural Heritage of Humanity. Therefore, due emphasis is required to be given to maintain the pristine and complete integrity of the centuries old Vedic Education (oral tradition/ recitation/ Veda knowledge Tradition). Keeping this aspect in view the MSRVVP and the Board have adopted unique type of Veda curriculum with modern subjects like Sanskrit, English, Vernacular language, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture etc. as well as skill and vocational subjects as prescribed by NEP 2020.

As per Vedic philosophy, any person can become happy if he or she learns both *Para-Vidya and Apara-Vidya.* The materialistic knowledge from the Vedas, their auxiliary branches and subjects of material interest were called *Apara-Vidya*. The knowledge of supreme reality, the ultimate quest from Vedas, Upanishads is called *Para-Vidya.* In all the total number of subjects to be studied as part of Veda and its auxiliaries are fourteen. There are fourteen branches of learning or *Vidyas* - four Vedas, Six Vedangas, Mimamsa (Purva Mimamsa and Uttara Mimamsa), Nyaya, Puranas and Dharma shastra. These fourteen along with Ayurveda, Dhanurveda, Gandharvaveda and Arthashastra become eighteen subjects for learning. All curriculum transaction was in Sanskrit language, as Sanskrit was the spoken language for a long time in this sub-continent.

Eighteen Shilpa-s or industrial and technical arts and crafts were mentioned with regard to the Shala at Takshashila. The following 18 skills/Vocational subjects are reported to be subjects of the study– (1) Vocal music (2) Instrumental music (3) Dancing (4) Painting (5) Mathematics (6) Accountancy (7) Engineering (8) Sculpture (9) Cattle breeding (10) Commerce (11) Medicine (12) Agriculture (13) Conveyancing and law (14) Administrative training (15) Archery and Military art (16) Magic (17) Snake charming (18) Art of finding hidden treasures.

For technical education in the above mentioned arts and crafts an apprenticeship system was developed in ancient India. As per the Upanishadic vision, the vidya and avidya make a person perfect to lead contented life here and liberation here-after.

Indian civilization has a strong tradition of learning of shastra-s, science and technology. Ancient India was a land of sages and seers as well as of scholars and scientists. Research has shown that India had been a Vishwa Guru, contributing to the field of learning (vidya-spiritual knowledge and avidya- materialistic knowledge) and learning centers like modern universities were set up. Many science and technology based advancements of that time, learning methodologies, theories and techniques discovered by the ancient sages have created and strengthened the fundamentals of our knowledge on many aspects, may it be on astronomy, physics, chemistry, mathematics, medicine, technology, phonetics, grammar etc. This needs to be essentially understood by every Indian to be proud citizen of this great country!

The idea of India like “Vasudhaiva Kutumbakam” quoted at the entrance of the Parliament of India and many Veda Mantra-s quoted by constitutional authorities on various occasions are understood only on study of the Vedas and true inspiration can be drawn only by pondering over them. The inherent equality of all beings as embodiment of “sat, chit, ananda” has been emphasized in the Vedas and throughout the Vedic literature.

Many scholars have emphasized that Veda-s are also a source of scientific knowledge and we have to look into Vedas and other scriptural sources of India for the solution of modern problems, which the whole world is facing now. Unless students are taught the recitation of Vedas, knowledge content of Vedas and Vedic philosophy as an embodiment of spiritual and scientific knowledge, it is not possible to spread the message of Vedas to fulfill the aspiration of modern India.

The teaching of Veda (Vedic oral tradition/ Veda recitation/ Veda knowledge Tradition) is neither only religious education nor only religious instruction. It will be unreasonable to say that Vedic study is only a religious instruction. Veda-s are not religious texts only and they do not contain only religious tenets; they are the corpus of pure knowledge which are most useful to humanity as whole. Hence, instruction or education in Veda-s cannot be construed as only “religious education/religious instruction.”

Terming “teaching of Veda as a religious education” is not in consonance with the judgment of the Hon'ble Supreme Court (AIR 2013: 15 SCC 677), in Civil Appeal no. 6736 of 2004 (Date of judgment-3rd July 2013). The Vedas are not only religious texts, but they also contain the knowledge in the disciplines of mathematics, astronomy, meteorology, chemistry, hydraulics, physics, science and technology, agriculture, philosophy, yoga, education, poetics, grammar, linguistics etc. which has been brought out in the judgment by the Hon'ble Supreme Court of India.

Vedic education through establishment of Board in compliance with NEP-2020

The National Education Policy-2020 firmly recognizes the Indian Knowledge Systems (also known as 'Sanskrit Knowledge Systems'), their importance and their inclusion in the curriculum, and the flexible approach in combining various subjects. Arts' and Humanities' students will also learn science; try to acquire vocational subjects and soft skills. India's special heritage in the arts, sciences and other fields will be helpful in moving towards multi-disciplinary education. The policy has been formulated to combine and draw inspiration from India's rich, ancient and modern culture and knowledge systems and traditions. The importance, relevance and beauty of India's classical languages and literature is also very important for a meaningful understanding the national aspiration. Sanskrit, being an important modern language mentioned in the Eighth Schedule of Indian Constitution, its classical literature that is greater in volume than that of Latin and Greek put together, contains vast treasures of mathematics, philosophy, grammar, music, politics, medicine, architecture, metallurgy, drama, poetry, storytelling, and more (known as 'Sanskrit Knowledge Systems').These rich Sanskrit Knowledge System legacies for world heritage should not only be nurtured and preserved for posterity but also enhanced through research and put in to use in our education system, curriculum and put to new uses. All of these literatures have been composed over thousands of years by people from all walks of life, with a wide range of socio-economic background and vibrant philosophy. Sanskrit will be taught in engaging and experiential as well as contemporary relevant methods. The use of Sanskrit knowledge system is exclusively through listening to sound and pronunciation. Sanskrit textbooks at the Foundation and Middle School level will be available in Simple Standard Sanskrit (SSS) to teach Sanskrit through Sanskrit (STS) and make its study enjoyable. Phonetics and pronunciation prescriptions in NEP 2020 apply to the Vedas, the oral tradition of the Vedas and Vedic education, as they are founded upon phonetics and pronunciation.

There is no clear distinction made between arts and science, between curricular and extra-curricular activities, between vocational and academic streams, etc. The emphasis in NEP 2020 is on the development of a multi-disciplinary and holistic education among the sciences, social sciences, arts, humanities and sports for a multi-disciplinary world to ensure the unity and integrity of all knowledge. Moral, human and constitutional values like empathy, respect for others, cleanliness, courtesy, democratic spirit, spirit of service, respect for public property, scientific temper, freedom, responsibility, pluralism, equality and justice are emphasized.

The NEP-2020 at point no. 4.23 contains instructions on the pedagogic integration of essential subjects, skills and abilities. Students will be given a large amount of flexible options in choosing their individual curriculum; but in today's fast-changing world, all students must learn certain fundamental core subjects, skills and abilities to be a well-grounded, successful, innovative, adaptable and productive individual in modern society. Students must develop scientific temper and evidence based thinking, creativity and innovation, aesthetics and sense of art, oral and written expression and communication, health and nutrition, physical education, fitness, health and sport, collaboration and teamwork, problem solving and logical thinking, vocational exposure and skills, digital literacy, coding and computational thinking, ethics and moral reasoning, knowledge and practice of human and constitutional values, gender sensitivity, fundamental duties, citizenship skills and values, knowledge of India, environmental awareness etc. Knowledge of these skills include conservation, sanitation and hygiene, current affairs and important issues facing local communities, the states, the country and the world, as well as proficiency in multiple languages. In order to enhance the linguistic skills of children and to preserve these rich languages and their artistic treasures, all students in all schools, public or private, shall have the option of learning at least two years in one classical language of India and its related literature.

The NEP-2020 at point no. 4.27 states that -"Knowledge of India" includes knowledge from ancient India and its contributions to modern India and its successes and challenges, and a clear sense of India's future aspirations with regard to education, health, environment, etc. These elements will be incorporated in an accurate and scientific manner throughout the school curriculum wherever relevant; in particular, Indian Knowledge Systems, including tribal knowledge and indigenous and traditional ways of learning, will be covered and included in mathematics, astronomy, philosophy, yoga, architecture, medicine, agriculture, engineering, linguistics, literature, sports, games, as well as in governance, polity, conservation. It will have informative topics on inspirational personalities of ancient and modern India in the fields of medicinal practices, forest management, traditional (organic) crop cultivation, natural farming, indigenous sports, science and other fields.

The NEP-2020 at point no. 11.1 gives directions to move towards holistic and multidisciplinary education. India emphasizes an ancient tradition of learning in a holistic and multidisciplinary manner, including the knowledge of 64 arts such as singing and painting, scientific fields such as chemistry and mathematics, vocational fields such as carpentry, tailoring; professional work such as medicine and engineering, as well as the soft skills of communication, discussion and negotiation etc. which were also taught at ancient universities such as Takshashila and Nalanda. The idea that all branches of creative human endeavour, including mathematics, science, vocational subjects and soft skills, should be considered 'arts', has a predominantly Indian origin. This concept of 'knowledge of the many arts' or what is often called 'liberal arts' in modern times (i.e., a liberal conception of the arts) will be our part of education system.

At point No. 11.3 the NEP-2020 further reiterates that such an education system “would aim to develop all capacities of human beings -intellectual, aesthetic, social, physical, emotional, and moral in an integrated manner. Such an education will help develop well-rounded individuals that possess critical 21st century capacities in fields across the arts, humanities, languages, sciences, social sciences, and professional, technical, and vocational fields; an ethic of social engagement; soft skills, such as communication, discussion and debate; and rigorous specialization in a chosen field or fields. Such a holistic education shall be, in the long term, the approach of all undergraduate programmes, including those in professional, technical, and vocational disciplines."

The NEP-2020 at point no. 22.1 contains instructions for the promotion of Indian languages, art and culture. India is a rich storehouse of culture – which has evolved over thousands of years, and is reflected in its art, literary works, customs, traditions, linguistic expressions, artifacts, historical and cultural heritage sites, etc. Traveling in India, experiencing Indian hospitality, buying beautiful handicrafts and handmade clothes of India, reading ancient literature of India, practicing yoga and meditation, getting inspired by Indian philosophy, participating in festivals, appreciating India's diverse music and art and watching Indian films are some of the ways through which millions of people around the world participate in, enjoy and benefit from this cultural heritage of India every day.

In NEP-2020 at point no. 22.2 there are instructions about Indian arts. Promotion of Indian art and culture is important for India and to all of us. To inculcate in children a sense of our own identity, belonging and an appreciation of other culture and identity, it is necessary to develop in children key abilities such as cultural awareness and expression. unity, positive cultural identity and self-esteem can be built in children only by developing a sense and knowledge of their cultural history, art, language and tradition. Therefore, the contribution of cultural awareness and expression is important for personal and social well-being.

The core Vedic Education (Vedic Oral Tradition / Veda Path / Veda Knowledge Tradition) of Pratishthan along with other essential modern subjects- Sanskrit, English, Mother tongue, Mathematics, Social Science,

Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, Indian Art, Socially useful productive work etc., based on the IKS inputs are the foundations/sources of texts books of Pratishthan and Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board. These inputs are in tune with the NEP 2020. The draft books are made available in pdf form keeping in view the NEP 2020 stipulations, requirements of MSRVVP students and the advice of educational thinkers, authorities and policy of Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain. These books will be updated in line with NCFSE in future and finally will be made available in print form.

The Teachers of Veda, Sanskrit and Modern subjects in Rashtriya Adarsh Veda Vidyalaya, Ujjain and many teachers of Sanskrit and modern subjects in aided Veda Pathshalas of Pratishthan have worked for last two years tirelessly to prepare and present Sanskrit and modern subject text books in this form. I thank all of them from the bottom of my heart. Many eminent experts of the national level Institutes have helped in bringing quality in the textbooks by going through the texts from time to time. I thank all those experts and teachers of the schools. I extend my heartfelt gratitude to all my co-workers who have worked for DTP, drawing the sketches, art work and page setting.

All suggestions including constructive criticism are welcome for the improvement of the quality of the text books.

आपरितोषाद् विदुषां न साधु मन्ये प्रयोगविज्ञानम्।
बलवदपि शिक्षितानाम् आत्मन्यप्रत्ययं चेतः॥

(Abhijnanashakuntalam 1.02)

Until the scholars are fully satisfied about the content, presentation, attainment of objective, I do not consider this effort to be successful, because *even the scholars are not fully confident in the presentation without feedback from the stakeholders.*

Prof. ViroopakshaV Jaddipal

Secretary

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Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board, Ujjain

FOREWORD

On the Indian subcontinent, there has been a rich tradition of mathematics. Since ancient times in history, Indian scholars and mathematicians have excelled in this field. The knowledge of mathematics has been given the highest place even since the Vedic period. For example, there is a clear statement in the Yajurveda-

यथा शिखा मयूराणां नागानां मणयो यथा ।
तद्वत् वेदाङ्गशास्त्राणां गणितं मूर्द्धनि स्थितम् ॥

(याजुषज्यौतिषम्, 4)

Meaning, just as the peacock's feather is the highest on the bird and the gem is positioned highest among the snakes, so too is mathematics the highest position among all the Vedangas, or limbs of the Vedas.

This textbook combines mathematical ideas from the Vedic mathematical system and the Sanskrit knowledge system in an effort of developing current mathematics by applying information from antiquity and analyzing historical accomplishments side by side. Vedic mathematics has attempted to make calculations simpler.

In the current global scenario, considering the changing environment and to provide proficiency in mathematics education to Vedic students across the entire Indian subcontinent, the National Education Policy 2020 has outlined - in accordance with the main principles such as discussion, analysis, examples, and application, the development of curricula and textbooks tailored to Vedic students while keeping in mind the perspective of key principles.

The textbook's straightforward wording will make it easier for pupils to understand. The tenth-grade textbook, Vedabhushan Pustakam, is nearly identical to the mathematics curriculum for Class 10 in all of India. The textbook makes allusions to various writings from the Sanskrit knowledge system, including Brahmasphuta Siddhanta, Shulbasutras, Aryabhatiyam, Lilavati, and Bijaganitam, as well as to Vedic mathematical principles in a number of its chapters. Due to this integration, Vedic students are able to experience the depth of their Indian ancestry and understand both contemporary and traditional mathematical ideas. Building on the experiences of earlier classes, the textbook has been structured into 14 chapters in line with the requirements of the Vedabhushan Pustakam Year curriculum. Chapter 1 provides a detailed description of the history of mathematics. Chapter 2 elaborates on different types of collections under the concept of Samuchchaya. Chapter 3 explores the relationships between multipliers and multiplicands within the concept of polynomials, as well as the division of polynomials. Chapter 4 presents linear equations with two variables solved using Vedic methods of simultaneous and successive equations. Chapter 5 introduces quadratic equations. Chapter 6 discusses coordinate geometry. Chapter 7 presents mathematical concepts through various sutras in Vedic mathematics. Chapter 8 provides a detailed description of parallel lines. Chapter 9 teaches the construction of angles and triangles within practical contexts. Chapter 10 covers the area and circumference of a circle. Chapter 11 discusses Bodhayana's theorem and the ratio of trigonometry. The computation of mean, median, and mode under statistics is covered in Chapter 12. How to calculate the probabilities of favourable events is covered in Chapter 13. Number and logical aptitude tasks, including series, blood relations, direction and distance, sequence determination, clock problems, calendar problems, and figure computation, are covered in Chapter 14 under numerical and logical aptitude.

The textbook has a number of exercises designed to help Vedic students learn mathematics and improve their ability to find information again. The "Do and Learn" activities are designed to raise students' awareness and engagement level. Furthermore, to enhance students' comprehension and involvement, significant ideas and results are emphasized as "What We Learned" at the conclusion of every chapter.

The textbook ends with a discussion of the contributions made by Indian mathematicians in order to increase students' understanding of the rich traditions and mathematical achievements made by India. Students will learn from this about the important contributions Indian mathematicians have made to the field of mathematics.

Vedic students will be well-prepared to take competitive exams if they comprehend the mathematical principles covered in the textbook. Students are advised to read the NCERT textbooks for Class 10 and other subject-specific books in addition to the aforementioned textbook in order to expand their knowledge.

The author would be grateful for your constructive feedback to improve the textbook and address any errors.

CONTENTS

Chapter – 1

History of Mathematics

The meaning and importance of mathematics

Since the beginning, in Indian traditions, mathematics has been considered the top most Shastra's (sciences) among all the Shastra's.

यथा शिखा मयूराणांनागानांमणयो यथा।

तद्वद् वेदाङ्गशास्त्राणांगणितंमूर्ध्निसंस्थितम् ॥ (वेदाङ्गज्योतिष 2)

Meaning, just as the crest of the peacocks and the gem in the snakes are found at the highest place, similarly mathematics has the highest place in all the Vedangashastras. Even Acharyas has said that

बहुभिर्विप्रलापैः किंत्रैलोक्येसचराचरे ।

यत्किञ्चिद्वस्तुतत्सर्वंगणितेनविना न हि ॥

It means that: What is the benefit of many things? Whatever exists in all three worlds (loka) and in the entire creation is nothing without mathematics.

1. The word mathematics in sanskrit is derived from the root word 'Gan', (गण्) and adding the suffix 'Kat'(क्त) to it. The meaning of the root word Gan is to count and hence it means that, in mathematics calculations are done, in this sense the words Mit = 'measured', **'Mitoganito'** (Kautilya Arthashastra p. 110) and Sankhyat = counted, **'Sankhyatam Ganitam'** (Amarkosh) have also

been used sometimes, but in scriptures its name has mostly been 'Mathematics' (गणित).

2. According to Kautilya's Arthashastra, in ancient times the synonyms of mathematics were –

(1) Calculation (2) Numbers (3) Numerology

Origin of Mathematics:

Mathematics is one of the ancient texts (शास्त्र) of India. Adequate knowledge of mathematics is available in Vedic literature. The use of mathematics is universal everywhere. Everyone uses mathematics in their own way, as per their need to carry out their daily lives. In almost all the countries of the world, mathematics started with numbers and calculations; after some time, the calculations transformed into arithmetic, and after a long time, many branches of mathematics like algebra, trigonometry, geometry, etc. came into existence.

In the Indian tradition, Yajnas have been the source of mathematics and drama. To perform various types of Yajnas and to construct Vedic altars, proper calculations related to astrology, movement, position, etc. are required. From the manifestation of humans, the study of astrological objects or celestial bodies started. But the development of astronomy was possible only when mankind progressed a lot in calculations. Hence, the study of geometry and astronomy was necessary.

Development of mathematics:

Mathematical contemplation in Vedic literature:

In terms of calculations, words like Ganak, Gan, Ganya, etc. are found in the Rigveda. Mathematics falls under the field of astronomy. By performing yajnas, only at the right time were auspicious results obtained, and evils were eliminated. To know the time, astrology was required, and its correct knowledge was possible only through the Nakshatras, Veda, and mathematics related to planets. **The biggest contribution to Vedic literature is the invention of numbers and the decimal number system.** Many mantras in Vedic literature are found referring to numbers. Besides, in many Veda mantras, many big numbers like एक (one), दश (ten), शत (hundred), सहस्र (thousand), अयुत (ten thousand), etc. have been mentioned under the decimal number system. After the biggest numbers, references to innumerable or even infinite numbers are found in the Veda mantras.

असंख्याता सहस्राणि ये रुद्रा अधि भूम्याम् ।
तेषा॑ सहस्रयोजनेऽव धन्वानि तन्मसि ॥ (यजुर्वेद.16-54)

The words Anakhya Sahasra are also found in the above-mentioned mantra. The words about fractions are found in the Vedas such as- Ardha$\left(\frac{1}{2}\right)$, pad $\left(\frac{1}{4}\right)$, Shaff $\left(\frac{1}{8}\right)$ and kushat $\left(\frac{1}{12}\right)$ etc. Even and odd numbers are mentioned in the Taittiriya Samhita of Yajurveda (verses 11-20) and tables up to 100 are also available: as follows -

$4 \times 1 = 4$	$100 \times 1 = 100$	
$4 \times 2 = 8$	$100 \times 2 = 200$	
$4 \times 3 = 12$	$100 \times 3 = 300$	etc.

The largest numbers are mentioned in the following mantra of Taittiriya Samhita -

शताय स्वाहा सहस्रायस्वाहाऽयुताय स्वाहा नियुताय स्वाहा प्रयुतायस्वाहाऽर्बुदायस्वाहा न्यर्बुदाय स्वाहा समुद्राय स्वाहा मद्धयाय स्वाहाऽन्ताय स्वाहा परार्द्धाय स्वाहोषसे स्वाहा व्युष्टयै स्वाहोदेष्यते स्वाहोद्यते स्वाहोदिताय स्वाहा सुवर्गाय स्वाहालोकाय स्वाहा सर्वस्मै स्वाहा।

(तैत्तिरीय संहिता- 7/2/20)

10^{2} = hundred	10^{3} = thousand	10^{4} = ayut
10^{5} = nyuth	10^{6} = prayuth	10^{7} = abruth
10^{8} = Nyrbud	10^{9} = Samudra	10^{10} = Madhya
10^{11} = Anta	10^{12} = Paradha	10^{13} = Ushas
10^{14} = Vyushti	10^{15} = Deshyat	10^{16} = Udyat
10^{17} = Udit	10^{18} = Suvarga	10^{19} = Lok

General arithmetic operations like addition or division are found in very sophisticated form only in Vedic literature. In Taittiriya Samhita, the names of numbers up to 10 lok are given in the decimal number system; thus, the list of Taittiriya Samhita is not only a proof of the knowledge of the decimal number system but is also a proof of the scientific need for nomenclature for larger numbers.

" Mathematics " in Vedangas :

The first use of the word mathematics is found in Vedanga astrology; the author of **Ganitam Murdhani Sansthitam (गणितं मूर्ध्नि संस्थितम्),** is considered to be Acharya 'Lagadh'. According to him, the Vedas were created to perform yajnas, which are conducted at regular intervals. From the study of Vedanga astrology, it is known that astrologers knew how to do addition, subtraction, multiplication, and division at that time around 800 BC.

तिथिमेकादशाभ्यस्तां पर्वभांशसमन्विताम्।
विभज्य भसमूहेन तिथिनक्षत्रमादिशेत्॥

Meaning, multiply the tithi by 11, and add the festivals number (पर्व के भांश). Divide by the stars (नक्षत्र संख्या)number. In this way use of tithi (तिथि) is ascertained.

Under Vedanga astrology, the basic concepts of mathematics and operations on fractions are refered in Shulabasutras which were used for geometrical constructions. Consequently, the knowledge of fundamental of arithmetic operations were in advanced stage.

Relationship between time and mathematics:

The description of formation of 360° at the center by the circumference of a circle is clearly reflected in the following mantra of Rigveda.

द्वादश प्रधयश्चक्रमेकं त्रीणि नभ्यानि क उ तच्चिकेत।

तस्मिन्त्साकं त्रिशता न शंकवोऽर्पिताषष्टिर्न चलाचलासः।।

(ऋग्वेद- 1.164.48)

Meaning, there is a wheel in which twelve spokes are connected to its axis. It has three divisions (navels), with three-hundred and sixty (360) dynamic nails attached, which only a mathematician understands.

The above mantra describes a circular wheel whose axis is connected by twelve spokes. It has three equal divisions, it is attached (from the center) by three hundred and sixty moving nails, each division of this wheel has an angle of one hundred and twenty degrees at the center.

In Vedic literature, mathematics was widely used for daily works such as the construction of various types of altars for yajnas and the calculation of time. In fact, the work of calculating time on this earth was first started by the Vedic sages, who clearly recognized that the creator of time is the sun. According to a mantra of Rigveda (1.155.6)

"चतुर्भिः साकं नवतिं च नामभिश् चक्रं न वृत्तं व्यतींरवीविपत्।"

(ऋग्वेद- 1.155.6)

Meaning, the Sun; a driving force of rotation for the all 94 parts of time, rotates all of them in a circular form (like a round wheel).

Meaning, the Earth rotates in space. Earth, along with its water (Neer), rotates on its own axis and also moves in an orbit. It revolves around its originator (the sun). The sun's rays divide the day into thirty parts. The sun alone is the basis of our lives.

The following mantra of Yajurveda clearly states,

आयं गौः पृश्निरक्रमीदस दन् मातरंपुरः । पितरं च प्रयन्त्स्वः।।

त्रिङशद्धाम विराजति वाक् पतंगाय धीयते ।प्रति वस्तोरह धुभिः ।

(यजुर्वेद 3.6-7)

Meaning, the sun, a driving force of rotation for all 94 parts of time, rotates all of them in a circular form (like a round wheel).

It is clear from the above verse that the Sun's rays divide a normal day into thirty parts. In the context of modern time calculation, the value of this one part is twenty-four minutes. Author of Vedanga Astrology, Mahatma Lagadha (1500 BC) called the oldest unit of time as'Nadi'. The description of the then calendar system is found in a mantra of the first chapter of the Rigveda -

द्वादशारं नहि तज्जराय ववर्ति चक्रं द्यामृतम्य।

आ पुत्रा अग्ने मिथुनासो अत्र सप्त शतानि विंशतिश्च तस्थुः।।

(ऋग्वेद 1.164.11)

Meaning, this (Sun) wheel consisting of twelve spokes keeps rotating in the celestial world. This cycle never gets blocked or worns out. O Agnidev's! Seven hundred and twenty (720) sons (day and night) rest on this wheel.

It is clear from the above mantra that in Rigveda, a year is divided into twelve months and three hundred and sixty Ahoratras (day and night), from which we can understand that one month is equal to thirty ahoratras. Since the beginning of the Vedic period, the calendar system has been given great importance in India.

Thus, the conclusion is that many facts related to time are found in Vedic literature, the basis of which is mathematics. Hence the relation between time and mathematics can be seen in the Vedas.

Methematical thinking in Shulbasutra:

Shulbsutra: Meaning of Shulb: 'Thread or Rope'. The method of arranging various types of altars, अग्निचिति (Agnichiti), मण्डपम् (pavilions), etc. with the help of rope, it is called Shulbasutra. So far, we are aware of 8 shulbsutras. There are seven Shulbsutras in the Krishna Yajurveda: Baudhayana, Apastamba, Satyashadha, Vaadhul, Manav, Maitra Yani, Varaha and Katyayana Shulbsutra is the eighth Shulbsutra under Shukla Yajurveda.

In the Shulba period, mainly information about the measurement and arrangement of altars and अग्निचिति (Agnichiti), to perform yajnas, various methods of arrangement of bricks and their designs are given, such as Angul, Purusha, etc. dimensions and their interrelationships, for the arrangement of altars, Chiti, Mandap, such as rope, bamboo, cones, principles of geometry and many geometrical constructions, size and number of bricks, construction of Agnichiti, information about rules, etc. are found in Shulbsutra.

The below given explanation are found about the Knowledge of geometry in Shulbasutra.

There are three forms of Agni (fire). These three are: Garhapatya, Ahavaniya and Dakshinagni. Garhapatya is circular, Ahavaniya is square and Dakshinagni is semi-circular in shape. The interesting thing is that the area of all three should be equal. The knowledge of geometry is required to construct the above-mentioned altars of Yajna.

To explain these rules, which were prevalent for centuries, the sages composed Shulbasutras in which altars were constructed using ropes (रज्जू). At that time, the measurements were done with ropes. Today, they are done with a scale and compass. The word Shulbvigyan is used in the Manav and Maitrayani Shulbsutras. The definitional names of the fundamental processes of mathematics are at their peak in the Shulbasutras, examples of which are as follows:

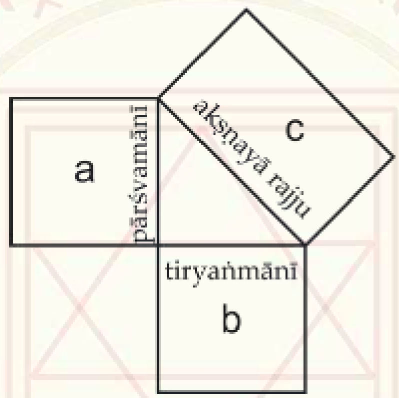
1. **Knowledge of Pythagoras theorem**:

Pythagoras theorem, which is basically known as the Baudhayan theorem. The statement of this theorem was first mentioned in the Baudhayana Shulbha Sutra (1-48).

दीर्घचतुरश्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी

च यत्पृथग्भूते कुरुतस्यद्उभयं करोति ।

(बौधायन शुल्बसूत्र 1.48)



Which means: In a long quadrilateral (rectangle), the area of the square dwran on its diagonal is equal to the sum of the area of the square dwran on its two sides. 460 years before Pythagoras (540 BC), the Baudhayana (1000 BC) fully propounded the above-mentioned theorem.

2. **Value of π:** The value of π becomes implicit in the context of drawing a circle equal to the square. It is said in the Manav Shulbasutra that a square of two hands, a circle of semi-diameter (radius), one hand, and three fingers are equal in area. If it is written in mathematical language, the following equation is obtained:

$$2^2 = \pi\left(\frac{9}{8}\right)^2$$

That is $$\pi = 4 \times \left(\frac{8}{9}\right)^2$$

$$= 4\times\frac{64}{81}$$

$$= 3.16049$$

Baudhayana in Shulbasutra π The value of (π) is given as 3.

यूपावटाः पदविष्कम्भाः त्रिपदपरिणाहानियूपोपराणीति ।

(बौधायनशुल्बसूत्र 1-82,83)

Converting a circle into a square is one of the major ancient problems. Baudhayan had given a rule to transform a circle into a square, in which

$$\pi = \left[1 - \frac{1}{8} + \frac{1}{8.29} + \frac{1}{8.29.6} + \frac{1}{8.29.6.8}\right]$$

$$= 3.0885$$

At another place, Baudhayan considered the value to be 3.088. This proves that there is a need to improve the precision of the values of (π) in its decimal digits. Still, this value was acceptable. After Archimedes gave the value of $\pi = \frac{22}{7} = 3.1428$. In 499 AD, aryabhatta had derived the value of pi (π) with greater precision from it.

Hence, $\pi = \frac{62,832}{20,000} = 3.1416$

3. The knowledge of Surd is mentioned in Apastamba Shulbasutra:

प्रमाणंतृतीयेनवर्धयेत्तच्चचतुर्थेनात्मचतुस्त्रिंशोनेनसविशेषः ।

(आपस्तम्बशुल्बसूत्र 1-12)

Meaning, $\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3.4} + \frac{1}{3.4.34}$

Till here, the knowledge of √ 2, √ 5 and so on had been acquired in doubling and multiplying squares.

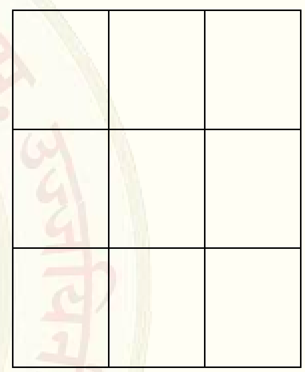
4. **Area of square:**

A reference to the area of a square is found in the Katyayana Shulbasutra, which is given below.

यावत्प्रमाणा रज्जुर्भवति तावतस्तावन्तो वर्गा भवति तान्त्समतस्येत् ।

(कात्यायन शुल्बसूत्र कण्डिका 3 -7)

meaning that the longer the rope is, the more units of rows are formed. By combining all of them, the area was calculated. For Example: In this picture, a 12-unit-long rope has made 3×3 squares (horizontally and vertically). So, by adding them, the area of the square became 9.



5. **Fraction:**

There was also knowledge about fractions during that time, which is given below.

अर्धप्रमाणेन पादप्रमाणं विधीयते, meaning

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Similarly - **अध्यर्धपुरूषा रज्जुर्द्वौ सर्पादौ करोति** i.e.

$$\left(1\frac{1}{2}\right)^2 = 2\ \frac{1}{4}$$

Invention of Zero:

The number zero is the base of arithmetic and has an important role. The biggest contribution of Vedic literature is the invention of numbers and the decimal number system, where the first use of '**zero**' is found in the following mantra of Atharvaveda.

शून्यैषी निर्ऋते याजगन्धोत्तिष्ठाराते प्रपत मेह रंस्थाः ।

अथर्ववेद (14/02/19)

The word **शून्यैषी** 'Shunyaishi' in the above-mentioned mantra has been used for someone who desires something that is not available. In Vedic literature, the largest number in terms of powers of ten is found in the mantras of Yajurveda (17/2), Taittiriya Samhita (7/2/20), and Atharvaveda (10/08/24), like Shat, Sahastra, Ayut, Niyut, Arbud, Nyarbud, etc. Gives the ten-fold value with respect to the previous numbers. On taking only the mantra of Taittiriya Samhita, according to local values, not only is there evidence of knowledge of the decimal number system, but it is also evidence for the scientific process of naming large numbers. In the decimal number system, even the largest numbers cannot be thought of without zero.

शताय स्वाहा सहस्रायस्वाहाऽयुताय स्वाहानियुताय स्वाहा प्रयुतायस्वाहाऽर्बुदायस्वाहा न्यर्बुदाय स्वाहा समुद्राय स्वाहा मद्ध्याय स्वाहाऽन्ताय स्वाहा परार्द्धाय स्वाहोषसे स्वाहा व्युष्ट्यै स्वाहोदेष्यते स्वाहोद्यते स्वाहोदिताय स्वाहा सुवर्गाय स्वाहालोकाय स्वाहा सर्वस्मै स्वाहा। (तैत्तिरीयसंहिता-(7/2/20)

Sign for Zero:

Regarding the **sign for Zero is found** in Rigveda under Vedic literature in the following mantra.

खे अराँ इव खेदया-   (ऋग्वेद : 8/77/3)

Meaning, in the Vedas, 'Kha' is used for a round hole in the sky. Understanding zero in mathematics is interesting. Whenever we look at the sky, we see the sky in the form of a sphere. According to the following shloka of the text Ganitasara sangrah, scholars, taking inspiration from the form of the sky, have defined the words synonymous with the void of the sky, giving the sign for zero:

आकाशं गगनं शून्यमम्बरं खं नभो वियत्   (गणितसारसंग्रह)

In mathematics, this zero, despite being in the form of nothing, is included in the number system. It has a different name and value. In the vedanga to understand the available knowledge, it is very important to know the Chanda shastra under the Vedanga which clarifies the Laghu and Guru Varnas of the Veda Mantras. first use of the '**sign** (symbol) **zero'** is found in the following sutras of Acharya Pingal's Chhand sutra.

रुपं शून्यम्   (पिंगल छन्द, अष्टमोऽध्याय: 29)

द्विः शून्ये   (पिंगल छन्द, अष्टमोऽध्याय:30)

According to the above formula, when the difference of an odd number in a circle is to be found or if an odd number appears while halving an even number, then on subtracting one number from the odd number, zero (0) is placed below the two digits written above. Do it. After dividing one number from the odd number and halving the even number, again write 2 (two) digits under the zero. Similarly, when an even number is reduced to half, two digits are added to it, and when an odd number is reduced, one is reduced to zero, until the desired number is reduced to one. After doing this, zero - in front of where zero is achieved. Double the number kept equal, i.e., on the right side.

It is clear from the facts of the above Vedic literature that the first use of the '**word zero**' is available in the Vedas, and the first use of the **'zero symbol'** is found in the Chhandasutra written by Acharya Pingal under the shad Vedangs.

Alphabetical (varnaak) system:

Note : It is necessary to know the alphabets of Hindi to understand these concepts.

When a number is expressed in the form of letters, it is called a '**code' or alphabet.** Mathematicians used this concept to express numbers. The use of alphabets or 'code numbers' in Sanskrit and Vedic literature is found in many places.

The alphabet system is explained in the following shloka of Aryabhattiyam (Dasgeetika Pada).

वर्गाक्षराणि वर्गेऽवर्गेऽवर्गाक्षराणि कात् ङ्मौ यः ।

खद्विनवके स्वरा नव वर्गेऽवर्गे नवान्त्यवर्गे वा ॥

(आर्यभट्टीयम्, दशगीतिकापाद -2)

Meaning, the square letters (which start with the letter 'क' till म) have been used in the square places and the non-square letters (य , र , ल , व , श , ष , स , ह) have been used in the non-square places. In this way ङ्मौ The value of (ङ + म) is the same as that of the letter य. Nine vowels have been used for the square of zeros and nine for the pairs having non-square space. Nine vowels (अ , इ , उ , ऋ , ऌ, ए , ऐ , ओ , औ) are used for nine square places (odd places) and nine non-square places, for places (even places) thus eighteen places have been mentioned for numbers. Aryabhatta propounded a uniquely original method of the alphabetic system, by using the letters of the alphabet to write large numbers in a shloka. According to this, aryabhatta gave the following values to the letters of the alphabet.

square (varg)

Character	No.	Character	No.	Character	No.	Character	No.	Character	no.
क	1	च	6	ट	11	त	16	प	21
ख	2	छ	7	ठ	12	थ	17	फ	22
ग	3	ज	8	ड	13	द	18	ब	23
घ	4	झ	9	ढ	14	ध	19	भ	24
ङ	5	ञ	10	ण	15	न	20	म	25

Non-square (Avarg)

य	र	ल	व	श	ष	स	ह
30	40	50	60	70	80	90	100

vowels	अ	इ	उ	ऋ	ऌ	ए	ऐ	ओ	औ
Square (varg)	10^0	10^2	10^4	10^6	10^8	10^{10}	10^{12}	10^{14}	10^{16}
Non-square (avarg)	10^1	10^3	10^5	10^7	10^9	10^{11}	10^{13}	10^{15}	10^{17}

Use of numerology in the form of alphabets:

It can be used only by those people who agree with each other and understand the codes, namely between the sender and the receiver. Let us determine the relation between word and the code derived from it. Let us confirm the code.

Corresponding vowel	क	ख	ग	घ	ङ	च	छ	ज	झ	ञ
Number	1	2	3	4	5	6	7	8	9	10

Use of words/characters for numbers is possible in the following ways-

(कमल = क्×अ + म्×अ + ल्×अ) $= 1 \times 10^0 + 25 \times 10^0 + 50 \times 10^1$

$= 1 \times 1 + 25 \times 1 + 50 \times 10$

$= 1 + 25 + 500 = 526$

ख्युघृ = खु +यु + घृ = ख्×उ + य्×उ + घ्×ऌ $= 2\times 10^4 + 30\times 10^4 + 4\times 10^6$

$= 2\times 10000 + 30\times 10000 + 4\times 1000000 = 43{,}20{,}000$

Aryabhatta, using Khayugra, has given the number of incarnations of Surya in one era as 43 lakh 20 thousand. In this method, the largest number is given in a few characters.

Do and learn:

Which number will be formed by converting the following words into numbers?

1. रमण =.............................. गणित = ………………….
2. Write the names of your family members in numbers and discuss with them.

katpayadi method:

This method was pervalent during 5th century. Suryadev, an interpreter of Aryabhatiyam, says that this Katpayadi method was known to aryabhatta (I) (473 AD). Aryabhatta, replaced this system with his own new system invented by him, which shows that this Katapyadi system existed before the fifth century. The formula for this method is found in the book 'Sadaratnamala' **-**

नञावचश्चशून्यानिसंख्या: कटपयादय: ।

मिश्रेतूपाऽन्त्यहल्संख्यान च चिन्त्यो हल: स्वरा: ।

Meaning, only (न, ञ) are indicative of vowel zero. Consonants starting with (क , त, प and य) Are represented by the numbers (1, 2, 3 etc.). In mixed consonants, only the last consonant with vowels is indicative of number. Do not consider the numbers for voiceless consonants. '**Ankana Vamato Gati' ('अंकाना वामतो गति:') means** Count the numbers from right to left. In this, place value processes (units, tens etc.) is also followed. Let us understand number 1 from the first letter of the square, number 2 from the second letter of the square etc. The form of Katpayadi system is:

Character		Number representation
क , ट , प , य	-	1
ख , ठ , फ , र	-	2
ग , ड , ब , ल	-	3

घ , ढ , भ , व	-	4
ड , ण , म , श	-	5
च , त , ष	-	6
छ , थ , स	-	7
ज , द , ह	-	8
झ , ध	-	9
ञ, न and only vowels	-	0

Examples of this method are found in records and donation papers etc. for example:

a) 2 4 4 1
रा घ वा य - 1442

b) 6 4 3 1
त त्वा लो के - 1346

भूतसंख्या पद्धति (Bhoot number System):

These are some common code words used for numbers in the system. (Their synonyms also have the same meaning.)

Number	Code-Word
0	शून्य, ख, अम्बर, गगन, नभ, वियत्, अनन्त
1	चन्द्र, इन्दु, विधु, सोम, अब्ज, भू, धरा, गो, रूप, तनु
2	यम, अश्विन, नेत्र, अक्षि, कर्ण, कर, पक्ष, द्वय, अयन, युगल
3	राम, गुण, त्रिगुण, भुवन, काल, अग्नि, त्रिनेत्र, लोक, पुर

4	वेद, श्रुति, सागर, वर्ण, आश्रम, युग, तुर्य, कृत, अय, दिश
5	बाण, शर, इषु, भूत, प्राण, तत्त्व, इन्द्रिय, विषय, पाण्डव
6	रस, अंग, ऋतु, दर्शन, अरि, तर्क, कारक, षण्मुख
7	नग, अग, पर्वत, ऋषि, मुनि, वार, स्वर, छन्द, द्वीप, धातु, अश्व
8	वसु, अहि, नाग, राज, सर्प, सिद्धि, भूति, अनुष्टुप्
9	अङ्क, नन्द, निधि, ग्रह, रन्ध्र, छिद्र, द्वार, दुर्गा
10	दिश्, दिशा, अंगुलि, पङ्क्ति, ककुभ्, रावणशिर, अवतार
11	रुद्र, ईश्वर, हर, ईश, भव, महादेव, अक्षौहिणी
12	रवि, सूर्य, अर्क, मास, राशि, व्यय, भानु, दिवाकर
13	विश्वेदेवा:, विश्व, काम, अतिजगती
14	मन, विद्या, इन्द्र, शक्र, लोक
15	तिथि, दिन, अहन्
16	नृप, भूप, भूपति, अष्टि, कला
17	अत्यष्टि
18	धृति, पुराण
19	अतिधृति
20	नख, कृति
21	उत्कृति, प्रकृति, स्वर्ग
22	आकृति
23	विकृति
24	गायत्री, जिन, अर्हत्, सिद्ध
27	नक्षत्र, उडु, भ

33	देव, अमर, सुर, त्रिदश
49	तान

Exercise 1.1

1. What is the meaning of the word mathematics?
2. The decimal number system is the contribution of which era?
3. What knowledge was required to make a Yajnavedi?
4. Give a brief introduction to the alphabetic system.
5. Throw light on the invention of zero.
6. What is the contribution of aryabhatta to the value of π ?
7. Explain the alphabetic number system and convert the names of five members of your family into numbers using the Aryabhatiyam number system.
8. Give a brief introduction to the Katpayadi method.
9. Write the name of the author of the text Vedic Mathematics and give a brief introduction of the text.
10. Match the following columns:

Aryabhatta	-	Vedic Mathematics
Bhaskaracharya	-	Pancha Siddhanta
Brahmagupta	-	Aryabhatiyam
Varahamihir	-	Siddhant Shiromani
Bharteeya Krishnatirtha	-	Brahmasphuta Siddhanta

Chapter -2

Sets

Dear students! In your Veda pathshala, after your morning prayers, you go to your respective classes. In each and every class, there are students. Such a collection of students, objects, or things is called a set. You must have observed many collections or sets in your daily life, such as

1 Set of vowels of the english alphabet
2 Constellation of planets
3 Set of days of the week
4 Set of natural numbers less than 15 etc.

Some sets which are not defined completly, such as.

1 Group of beautiful flowers
2 Group of tall students etc.

Set:

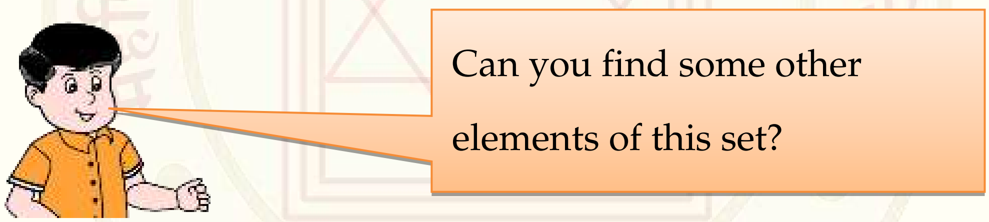
Set is defined as a well-defined group of objects are things. The meaning of a well-defined collection of objects is that we can tell whether an object belongs to that collection or not. For example the latter 'b' in not present in the set of vowels of the English alphabets, where as latter 'a' is present in the set. The number of vowels and their characteristics are well defined. Hence the group of vowels is well defined. Thus, it is a set.

We indicate the set by the capital letters of the English alphabet A, B, C, D, by … etc to represent a set. To represent the elements of a set we use small case letters such as a, b, c... etc.

If A is a set, every member is called its **element.** Suppose A is the set of all natural numbers smaller than 8, which means this set has 1, 2, 3, 4, 5, 6, 7 as its elements, then 5 is the element of this set. The representation of an element belonging to a set is done as $a \in A$

For example $7 \in A$ when A= {1, 2, 3, 4, 5, 6, 7}.

Note: The elements of a set are written in the brackets { }.



Is 10 an element of this set? The number 5 is an element of the set A. To represent this fact through a symbol, we write :

$$5 \in A$$

This is read as 5 elment of A which means 5 is an element of set A.

As we can see, 10 is not an element of this set, to represent this fact with a symbol we write.

$$10 \notin A$$

This is read as, " Set A does not have 10 as an element" or "10 is not an element of A''.

In general, if 'a' is an element of set A we write it as, $a \in A$ and read it as

"a is a element of A". If 'a' is not a element of the set A then we write it as $a \notin A$ and read it "a is not a element of A".

Representation of set –

To represent a set, the two methods given below are usually used.

1) roaster form
2) set builder Form

1) Roaster form –

To present a set in roaster form, we write all its elements individually sepraped by a comma (,) in brackets{}.

Example : If 1,3,5, 7, 9 are elements of the set A, we write –

$A = \{ 1,3,5, 7, 9 \}$

While expressing a set in roaster form, it is kept in mind that each element is written only once and no importance is given to the order in which the elements are written, thus the set {3,2,1},{2,1, 3}, {1, 2, 3}, {3,1, 2}represent the same set.

2) Set builder form – We use the set builder form only whine there is a common property for all the elements.

Example: For the set { a, e, i, o, u }The common property of each element of A is each one of them is a vowel of the English alphabets. We write this set by representing it with the symbol'V'.

$$V = \{x : x \text{ is a vowel in the English alphabet}\}$$

It is read "V is the set of all x such that x is a vowel of the English alphabet."Symbol: or / is read as " such that ".

Example :Some sets are given below in roaster form.

1. $A = \{ x \,/\, x$ is a natural number; and $5 < x \leq 10 \}$

   In this $A = \{ 6, 7, 8, 9, 10 \}$

2. $A = \{ x \,/\, x$ positive odd numbe $\}$

   In this $A = \{ 1, 3, 5, 7..... \}$

Finite set – A set which has finite (countable) number of elements is called a finite set.

Example :

1. $A = \{ 1, 2, 3, 4 \}$
2. B = { Prime Ministers of India }

infinite set:

An infinite set is a set which is not a finite set, that is, in which the number of elements is not uncountable (infinite).

Example: Set of all integers

$$Z = \{\ldots\ldots -2, -1, 0, 1, 2\ldots\ldots\}$$

$$P = \{ x : x \text{ is any number} \}$$

Empty Set or Null Set:

The set which does not have any element is called **an empty set,** it is represented by the symbol '$\emptyset$' (Phi). The empty set is expressed in roaster form with the symbol { }.

Example:

1 $A = \{ x : x \text{ is an odd natural number and } 7 < x < 9 \}$

This is the empty set because there is no odd natural number between 7 and 9.

2 $A = \{ x : x \text{ is the intersection point of two parallel lines} \}$

Subset:

When a set is formed using the elements of the given set, the set so formend is called subset of the given set.

Example:

$A = \{ a, e, i, o, u \}$ and $B = \{ a, b, c, d, e \ldots\ldots x, y, z \}$

We observe that Every element of the set A is an element of set B, which means the set A is a part of the set B. Hence, set A is a subset of set B. It is denoted by $A \subset B$, it is read as "A is a subset of B" and B is called **a superset** of A.

remember :

1. An empty set is a subset of every set.
2. Every set is a subset of itself.

Example : Write all the subsets of A = { 2, 3 }.

Solution : set A = { 2, 3 }

Subset ={ 2 } , { 3 } , { 2 , 3 } , { }

Example : If A = { a, b, c } and B = { b, c, a, d } then which one is correct

$A \subset B$ or $B \subset A$?

Solution : Given: set A = { a, b, c } and B = { b, c, a, d }

Observe that every element of set A is present in set B.

Hence, $A \subset B$

Union of Sets:

The union of two sets is, a set of elements whose every element is present in at least one of the two sets.

Suppose, A and B are two sets, now let us form a new set in which all the elements of A and B are included and apart from these there are no other elements, this new set is called the **union set** of A and B. It is called 'Union of sets'.

$$\mathbf{A \cup B = \{ x / \; x \in A, x \in B \}}$$

of A and B is denoted as '$A \cup B$' and it is read as 'A union B':

Example : If A = { 2 , 3 , 4 } andB = { 2 , 4 , 5 , 6 the union of the sets,

$$A \cup B = \{ 2 , 3, \; 4 , 5 , 6 \}$$

Intersection of Sets:

Let A and B be two sets. The set of common elements of both the sets is called their intersection set.The intersection of A and B is symbolized as 'A ∩B' and It is read as 'A intersection B'.

Thus,

$$A \cap B = \{ x \ / \ x \in A \text{ and } x \in B \}$$

Example : A = { 1 , 2 , 3 , 4 , 5 } and B = { 2 , 3 , 5 , 7 , 8 } find A ∩ B.

Solution: Given: A = { 1 , 2 , 3 , 4 , 5 } and B = { 2 , 3 , 5 , 7 , 8 }

We know that A∩ B is the set of common elements of both A and B.

in this the common elements of sets A and B are 2, 3, 5 .

hence,

A ∩ B= { 2, 3, 5 }

Universal set:

Consider the following sets.

Universal set (∪) = { 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 }

A = { 1 , 3 , 4 , 5 , 6 }   C = { 7 , 8 , 9 , 6 }

B = { 2 , 3 , 5 , 7 , 8 }   D = { 6 , 7 , 1 , 9 , 5 }

We observe that sets A, B, C, and D are the subsets of U. The set ∪ is called **the Universal set** of sets A, B, C and D.

The universal set of given sets is the set that contains all the elements or all the elements and more, which means all the given sets or the subset of the universal set. It is represented by the U.

Example : set A = { 2, 3 }, B = { 3, 4, 5 }, C = { 5, 6, 8, 11 } D = { 15, 17, 19, 2 }.

Write the universal set of A, B, C and D.

Solution : Given: set

A = { 2 , 3 }　　　B = { 3 , 4 , 5 }

C = { 5 , 6 , 8 , 11 }　　　D = { 15 , 17 , 19 , 2 }

we know that The sets A, B, C and D are subsets of the universal set ∪.

that is

∪ = { 2 , 3 , 4 , 5 , 6 , 7 , 8 , 11 , 15 , 17 , 19 }

Exerc ise 2.1

1. Select the correct option for the following multiple-choice questions.

(a) If A = {1, 3, 5, 7, 8} and B = {5, 7, 9, 11} the value of $A \cup B$ will be.

(I) {1, 3, 7, 9} (II) {1, 3, 5, 7, 8, 9, 11}

(III) {5, 7} (IV) {1, 3, 8, 11}

(b) If A = {2, 4, {5, 6}, 8} which of the following is false?

(I) $\{5, 6\} \subset A$ (II) $\{5, 6\} \in A$

(III) $8 \notin A$ (IV) $2, 4, 8 \in A$

(c) What is the number of subsets of the set A = {a, b}?

(I) 3 (II) 2 (III) 1 (IV) 4

(d) Sets A and B have 5 and 10 elements respectively. What will be the minimum number of elements in $(A \cup B)$?

(I) 15 (II) 10 (III) 8 (IV) 5

2. If A = { 3, 4, 6 } and B = { 1, 2, 3, 4, 5, 6 } out of $A \subset B$ or $B \subset A$ which one is correct?

3. If the set A = { 1,2,3 } and B = { 1, 2, 3, 4, 5, 6 } find the union and intersection of the sets?

4. Write all the subsets of the sets given below.

(a) {1, 2} (b) {3, 4, 5} (c) {5}

5. Write a set whose subset are given below.in

A) { a }, { a, b }, { b }, { ∅ }

b) { b }, { ∅}

c) { 1 }, { 4 }, { 5 }, { 4, 5 }, { 1, 5 }, { 1, 4 }, { 1, 4, 5 }, { ∅}

6. Find the union of the following sets.

a) Find (A ∪B) if the set A = { a, e, i, o, u } and B = { a, b, c }

b) Find (X∪Y) if the setX = { 1, 3, 5 } and Y = { a, b }

c) Find (A ∪B)if the set A = { a, 4, 5, 2, c } and B = { 1, 4, c }

d) Find (X∪Y) if the set X = { 1, 3, 5, 7, 7} and Y = { 1, 8, 9 }

e) If the set X = { 2, 0, 5 , 8, 8} and Y = { 1, 6 } then Find (X ∪Y).

7. Let A = { a, b }, B = { a, b, c } Is A⊂ B and A ∪ B = B?

8. Find the intersection set of the following sets.

a) A = { 1 , 3 , 4 , 5 , 6 } , B = { 1 , 3 , 5 , 7 }

b) A = { a , b , c , e , d , f }, B = { e , f , a , i , b }

c) A = { 4 , b , 3 , e , 5 , f }, B = { e , 2 , a , 4 , b }

d) A = { 1 , b , 3 , e , 5 , f }, B = { 1 , 2 , 3 , 4 , 5 , 6 , 7 ,f }

9. Find the universal set of the following sets.

a) A = { 1 , 3 } , B = { 2 , 3 , 4 } , C = { 3 , 4 , 5 , 6 }

b) X = { a , b } , Y = { 1 , 3 , 6 } , Z = { 5 , 7 , 6 }

c) A = { 0 , 5 } , B = { a , b , c , e } , C = { 3 , e , 5 , f }

d) X = { a , b } , Y = { 8 , 0 , 6 } , Z = { 5 ,e , 2 , a, 6 }

10. If the universal set U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }, B = { 6, 7, 8 } and A∪C = {1,2,3, 4,5,6} find the set (A ∪ B ∪ C).

11. If universal set U = {1, 2, 3, 4, 5, 6} subset A = {1, 2, 5}, B= { 3, 4, 5, 6} find the number of elements in the set (A ∪ B) ∪ (A ∩ B) .

12. If A = { 3, 4, 7, 8 }, B = { 1, 5, 6, 4, 3 }, C = { 4, 5, 9, 3, 8, 6 } find A∪B∩C .

We learned -

1) Collection of well-defined of objects is called a set.

2) **Representation of set -**

   a) Roaster form b) Set buildet form

3) **Finite set –** A set in which the number of members (elements) is finite is called a finite set.

4) **Infinite set –** A set which is not a finite set is called an infiniteset.Number of elements in it are infinite.

5) **Empty set of -**a set that has no members (elements) is called an empty set.It is represented by ∅ or { }.

6) **Subset -** If a new set is formed by choosing some or all the elements of the given set, the new set is called a subsetof the given set.

7) **Union of sets –**The union of two sets is a set whose every element is an element of at least one of the two sets.

   Example: A = { 2, 3 }, B = { 3, 4, 5 }

Then, $(A \cup B) = \{ 2, 3, 4, 5 \}$

8) **Intersection of sets:**The intersection of two sets is a setwhose every element is in both the sets.

Example : $A = \{ 3, 4, 5, 6 \}$, $B = \{ 5, 6, 7, 8 \}$

Then, $(A \cap B) = \{ 5, 6 \}$

9) **Universal set:** A set whose subsets are the given sets, is called the universal set of the given sets, it is represented by'$\cup$'.

Example : $A = \{ 1 , 2 , 3 \}$, $B = \{ a , b , c \}$

$C = \{ e , f , g \}$, $D = \{ 4 , 5 , 6 \}$

Then, $\cup = \{ 1 , 2 , 3 , 4 , 5 , 6 , a , b , c , e , f , g\}$

Chapter 3

Polynomial

Vedic students! You may remember that you studied polynomials in your previous classes. The algebraic expression in which the degree of the variable is a whole number is called **a polynomial**. For any polynomial the highest degree of the variable x in the polynomial is called the degree of the polynomial.

We can identify the polynomials on the basis of their degrees. For examplae:

1) In $4x + 3$, for variable x,is a polynomial of degree 1.
2) In $2x^2 + 3x + 4$, for the variable *x*,is a polynomial of degree 2.
3) In $4x^3 + 4x^2 + 5$, for the variable *x* is a polynomial of degree 3.

Linear polynomial:

The polynomial of degree 1, is called a linear polynomial.

Example : $3x + 5$, $4x$, $5x$, $6x + 7$

general form of linear polynomial: $ax + b$. where a, b is a real number and $a \neq 0$.

Quadratic polynomial:

The polynomial of degree 2 is called a quadratic polynomial.

Example : $5x^2$, $5x^2 + 3$, $4x^2 + 5x + 4$

The general form of the quadratic polynomial is $ax^2 + bx + c$. where a, b and c are real numbers and $a \neq 0$.

Cubic polynomial:

The polynomial of degree 3 is called a cubic polynomial.

Example: $5x^3$, $4x^3 + 2x + 1$, $4x^3 + 5x^2 + 4x + 1$

The general form of the cubic polynomial is $ax^3 + bx^2 + cx + d$. where a, b, c and d are real numbers and $a \neq 0$.

Zeros of polynomial

Now consider the value of x in the polynomial $P(x) = 3x + 2$.

$$3x + 2 = 0$$
$$3x = -2$$
$$x = \frac{-2}{3}$$

If we substitute $x = \left(\frac{-2}{3}\right)$in the polynomial $3x + 2$, we get $P(x) = 0$,

$$P\left(\frac{-2}{3}\right) = 3\left(\frac{-2}{3}\right) + 2$$
$$= -2 + 2$$
$$= 0$$

Since, $P\left(\frac{-2}{3}\right) = 0$, $\left(\frac{-2}{3}\right)$ is called the zero of the given polynomials (3x+2).

Hence, the zero of the linear polynomial $ax + b$ is -

$$\frac{-b}{a} = \frac{-\text{(constant term)}}{\text{cofficient of } x}.$$

Relationship between zeros and coefficients of a polynomial:

You have seen earlier that the zero of the linear polynomial (ax + b) is $\left(\frac{-b}{a}\right)$. The number of zeroes of a linear polynomial is only one.

In the quadratic polynomial ($ax^2 + bx + c$), the number of zeros is two. comprehensively if α,β are the zeros of quadratic Polynomial P(x) = $ax^2 + bx + c$ for $a \neq 0$,

Sum of zeros (α+ β) = $\frac{-b}{a}$

product of zeros(α β)= $\frac{c}{a}$

In this α and β are Greek letters.

Example: Find the zero of the polynomial $P(x) = 2x^2 - 8x + 6$.

Solution: Polynomial on factoring the polynomia, $P(x) = 2x^2 - 8x + 6$

$$2x^2 - 2x - 6x + 6$$

$$2x(x-1) - 6(x-1)$$

$$(2x-6)(x-1)$$

Then $x - 1 = 0$, $2x - 6 = 0$ or $x - 3 = 0$

when $x = 1$ or $x = 3$

Hence

$P(1) = 2(1)^2 - 8(1) + 6 = 2 - 8 + 6 = 0$

$P(3) = 2(3)^2 - 8(3) + 6 = 24 - 24 = 0$

Hence, the zeros of $2x^2 - 8x + 6$ are 1 and 3.

Remember:

In a quadrtic polynomial

1) Sum of zeros = $\frac{-\text{cofficient of x}}{\text{cofficient of } x^2}$

$$1 + 3 = \frac{-(-8)}{2}$$

$$4 = 4$$

2) Product of zeros = $\frac{\text{constant term}}{\text{cofficient of } x^2}$

$$1 \times 3 = \frac{6}{2}$$

$$3 = 3$$

Example : Find the zeros of (4x + 5).

Solution : Comparing with linear polynomial (ax + b) a = 4, b = 5.

We know, zero of linear polynomial is $\left(\frac{-b}{a}\right)$.

then,zero of $(4x + 5)$ is $\left(\frac{-5}{4}\right)$

Example: Find the zeros of 10x + 7.

Solution : Comparing with linear polynomial (ax + b) a = 10, b = 7.

We know,$\left(\frac{-b}{a}\right)$ is the zero of a linear polynomial.

Hence $\left(\frac{-7}{10}\right)$ is zero of the given polynomial.

Example: Find the sum and product of zeros of a following quadratic polynomials.

1) $2x^2 + 5x + 3$ 2) $3x^2 + 4x + 7$

1) $(2x^2 + 5x + 3)$

Solution : Comparing with general form $ax^2 + bx + c$, a = 2, b = 5, c = 3

we know that,

Sum of zeros $(\alpha + \beta)$ = $\frac{-b}{a}$ = $\frac{-5}{2}$

Product of zeros $(\alpha\beta)$ = $\frac{c}{a}$ = $\frac{3}{2}$

Hence, the sum of zeros of the given quadratic polynomial is $\left(\frac{-5}{2}\right)$and product of zeros is $\left(\frac{3}{2}\right)$.

2) $(3x^2 + 4x + 7)$

Solution: Comparing with general form $ax^2 + bx + c$, a= 3, b= 4, c = 7

we know,

Sum of zeros $(\alpha + \beta) = \frac{-b}{a} = \frac{-4}{3}$

Product of zeros $(\alpha\beta) = \frac{c}{a} = \frac{7}{3}$

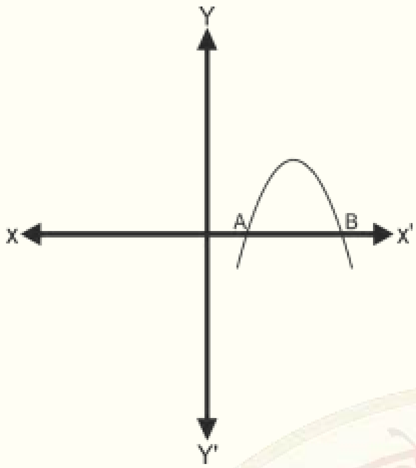
Hence,in the given quadratic polynomial, the sum of zeros is $\left(\frac{-4}{3}\right)$ and produc of the zeros is $\left(\frac{7}{3}\right)$.

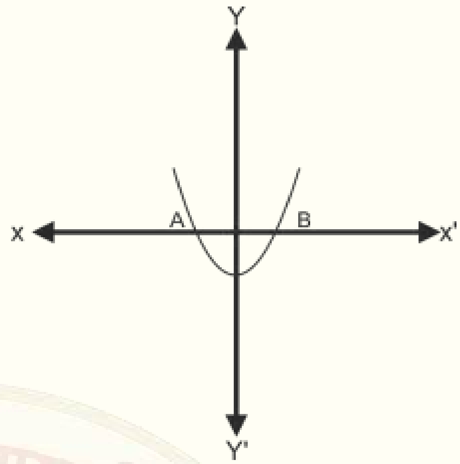
Do and learn

Find the sum of zeros of the Polynomial $(3x^2 + 4x + 7)$.

Geometrical meaning of zeros of polynomials:

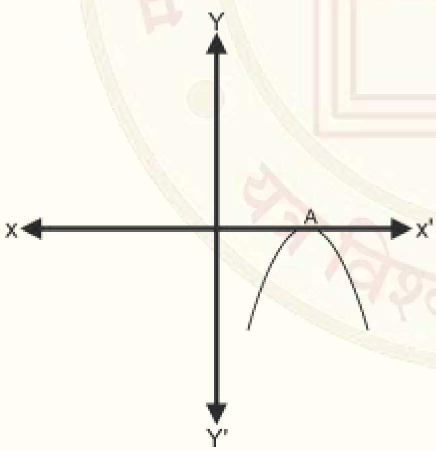
1. If the quadratic polynomial $p(x) = ax^2 + bx + c$ has two distinct zeros, the graph of the polynomial intersects the x - axis at two distinct point as shown below.

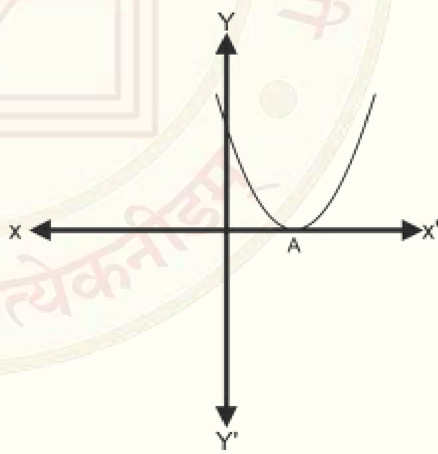




In this case the graph cuts the x- axis at two different points A and B.

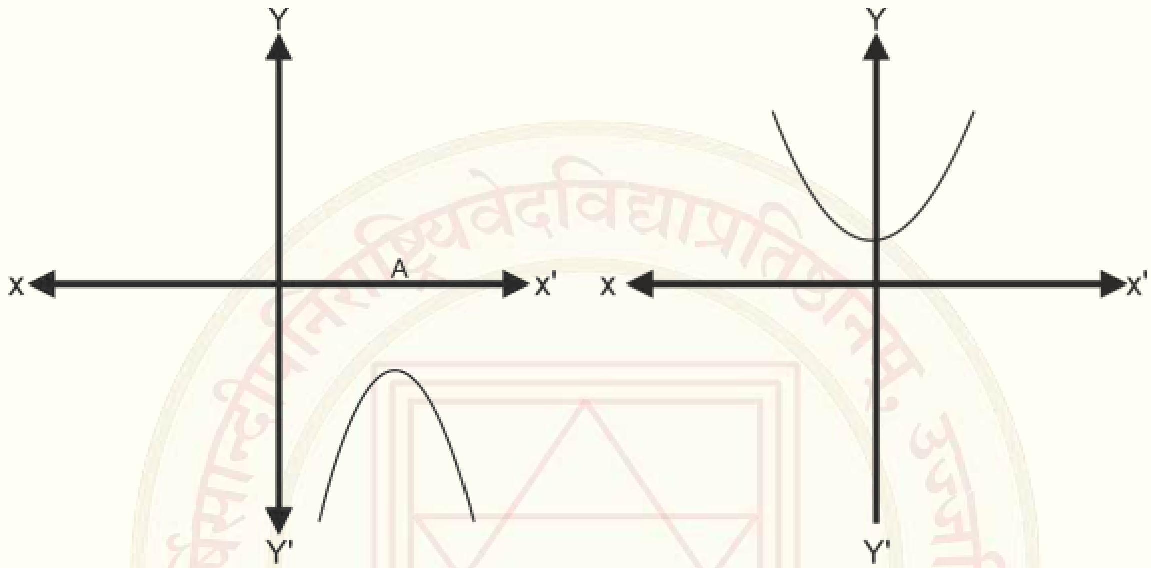
2. If the quadratic polynomial $p(x) = ax^2 + bx + c$ has two equal zeros, the graph of the polynomial intersects the x - axis at one point as shown below.





In this case the graph touches the x - axis at only one point A.

3. If quadratic polynomial $P(x) = ax^2 + bx + c$ has no real zero, the graph of the polynomial will not intersect the x −axis at any point.



the graph is either completely above the x -axis or completely below the x –axis, as it does not intersect the x -axis anywhere.

Exercise 3.1

1. Select the correct option for the following multiple-choice questions.

(a) A polynomial of degree one is called,

(I) Linear Polynomial (II) Quadratic polynomial

(III) Cubic equation (IV) None of these

(b) What is the number of zeroes of a quadratic polynomial?

(I) 2 (II) 3 (III) 4 (IV) 6

(c) If the zeros of the polynomial p (x) = $x^2 - 2x + 5$ are α and β, the value $\alpha\beta$ will be

(I) 5 (II) – 5 (III) 2 (IV) –2

(d) If the zeroes of the polynomial f (x) = $x^2 - 3x + 5$ are α And β, then $4(\alpha + \beta)$ = ----------

(I) –12 (II) 12 (III) 20 (IV) –20

(e) If the sum of zeroes of the polynomial $x^2 - nx + 5$ is 7, what is the value of n?

(I) –1 (II) 7 (III) -5 (IV) 3

(f) If the quadratic polynomial P (x) = $x^2 - x + 5$ has zeros α And β the value of $(\alpha + \beta)$ will be?

(I) –1 (II) 5 (III) 1 (IV) 0

(g) The sum of zeroes of the quadratic polynomial $4x^2 + 4x + 1$ will be–

(I) $-1/4$ (II) 1 (III) -1 (IV) $1/4$

2. Find the zeros of the linear polynomial.

   1) $3x + 7$ 2) $5x + 4$ 3) $8x + 5$ 4) $10x - 3$

3. Find the zeros, the sum of zeros and product of zeros of the following quadratic polynomials.

   1) $5x^2 + 3x + 8$ 2) $3x^2 + 10x + 7$

   3) $4x^2 + 7x + 3$ 4) $4x^2 + 9x + 5$

   5) $7x^2 + 11x + 4$ 6) $7x^2 + 14x + 2$

Operations on Polynomials (+, −, ×, ÷)

In our last classes, we learned about addition, subtraction, multiplication and division of polynomials. Let us once again revise these concepts. You will remember that in addition and subtraction, like terms are added or subtracted. In multiplication, the powers of the variables of the terms are added.

Addition:

In the following verse from beejaganitam, explains the addition and subtraction of algebraic expressions.

योगोऽन्तरं तेषु समानजात्यो-र्विभिन्नजात्योस्तु पृथक् स्थितिश्च ।

(बीजगणितम्,अव्यक्तसंकलनव्यकलने करणसूत्रं वृत्तार्धम्,पृ.20)

Meaning, letters are used to represent the variables quantities; the sum and difference of the variables quantities are between homogeneous terms; the terms that are not homogeneous (different) remain as they are.

Example: Find the sum of $5x + 3$ and $4x + 2$.

Solution: $(5x + 3) + (4x + 2)$

$= 5x + 3 + 4x + 2$

$= (5x + 4x) + (3 + 2)$

$= 9x + 5$

Example: Add the polynomial $(x^2 + 2x + 3)$ and $(x + 4)$.

Solution : $(x^2 + 2x + 3) + (x + 4)$

$= x^2 + 2x + 3 + x + 4$

$= x^2 + (2x + x) + (3 + 4)$

$= x^2 + 3x + 7$

Subtraction (difference)

Example : Subtract $(x^2 - x - 2)$ from the polynomial $(2x^2 + 5x + 4)$.

Solution : $(2x^2 + 5x + 4) - (x^2 - x - 2)$

$= 2x^2 + 5x + 4 - x^2 + x + 2$

Since :$-(-a) = a$

$= (2x^2 - x^2) + (5x + x) + (4 + 2)$

$= x^2 + 6x + 6$

Multiplication

Example : Multiply $(x - 7)$ by $(x + 5)$.

Solution: $(x + 5) . (x - 7)$

$$= x (x - 7) + 5 (x - 7)$$

$$= x^2 - 7x + 5x - 35$$

$$= x^2 - 2x - 35$$

Remember:

In addition and subtraction of polynomials, like terms are kept together and in multiplication, powers of the terms are added together.

Exercise 3.2

1. Solve the following polynomial.

   a) $(7x + 3) + (4x + 4)$

   b) Add the polynomials $(4x + 9)$ and $(3x + 3)$.

   c) Subtract$(x^2 - 3x - 4)$ from the polynomial $(3x^2 + 2x - 3)$.

   d) Subtract $(x^2 + x + 2)$ from the Polynomial $(2x^2 + 3x + 5)$.

2. Multiply the Polynomials –

   a) Multiply $(2x + 3)$ by $(4x + 5)$.

   b) Multiply $(x + 3)$ by $(4x^2 + 2x)$.

Division:

The following sholka related to division of polynomials is found in the Bijaganitam (Bhaghare Karansutra).

भाज्याचेद ः शुद्यायती प्रच्युत ः सं स्वेषु स्वेषु स्थानकेषु क्रमेन।

यैर्यायर्वर्नाई ः संगुनो याश्च रूपैर्भागाहारे लब्ध्यास्ता ः स्युरत्र॥

(बीजगणितम्,भागहारे करणसूत्रं वृत्तम्,पृ. 38)

In the method of division, the division method is clear according to this rule: "भाज्याध्दर शुद्धयति". When the divisor is subtracted from the dividend in the division of latent quantity, there is no remainder; hence, whatever quantity is included in the divisor is multiplied. That amount, is obtained as a quotient. We can also divide polynomials. How will you keep track of the terms and their powers while dividing?

Before thinking about all this, let us see when we need to divide polynomials. Consider the following situation:

Example : A car covers x km in 4 hours. Find the speed of the car?

Hint–

Speed= $\frac{\text{Distance}}{\text{Time}}$

Solution: Distance traveled by a car is x km.

And time taken to cover this distance = 4 hours

Since, **speed** = $\frac{\text{Distance}}{\text{Time}}$

speed = $\frac{x}{4} \frac{\text{KM.}}{\text{Hour}}$

This part is easy because the polynomial of single term is divided by a constant polynomial of single term

Example : Divide the polynomial $18x^2 + 9x$ by 3x.

Solution : To divide the polynomial $18x^2 + 9x$ x by 3x,we can write it as follows.

$$= \frac{18x^2}{3x} + \frac{9x}{3x}$$

$$= \frac{18x.x}{3x} + \frac{3x}{x}$$

$$= 6\ x + 3$$

In dividing a multi-term polynomial by a one-term polynomial, we can divide each term separately. using factorisation divide the polynomial $(18x^2 + 9x)$ by $3x$.

Division of$(18x^2 + 9x)$by 3xcan be written as follows.

$$= \frac{18x^2 + 9x}{3x}$$

$$= \frac{9x(2x+1)}{3x}$$

$$= 3\ (2x + 1) \quad = \quad 6x + 3$$

Let us take another example.

Example: Factoring the polynomial ($4x^4 + 12x^2$),divide it by $4x^2$.

Solution : Writing the division as follows-

$$= \frac{4x^4+12x^2}{4x^2}$$

$$= \frac{4x^2(x^2+3)}{4x^2}$$

$$= x^2 + 3$$

While dividing by factorisation, many times we are not able to recognize the factors.In such situations we use the long division method. In arithmetic, you know what it means to divide 25 by 4.

$$= \frac{25 \leftarrow}{4 \leftarrow} \frac{\text{Numerator}}{\text{Denominator}}$$

Divisible

$$4 \overline{)\,25\,}(6 \quad \text{Quotient}$$

Divisor

$$\underline{-\,24}$$

Remainder

in this $25 = 4 \times 6 + 1$

That is, **Dividend = divisor $\times$ Quotient+ remainder**

Remember : Dividing the dividend by the divisor we get a quotient and a remainder. If the division is complete, the remainder will be zero.

Example:

Divide the Polynomial ($2x^2 + 5x - 3$) by the polynomial ($x - 2$).

Solution:

$$
\begin{array}{c|c|c}
\text{divisor} & \text{Dividend} & \text{Quotient} \\
x - 2 & 2x^2 + 5x - 3 & 2x + 9 \\
 & {}_{-}2x^2 {}_{+} - 4x & \\
\hline
 & 9x - 3 & \\
 & {}_{-}9x {}_{+} - 18 & \\
\hline
 & 15 & \\
 & \text{remainder} &
\end{array}
$$

In this we get quotient = ($2x + 9$) and get remainder = 15.

Hint:

Steps of division of polynomials by a polynomial

step:1 Dividend and divisor will be written in descending order of their powers.

step:2 Dividing the first term of the dividend by the first term of the divisor.

In this $\frac{2x^2}{x} = 2x$

This will be the first term of the quotient.

step: 3 Multiply the divisor by this quotient and subtract the product from the dividend.

$2x(x-2) = 2x^2 - 4x$

$(2x^2 + 5x - 3) - (2x^2 - 4x)$

$= 2x^2 + 5x - 3 - 2x^2 + 4x$

$= 9x - 3$

Step 4 – Divide the first term of the result obtained after subtraction by the first term of the divisor.

$= \frac{9x}{x} = 9$ This will be the second term of the quotient.

Step 5- Again this quotient will be multiplied with the divisor.

That is $= 9(x-2)$

$= 9x - 18$

Now subtract $9x - 18$ from $9x - 3$.

$= (9x - 3) - (9x - 18)$

$= 9x - 3 - 9x + 18$

$= 9x - 9x - 3 + 18$

$= 15$

This process is repeated until the remainder becomes zero or the degree of the variable in the remainder becomes less than the degree of the variable in the divisor,in this example the remainder is 15, the degree of the variable in 15 is less than the degree of variable of $(x-2)$, The brief form of the long division is as follows-

$(2x^2 + 5x - 3) = (x - 2)(2x + 9) + 5$

Dividend= divisor ×quotient + remainder

Example :

Divied the Polynomial ($2x^3 - 12x^2 + 5x - 11$) by the polynomial($x - 5$).

Solution :

$$
\begin{array}{r|l|l}
x-5) & 2x^3 + 12x^2 + 5x - 11 & (2x^2 + 22x + 115 \\
 & 2x^3 - 10x^2 & \\
 & \underline{(-) \qquad (+)} & \\
 & \qquad 22x^2 + 5x - 11 & \\
 & \qquad 22x^2 - 110x & \\
 & \qquad \underline{(-) \qquad (+)} & \\
 & \qquad 115x - 11 & \\
 & \qquad 15x - 575 & \\
 & \qquad \underline{(-) \qquad (+)} & \\
 & \qquad \qquad \underline{564} &
\end{array}
$$

Thus, On dividing ($2x^3 - 12x^2 + 5x - 11$) by ($x - 5$), we get the quotient ($2x^2 + 22x + 115$) and the remainder (564) and we can write it in the following form.

$(2x^3 - 12x^2 + 5x - 11) = (x - 5) \times (2x^2 + 22x + 115) + (564)$

(which means dividend = divisor × quotient + remainder)

Another Vedic method for division of polynomials:

For the operation of division in polynomials, we can easily do the division of polynomials like the division of numbers using the formula of Vedic mathematics,'**Paravartya Yojayet**'.Let us understand with the following example.

Example: Divide $(x^4 + 2x^3 - 3x^2 + x - 1)$ by $(x - 2)$.

Solution: First construct the modified divisor K which is found by changing the sign of the constant term.

$$(x^4 + 2x^3 - 3x^2 + x - 1) \div (x - 2)$$

Dividend- $x^4 + 2x^3 - 3x^2 + x - 1$ divisor - $x - 2$

denominator	$x - 2$	+1	+2	-3	+1	-1
Reviseddenominator	+2	+2	+8	+10	+22	
		+1	+4	+5	+11	+21

Step 1: Add first, the divisor is revised by changing the sing of the constant term of the polynomial.

Step2: Write the coefficients of the terms of the divisor with their signs in descending order of exponents.

Step 3: Modified divisor is (+2). It has a single digit. Hence, draw a vertical line after the constant term (last term -1) of the dividend.

Step 4: The first term from the left of the dividend is the quotients first term. Write in the quotient.

Step 5: Find the product of the first term of the quotient and the modified divisor to write below the second term from the left of the dividend.

Step 6: +2 +2 = 4

Add the second terms and multiply with the modified divisor to write in the next term of the dividend. We write (+4) ×(+2) below −3.

Step 7: Now write (−3 + 8 = 5)below the third term.

Step 8: (+ 5) ×(+2) = + 10 will be written below +1

Add both and write below them (+ 1 + 10 = + 11).

Step 9: + 11× (+2) = 22 Is written below the number (–1) after the vertical line. Add the numbers (–1+22 = 21) is written below them.

Step10: The coefficients of the quotient are the numbers found between the two vertical lines.

Hence, +1, 14, + 5, +11are the coefficients of the terms of the quotient.

Thus, $x^3 + 4x^2 + 5x + 11$ is the quotient.

Step 11: The remainder is 21 which is found in the last column (after the vertical line).

Check :

Coefficients of dividend = Coefficients of divisor × Coefficients of Quotient + Coefficients of Remainder

In this left hand side = $1 + 2 - 3 + 1 - 1 = 0$

and right hand side = $(-1 \times 3) + 3$

$0 = -3 + 3$

$0 = 0$

Hence,left hand side = right hand side

Thus, the quotient is checked.

Example : Divide the polynomial $(4x^3 - 5x - 9)$by $(2x + 1)$

Solution:

In this the divisor : $2x + 1$

Dividend: $4x^3 - 5x - 9$

Writing the dividend in descending powers of the variable:

$4x^3 + 0x^2 - 5x - 9$

Dividing the divisor by 2, as in this method the highest coefficient of the variable of the divisor should be 1.

Therefore, $\frac{2x+1}{2} = x + \frac{1}{2}$

New divisor: $x + \frac{1}{2}$, Coefficients of the dividend in descending powers of x with the signs.

Divisor	$x+\frac{1}{2}$	+4	+0	−5	−9
Revised divisor	$-\frac{1}{2}$		−2	+1	+2
		+4	−2	−4	−7

Step 1: Modify the divisor.

Step2: Quotient's First term is +4

Step3: Revised divisor× quotient's first term

$-\frac{1}{2}\times 4=-2$ will be written below 0.

Step4: The quotients second term = +0 – 2 = – 2.

Step5: Write $-\frac{1}{2}\times(-2) = 1$ below the next term of the divident −5.

Step6: Quotients third term = −5 + 1 = −4.

Step7: Reviseddivisor × quotients third term

$= -\frac{1}{2}\times(-4)=+2$. Write below the number after the vertical line (-9).

Step 8. Is the remainder −9 +2 = −7.

Step9. +4, −2,−4 quotientof will be by divided 2 as the divisor was divided by 2.

Hence, the quotient is $\frac{1}{2}(+4 -2 -4) = 2 - 1 - 2$

Thus, quotient is $2x^2 - x - 2$ there and the remainder is −7.

Exercise 3.3

Divide:

1. Find the quotient and remainder by dividing $(x^2 - x + 1)$ by $(x + 1)$.
2. Find the quotient and remainder by dividing $(6x^2 - 5x + 1)$ by $(2x - 1)$.
3. Divide the Polynomial $(2x^2 + 3x + 1)$ by $(x + 2)$.
4. Divide the Polynomial $(x^3 - 4x^2 + 7x + 10)$ by$(x - 1)$.
5. Divide the Polynomial $(3x^2 + x^2 + 2x + 5)$ by $(x^2 + 2x + 1)$.
6. Prove that the remainder is zero when the polynomial $(2x^3 + x^2 - 5x + 2)$ is divided by $(x + 2)$.

We learned -

1) If the highest degree of the variable in a polynomial is 1, 2 and 3, they are called linear polynomial,quadratic polynomial and cubic polynomials respectively.
2) The general form of a quadratic polynomial is $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.
3) A quadratic polynomial will have at most two zeros.
4) If α and β are the zeros of the quadratic polynomial $ax^2 + bx + c$,

Sum of zeros $(\alpha + \beta) = \frac{-b}{a}$

Product of zeros $(\alpha\,\beta) = \frac{c}{a}$

5) The process of division in polynomials is slightly different from arithmetic. In this, the degree of the variable has to be kept in mind.

6) To divide polynomials, the powers of the dividend and divisor are written in descending order.

7) Long division method is used to divide polynomials.

8) In the long division method, the process of division is repeated until the remainder becomes zero or the degree of the variable of the remainder becomes less than the degree of the variable of the divisor.

Brief form of Division

$$\textbf{Dividend} = \textbf{divisor} \times \textbf{quotient} + \textbf{remainder}$$

Chapter 4

Pair of linear equations in two variables

In ancient times, equations with many variables were studied in detail in algebra. Mathematicians of that time like Brahmagupta, Mahaviracharya, Bhaskaracharya, etc. had provided solutions to the question. The great mathematician Bhaskaracharya II has said:

पूर्वं प्रोक्तं व्यक्तमव्यक्तबीजं प्रश्ना नोविनाऽव्यक्तयुक्त्या ।

ज्ञातुं शक्या मन्धीभिर्नितान्तं यस्मात्तस्माद्वच्मि बीजक्रियां च ॥

(बीजगणितम्,मङ्गलाचरण: 3)

Meaning, the problems which cannot be solved using arithmetic, can be solved using algebra. In other words, algebra helps us to simplify the problems compared to arithmetic.

In previous class we discussed solving equations in one variable.

For example:

$$\underset{\substack{\uparrow \\ \text{variables}}}{ax} + b = c$$

Where a, b and c are real numbers and $a \neq 0$. You are familiar with this.

So, let us revise the consept to recall .

Quantities whose value varies according to given conditions are called **Variable quantities.**

For example: $x + 1 = 3 \rightarrow \quad x = 2$

$x + 2 = 5 \rightarrow \quad x = 3$

In these x has different values in both the equations, in the first equation $x=2$ and in the second equation $x = 3$, hence x is the variable.

In this chapter, we will learn to solve the pair of linear equations in two variables by Vedic methods.

Let x and y be two variables, the equation formed by them is

$$ax + by = c.$$

Where a, b and c are real numbers and $a \neq 0$ and $b \neq 0$.

It is a linear equation with two variables because the variables x and y have the highest degree (one) and it is an equation made up of two variables. The values of and in the above equation determine the solution of the equation. The value for which both sides of the equation are equal is the solution. A pair of linear equations in two variables is called a **simultaneous equation.**

Example: Solve the following equations.

$2x + 3y = 5$

$4x + 2y = 3$

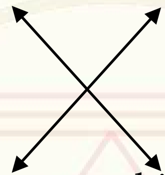
The general form of system of simultaneous linear equations in two varibles *x* and *y*.

$a_1 x + b_1 y + c_1 = 0$

$a_2 x + b_2 y + c_2 = 0$

Where a_1, b_1, c_1, a_2, b_2 and c_2 are all real numbers, a_1, b_1 and a_2, b_2 are not equal to zero. Both of the above linear equations have two variables, x and y. Thus, a pair of linear equations with two variables is called a system of linear equations. The pair of equations in two variables will be graphically represented as follows: If there are two lines in a plane, then only one of the following is possible:

2. When both lines meet or intersect at a point.



2.When both lines do not intersect, which means, they are parallel.

3. Both the lines coincide.

Assume that there is a pair of linear equations in two variables:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0 \text{ then,}$$

Serial number	Proportions	Graphical representation	Algebraic representation
1.	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	lines intersect each other	Only One solution (unique)
2.	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	lines overlap (coincident) each other	infinite Solution (many)
3.	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	lines are parallel to each other.	No Solution

When the number of solution of a pair of linear equations is infinite or unique, then the system is consistent and when a system has no solution then we say the system may be inconsistent.

Example : check for the solutions of the following system of linear equations by comparing the coefficients with the general form ($a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$). Both graphically and algebraically.

1. Linear equation: $2x + 3y - 10 = 0$ and $4x + 6y - 20 = 0$

Solution:

General form ($a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$) when compared

$a_1 = 2$, $b_1 = 3$, $c_1 = -10$ and $a_2 = 4$, $b_2 = 6$, $c_2 = -20$ Then,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{4} = \frac{3}{6} = \frac{(-10)}{(-20)}$$

or $$\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$$

which means , the pair of linear equations in two variables have infinitely many solutions and the lines will coincide.

2. linear equations: $2x + 7y + 5 = 0$ and $4x + 3y + 5 = 0$

Solution:

General form ($a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$) when compared $a_1 = 2$, $b_1 = 7$, $c_1 = 5$ and $a_2 = 4$,$b_2 = 3$,$c_2 = 5$Then,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{2}{4} \neq \frac{7}{3}$$

Which means, the system of equations has unique solution. and lines intersect each other.

3. linear equations: $5x + 2y + 4 = 0$ and $15x + 6y + 16 = 0$

Solution:

General form ($a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$) when compared

$a_1 = 5$,$b_1 = 2$,$c_1 = 4$ and $a_2 = 15$,$b_2 = 6$,$c_2 = 16$ Then,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{5}{15} = \frac{2}{6} \neq \frac{4}{16}$$

or $$\frac{1}{3} = \frac{1}{3} \neq \frac{1}{4}$$

Which means, the pair of linear equations in two variables will have no solution and the lines will be parallel.

Algebraic solution of linear equation of two variables :

Substitution method:

Solve the following equations.

Example: Solve.

$$2x - y = 3$$

$$4x - y = 5$$

Solution: $2x - y = 3$ ....(1)

$4x - y = 5$.........., (2)

From equation (1)

$$2x - y = 3$$

$$2x - 3 = y \qquad \text{..........,(3)}$$

By substituting the value of y from equation (3) into equation (2)

$$4x - (2x - 3) = 5$$

$$4x - 2x + 3 = 5$$

$$2x + 3 = 5$$

$$2x = 5 - 3$$

$$2x = 2$$

$$\boxed{x = 1}$$

substituting the value of $x = 1$ in equation (3)

$$y = 2 \times 1 - 3$$

$y = 2 - 3$

$y = -1$

Hence x = 1 and y = (– 1) The solution of the equations are

$x = 1, y = (-1)$

Solution of simultaneous equation by Vedic method:

कल्प्यते अ$_1$ या + व$_1$ का + र$_1$ नी = 0 तथा

अ$_2$ या + व$_2$ का + र$_2$ नी = 0 तदा

$$\frac{\text{या}}{\text{व}_1\text{र}_2 - \text{व}_2\text{र}_1} = \frac{\text{का}}{\text{र}_1\text{अ}_2 - \text{र}_2\text{अ}_1} = \frac{\text{नी}}{\text{अ}_1\text{व}_2 - \text{अ}_2\text{व}_1}$$

(बीजगणितम् , वज्राभ्यासगुणानरीतिः , पृ. 219)

Vedic formula: 'UrdhvatiryagbhayamSutra' The meaning of this method is cross multiplication. It is very easy involving less numer of calculations to solve system of linear equations of two variables.

Consider the system of linear simultaneous equations.

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

By solving these, the values of x and y are obtained.

Then from Urdhvatiryagbhyam Sutra

$$X = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$Y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

Example: Find the value of x and y in the given system.

$5x + 2y + 6 = 0$

$7x + 4y + 3 = 0$

Solution : Comparing the general form of linear equation in two variables, in this equation (1) is compared to $a_1x + b_1y + c_1 = 0$ and equation (2) is compared to $a_2x + b_2y + c_2 = 0$.

$a_1 = 5, b_1 = 2, c_1 = 6$

$a_2 = 7, b_2 = 4, c_2 = 3$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} = \frac{2\times2 - 4\times6}{5\times4 - 7\times2} = \frac{6-24}{20-14} = \frac{-18}{6} = -3$$

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} = \frac{6\times7-5\times3}{5\times4-7\times2} = \frac{42-15}{20-14} = \frac{27}{6} \text{ or } \frac{9}{2}$$

Example: Find the values of x and y in the given system.

$2x + 3y + 6 = 0$…………(1)

$5x + 4y + 2 = 0$ ………… (2)

Solution : Comparing the general form of linear equation in two variables, in this equation (1) is compared to $a_1x + b_1y + c_1 = 0$ and equation (2) is compared to $a_2x + b_2y + c_2 = 0$.

$a_1 = 2, b_1 = 3, c_1 = 6$

$a_2 = 5, b_2 = 4, c_2 = 2$

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} = \frac{3\times2 - 6\times4}{2\times4 - 5\times3} = \frac{6-24}{8-15} = \frac{-18}{-7}$$

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} = \frac{5\times6-2\times2}{2\times4-5\times3} = \frac{30-4}{8-15} = \frac{26}{-7}$$

Hence, $x = \frac{-18}{-7}$ or $\frac{18}{7}$ and $y = \frac{26}{-7}$ or $\frac{-26}{7}$.

Reflection method of Vedic mathematics:

To solve simultaneous equations in two variables, the reflex formula is used. The answer can be given by calculating mentally by using the reflex rule. The cyclic rule is used to solve the question. Using the cyclic rule from the reflection formula, the system of equations is solved as follows:

If the general form of the equations is as follows, then

$$a_1 x + b_1 y + c_1 = 0$$

$$a_2 x + b_2 y + c_2 = 0$$

Then for the value of x:

In this method, to find the value of x, find the quotient of the difference between the cross products of coefficients of y and constant terms, starting from the first equation to the second equation. Consider this as the numerator, and the denominator will be the difference between the cross products of the coefficients of x and y starting from the first to the second equation.

Similarly, to find the value of y, find the quotient of the difference between the cross products of constant terms and coefficients of x, starting from the first equation to the second equation. Consider this as the numerator; the denominator will be the difference between the cross products of the coefficients of x and y, starting from the first to the second equation.

$$x = \frac{b_1c_2 - b_2\,c_1}{a_1b_2 - a_2b_1}$$

x is obtained.

$$x = \frac{b_1c_2 - b_2\,c_1}{\text{Denominator}}$$

To get the denominator of the value of

$$\text{Denominator of } x = \frac{\textit{Numerator}}{a_1b_2 - a_2b_1}$$

Hence, $x = \frac{\textit{Numerator}}{\text{Denominator}} = \frac{b_1c_2 - b_2\,c_1}{a_1b_2 - a_2b_1}$

For the value of y:

Follow the cyclic rule, that is, starting from the independent term of the top row and multiplying it by the x coefficient of the bottom row, thus-

Hence, $y = \frac{\textit{Numerator}}{\text{Denominator}} = \frac{c_1a_2 - c_2\,a_1}{a_1b_2 - a_2b_1}$

Remember :

The denominator remains the same in the above values of x and y.

Example: Solve the following equation by cross multiplication formula.

$5x + 3y + 11 = 0$

$6x + 5y + 9 = 0$

Solution: Comparing the general equation form of linear equation in two variables, here equation (1) is compared to $a_1\,x + b_1\,y + c_1 = 0$ and equation (2) is compared to $a_2\,x + b_2\,y + c_2 = 0$.

x, it will be obtained by cross multiplying the numerator and independent terms and cross multiplying the coefficients of the denominator y and x.

$$x = \frac{\mathbf{b_1c_2 - b_2c_1}}{\mathbf{a_1b_2 - a_2b_1}}$$

$$x = \frac{3\times9 - 5\times11}{5\times5 - 6\times3} = \frac{27-55}{25-18} = \frac{-28}{7} = -4$$

To get the value of y, we will cross multiply the free term and coefficient of x for the numerator and the denominator remains the same. Hence

$$y = \frac{c_1a_2 - c_2\,a_1}{a_1b_2 - a_2b_1}$$

$$y = \frac{11\times6 - 9\times5}{5\times5 - 6\times3}$$

$$= \frac{66-45}{25-18}$$

$$= \frac{21}{7} = 3$$

Therefore, the values of x and y are x = -4 and y = 3 respectively.

Exercise 4.1

1. Select the correct option for the following multiple-choice questions.

(a) The solution of the system of simultaneous equations $2x + 3y = 5$, $4x + 6y = 9$ will be -

I) Inconsistent II) Unique solution

III) Infinitely infinite solution IV) None of these

(b) The graphs of two linear equations are intersecting lines,then the solution of the pair of linear equations is

I) There is no solution II) There is a unique solution

III) There are two solutions IV) There are infinite solutions

(c) The graphs of two linear equations should be parallel lines,Then the solution of the pair of linear equations :

I) There is no solution II) There is a solution

III) There are two solutions IV) There are infinite solutions

(d) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Solution of the system of equations $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$:

I) There is no solution II) There is a unique solution

III) There are infinite solutions IV) None of these

(e) The system of equations $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ will have unique solution, when

I) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ II) $\frac{a_1}{a_2} = \frac{c_1}{c_2}$

III) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ IV) $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$

(f) When the graphs of equations in two variables are coincident, then their solutions will be –

I) One II) Two III) Three IV) Infinite

(g) If the solutions of two linear equations are infinite,then their graphs will be –

I) Two parallel lines II) Two intersecting lines

III) Two coincident lines IV) none of these

2. Find the solution for the following systems by cross multiplication method.

1) $x + y = 7$
$3x - 2y = 1$

2) $4m + 2n = 1$
$3m + 2n = 2$

3) $7x + 11y = 1$
$8x + 13y = 3$

4) $p + q = 1$
$p + 2q = 1$

5) $2x + y = 5$
$3x - 4y = 2$

6) $m + 2n = 3$
$3m + n = 4$

7) $x + y = 5$
$8x - 3y = 1$

8) $5x + 2y = 3$
$4x - 3y = 2$

We learned -

1) A pair of linear equations in two variables is called a system.

2) The general equation of the system of equations is $a_1 x + b_1 y + c_1=0$ and $a_2 x + b_2 y + c_2 = 0$.

3) The algebraic solution of the linear equation of two variables can be solved with the Vedic sutra (formula) of cross multiplication.

For the value of x= $\frac{Numerator}{\text{Denominator}}$

In general form $x = \frac{\mathbf{b_1c_2 - b_2c_1}}{\mathbf{a_1b_2 - a_2b_1}}$

For the value of y = $\frac{Numerator}{\text{Denominator}}$

In general form $y = \frac{c_1a_2 - c_2 a_1}{a_1b_2 - a_2b_1}$

Chapter 5

Quadratic equation

Dear Students! In ancient times, there were many references to solving quadratic equations in Vadic literature. For example, the following shloka is found in Brahamsphoot Siddhant, written by Indian mathematician Brahmagupta, explaining the method of solving equations.

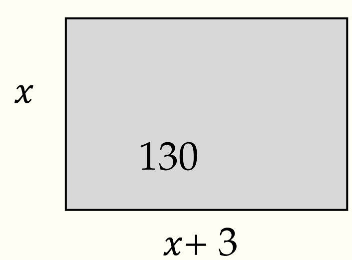
वर्गाहतरूपाणाम् अव्यक्तार्धकृतिसंयुतानां यत् ।

पदमव्यक्तार्धोनं तद्वर्गविभक्तमव्यक्तः ।।

(ब्राह्मस्फुट सिद्धान्त,कुट्टकाध्याय : 45)

That is, add the square of half of the latent amount's coefficient (b), i.e. $\left(\frac{b}{2}\right)^2$ to the expressed amount (c), then subtract half of the latent quantity coefficient from its square root. Again, the latent quantity is calculated by dividing the unknown amount by the coefficient (a).

We use quadratic equations to solve various problems related to real life. Suppose the length of a room is 3 meters more than its width, and the floor area of the room is 130 square meters. Find the length and width of the room.



$$\text{Area of room} = \text{length} \times \text{width}$$
$$130 = (x + 3) \times x$$
$$130 = x^2 + 3x$$

Consider the following equation: The unknown quantity in this equation is x, and the maximum power of x is 2. This type of equation is known as a quadratic equation. In order to solve such problems, we will discuss quadratic equations, also known as second degree equations, in this chapter.

Quadratic equation:

The equation in which the maximum degree of the unknown quantity is 2 it is called a **quadratic equation.** The general form of the quadratic equation is as follows:

$$ax^2 + bx + c = 0$$

Where a, b and c are real numbers and $a \neq 0$. On solving any quadratic equation, at the maximum we get two values of the variables as solutions, which are called the roots of the quadratic equation. We represent roots of the equation by Greek letters α and β.

Method to solve quadratic equation:

Formula Method or Sridharacharya:

Brief introduction of Shridharacharya and his formula:

Shridharacharya was born in 750 AD (approximately). Texts of Shridharacharya are very valuable in the field of mathematics, which contain many formulas, which were unknown before. Two of his books are available namely -

1) Patiganit **2) Trishatika**

Sridharacharya's method of solving quadratic equations is known in the present era as the perfect seuqring method of solving quadratic equations.

चतुराहतवर्गसमैरूपै: पक्षद्वयं गुणयेत् ।

अव्यक्तवर्गरूपैर्युक्तौ पक्षौ ततो मूलम् ॥

(बीजगणितम्,एकवर्णमध्यमाहरण, 4, पृ. 315)

Let the quadratic equation be $ax^2 + bx + c = 0$. Multiplying both the sides by 4a and on transposing 4ac to the other side

$$ax^2 + bx + c = 0$$

$$4a\,(ax^2 + bx + c) = 4a\,(0)$$

$$4a^2x^2 + 4abx + 4ac = 0$$

$$4a^2x^2 + 4abx = -4ac$$

Adding b^2 on both sides

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$

$$(2ax + b)^2 = b^2 - 4ac$$

$$(2ax + b) = \pm\sqrt{b^2 - 4ac}$$

$$2ax = -b \pm \sqrt{b^2 - 4ac}$$

On simplifying it, we will get -

Sridharacharya formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Which are the real roots of the quadratic equation, these roots are depicted byαand β.

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Shridharacharya's books have been like columns of light, based on which later scholars presented their works. Sridharacharya (750 AD) was between Brahmagupta (628 AD) and Bhaskaracharya (1105 AD).

Remember: We can identify the nature of roots of a quadratic equation from its discriminant.

Discriminant of quadratic equation = $b^2 - 4ac$

1. If the value of $(b^2 - 4\text{ac})$ is positive, both the roots $\alpha$and β are real and different.
2. When $b^2 - 4\text{ac} = 0$, then the values of both the roots α and β of the quadratic equation are real and equal.
3. When the value of $(b^2 - 4\text{ac})$ is less than zero, then both the roots α and β of the quadratic equation are not real.

Example: Solve the equation $2x^2 + 3x - 2 = 0$ by formula method (Shridharacharya formula).

Solution : $2x^2 + 3x - 2 = 0$

Compare this with the general form of the quadratic equation

$ax^2 + bx + c = 0$

$$a = 2, b = 3, c = -2$$

We know that Sridharacharya formula

$$\mathbf{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-2)}}{2 \times 2}$$

$$x = \frac{-3 \pm \sqrt{9 + 16}}{4}$$

$$x = \frac{-3 \pm \sqrt{25}}{4}$$

$$x = \frac{-3 \pm 5}{4}$$

Taking positive sign :

$$x = \frac{-3 + 5}{4}$$

$$x = \frac{2}{4}$$

$$x = \frac{1}{2}$$

On taking negative sign :

$$x = \frac{-3 - 5}{4}$$

$$x = \frac{-8}{4}$$

$$x = -2$$

Hence $x = -2, \frac{1}{2}$ is the solution of the equation.

Example: Solve the equation $3x^2 - 5x + 2 = 0$ using the formula method.

Solution: Compare this with the general form of the quadratic equation

$ax^2 + bx + c = 0$

$$a = 3, b = -5, c = 2$$

We know that Sridharacharya formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4\times3\times2}}{2\times3}$$

$$x = \frac{5 \pm \sqrt{25-24}}{6}$$

$$x = \frac{5 \pm \sqrt{1}}{6}$$

$$x = \frac{5 \pm 1}{6}$$

Taking positive sign :

$$x = \frac{5+1}{6}$$

$$x = \frac{6}{6}$$

$$x = 1$$

On taking negative sign :

$$x = \frac{5-1}{6}$$

$$x = \frac{4}{6}$$

$$x = \frac{2}{3}$$

Hence $x = 1, \frac{2}{3}$ is the solution of the equation.

Example: Find the values of a, b and c by comparing the following equation with the general form of the quadratic equation **($ax^2 + bx + c = 0$).**

1) $x^2 + 3x + 4 = 0$

Solution: We know that by comparing with the general form of quadratic

equation $ax^2 + bx + c = 0$,

$a = 1, b = 3, c = 4$

2) $5x^2 - 4x - 7 = 0$

Solution: We know that comparing with the general form of quadratic

equation $ax^2 + bx + c = 0$,

$a = 5, b = -4, c = -7$

3) $x^2 - 9x + 4 = 0$

Solution : We know that comparing with the general form of quadratic equation $ax^2 + bx + c = 0$,

$a = 1, b = -9, c = 4$

Exercise 5.1

1. Select the correct option for the following multiple-choice questions.

(a) If the value of discriminant of a quadratic equation is greater than zero, both the roots will be-

(I) Real and different (II) Real and similar

(III) Unreal and different (IV) Unreal and similar

(b) quadratic equation $ax^2 + bx + c = 0$ will have the same roots,if–

(I) $b^2 = 4ac$ (II) $ac = 0$

(III) $b^2 + 4ac = 0$ (IV) $b^2 + ac = 0$

(c) The discriminant of the quadratic equation $bx^2 + ax + c = 0$ will be-

(I) $b^2 - 4ac$ (II) $a^2 - 4bc$

(III) $c^2 - 4ab$ (IV) $b^2 - 2ac$

(c) The solution of the quadratic equation $x^2 + 5x + 6 = 0$ is –

(I) $-1, -2$ (II) $-5, -1$ (III) $-3, -2$ (IV) $-1, 5$

(d) The discriminant of the quadratic equation $2x^2 - 7x + 6 = 0$ is equal to –

(I) 2 (II) -3 (III) 1 (IV) 3

2. Compare the following equations with the general forms of quadratic equations.($ax^2 + bx + c = 0$) find the value of a, b and c by comparing with-

a) $3x^2 - 7x + 4 = 0$ b) $-x^2 - x + 4 = 0$

c) $-x^2 + 8x + 9 = 0$ d) $-3x^2 - 4x - 5 = 0$

e) $x^2 + 5x - 7 = 0$ f) $12x^2 + 15x - 3 = 0$

3. Solve the following equations by the formula method (Shridharacharya formula). $\left[\textbf{Hint : x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$

a) $x^2 - 5x - 6 = 0$ b) $x^2 - 4x + 3 = 0$

c) $x^2 - 3x + 2 = 0$ d) $5x^2 - 20x + 2 = 0$

4. Write Sridharacharya formula.

5. Check whether the followingequations are quadratic equations?

a) $x^2 - 2x = (-2)(3 - x)$

b) $(x - 2)(x + 1) = (x - 1)(x + 3)$

c) $(x + 1)^2 = 2(x - 3)$

d) $(x - 3)(2x + 1) = x(x + 5)$

We learned -

1) The equation in which the maximum degree of the variable is two is called **a quadratic equation.**

2) The general form of a quadratic equation in variable x is

   $ax^2 + bx + c = 0$

   where a, b and c are real numbers and $a \neq 0$.

3) Shridharacharya's formula for solving quadratic equations:

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

4) Solving the quadratic equation $ax^2 + bx + c = 0$, two (root) solutions are obtained, which are respectively if the roots of the equation are α and β.

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Chapter 6

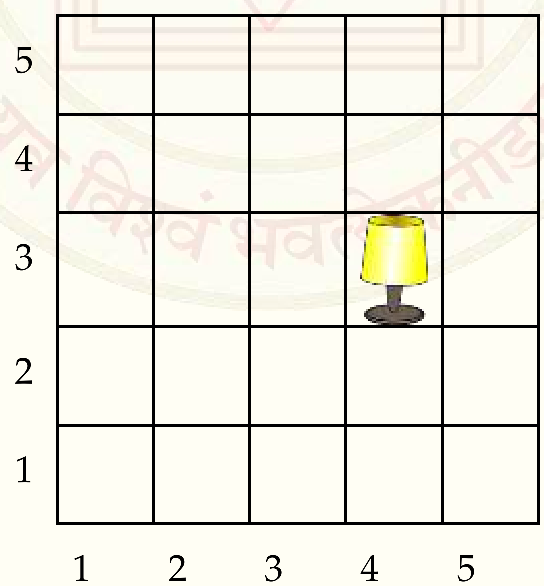
Coordinate Geometry

Dear Students! Coordinate geometry an algebraic method through which the geometry of shapes is studied. Coordinate geometry helps us understand shapes using algebra, that's why coordinate geometry is widely used in various fields like physics, engineering, maritime transportation, seismology etc.

Let us consider the following situation as an example.

Activity:

On a table (flat area) designed in the form of square shape, a lamp is placed on it inside the square.

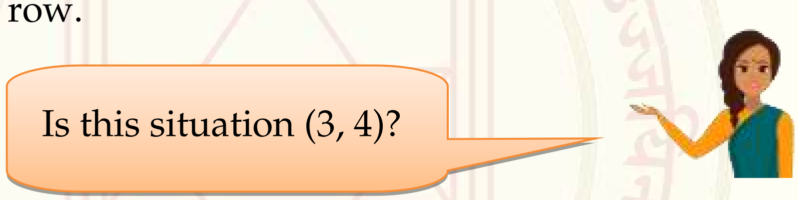


Students! If you decide to find the exact position of the lamp kept on the table, it is determined with the help of the following two informations.

1) The column in which the lamp is kept.

2) The row in which that lamp is kept.

If you look at the position of the lamp which is in the fourth column and third row, It is shown in the shaded square (brackets). In this the position of the lamp can be expressed as (4, 3), where the first number represents the column and the second number represents the row.



Aradhaya: This is not the situation because (4,3) represents 4 columns and 3 rows. Whereas in (3, 4) it is different, here there is 4 row of 3 column which is opposite.

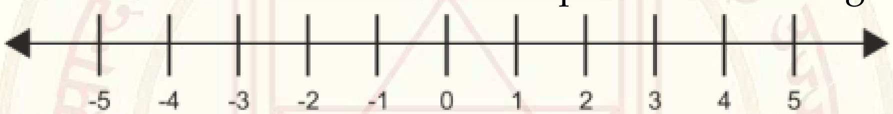
Guru ji - Correct ! very Good Aradhaya

If the lamp was placed in the fifth column and second row, then (5,2) is written for it. How will you write the position of the lamp, which is kept in the 3rd column and 5th row of the table? In relation to this activity of table and lamp, we need to know the number of columns and rows. This simple idea has far-reaching consequences, which is called ‘**Coordinate Geometry’**, an important branch of mathematics.

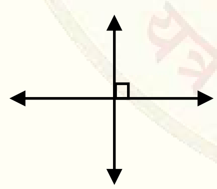
In this chapter, Let us discuss the concepts of coordinate geometry in detail.

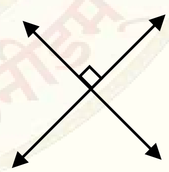
Cartesian coordinates:

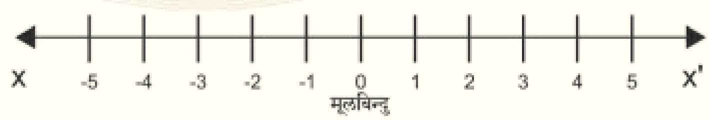
You have read about number lines in your earlier classes. From a fixed points (0) the positive numbers are written to its right and negative numbers on its left such that the distance between any two successive numbers is the same. The fixed point is called origin.



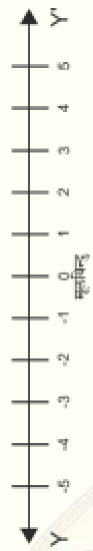
Descartes observed the relative position of two numbers when they are perpendicular to each other.The idea of determining the location of points in the plane is presented below.







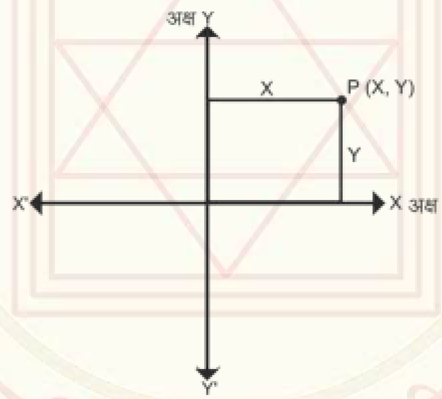
Two number lines named as xx’and’yy’. xx’. Place the number line xx’ horizontally line and the yy’number line vertically such that



their origins intersect each other.

The difference between the two number lines, is that xx′ isthe horizontal line and yy′ is the vertical line. is the horizontal line xx′ is called x -axis and vertical yy′ is called y -axis, where both the lines meet or intersect each other at the origin, which is denoted by (0, 0)

A pair of coordinate axes enables us to determine the position of a point on a plane.



Remember :-the distance of any point in the plane from the y axis is called the x coordinate or abscissa and the distance of any point from the x axis is called the y coordinate or ordinate of that point.

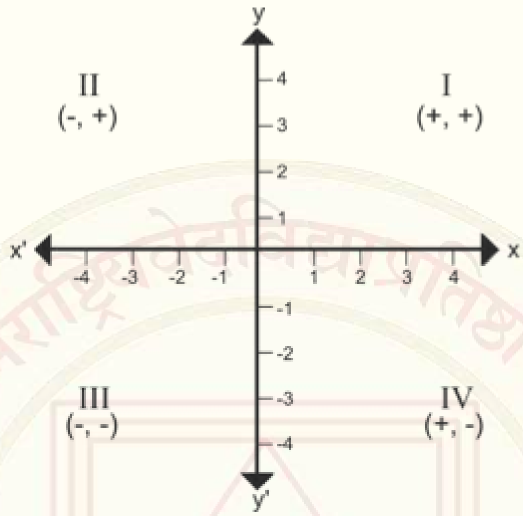
If (3, 4) is the coordinate

↑ ↑

(x) abscissa (y) ordinate

The coordinate of x is also known as the "abscissa" and the coordinate of y is known as the "ordinate."

Symbols of quadrant and coordinates:



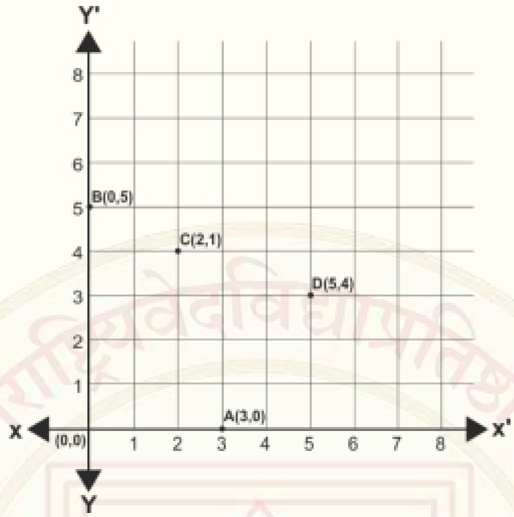
1. The y coordinates above the x axis are positive and the y coordinates below the x axis are negative.

   Similarly,

2. x coordinates of the points on the right of the y axis are positive and the x coordinates to the left of the y axis are negative.

Some examples of coordinates:

In the given graph –



1) Point 0 is the origin of the plane; the coordinate number of the origin of any plane is (0, 0); this means that the distance of the origin from the x axis = 0 and the distance of this origin from the y axis is = 0.

2) Given point A is on the x axis, the coordinate of point A = (3,0), it means that the distance of point A from the origin on the x-axis is 3 and on the y - axis is 0.

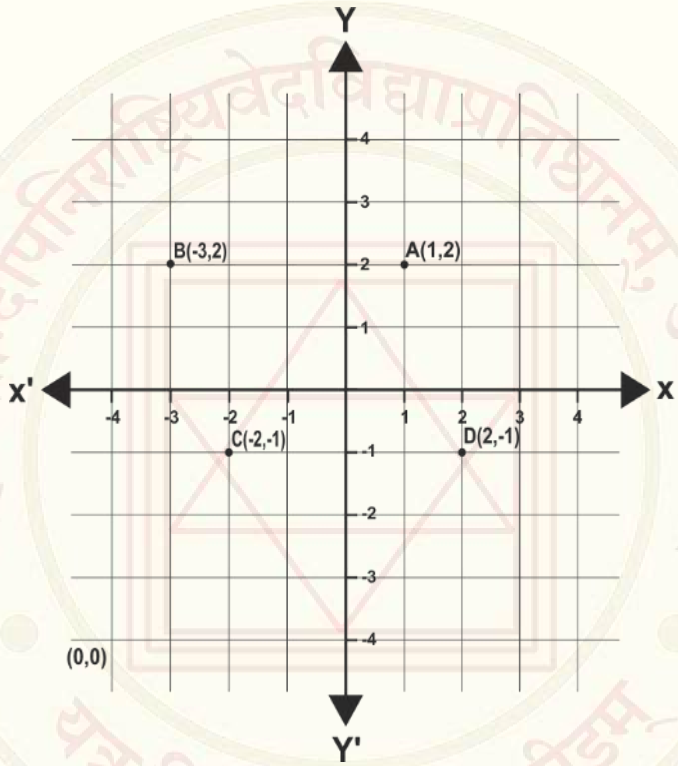
3) Point B lies on y axis. Hence, the coordinates of point B are = (0,5), which means that the distance of point B from the origin is on the x axis = 0 and on the y axis = 5.

4) If the coordinate number of point C = (2, 4), it means that the distance of point C from the origin on the x axis = 2 and on the y axis is 4.

5) The coordinates of point D are (5, 4), which means that the distance of point D from the origin on the x axis = 5 and the distance on the y axis = 4.

Positive and negative coordinates:

Example : Let us understand from the following graph.



In the above graph the coordinates of A, B, C and D are located.

1) Coordinate point A = (1, 2) means that its position is at a distance of 1 unit to the right of the y axis and is fixed at a distance of 2 units above.

2) Coordinate point B = (3, 2) means that point B is at a distance of 3 units to the left of the y axis and above x. Is at a distance of 2 units.

3) Coordinate point C = (2, 1), which means that point C is at a distance of 2 units to the left of the y axis and at a distance of 1 unit below the x axis.

4) Coordinate point D = (2, 1) means that to the right of the y axis at a distance of 2 units, it is situated at a distance of 1 unit below the x axis.

Remember:

We can find the quadrant in which the given point lies, just by looking at the signs of its coordinates.

Example : Coordinate point P (x, y)

$$\underset{\text{abscissa}}{\overset{\uparrow}{x}}\quad \underset{\text{ordinate}}{\overset{\uparrow}{y}}$$

A coordinate point is made up of abscissa and ordinate.

1) In the first quadrant, both the abscissa and ordinate are positive, hence the coordinates are (+, +).

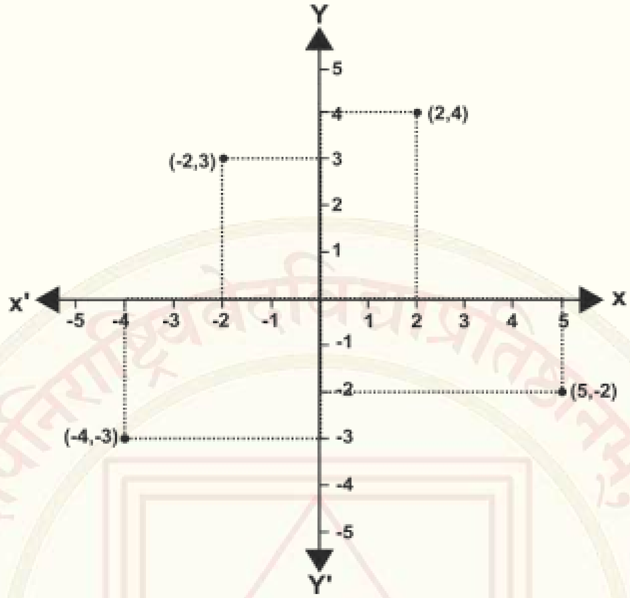
2) In the second quadrant, the abscissa is negative and the ordinate is positive, hence the coordinates are (- , +)

3) In the third coordinate number, both the abscissa and ordinate are negative, hence the coordinate number (-, -)

4) In the fourth quadrant, the abscissa is positive and the ordinate is negative,hence the coordinate number(+, -).

Example : In the rectangular coordinate system plot the points (2, 4),(−2, 3), (−4,−3) and (5, −2).

Solution:



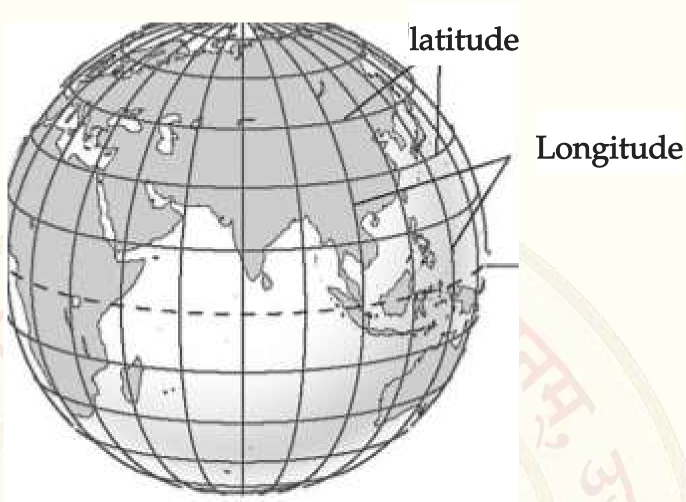
On the preceding figure, rectangular coordinates xox' and yoy' were drawn. Let us plot the coordinate points (2, 4), (-2, 3), (-4, -3) and (5, -2).

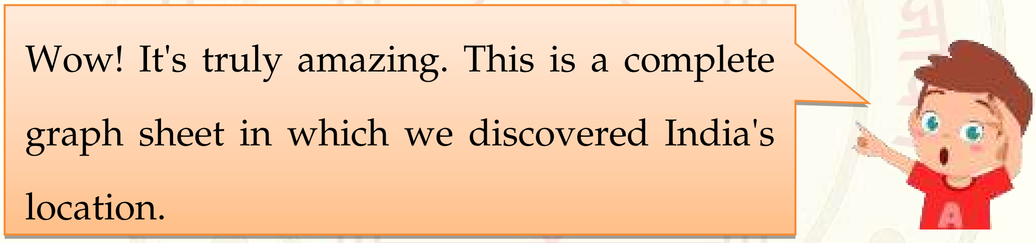
Do and learn:

Plot the coordinates (2, 5) (2, 3) and (-1, 3) on the graph paper.

Vedic students! The latitude and longitude lines on our planet also look like graph paper, but we determine the position of a point in the same way. Which we use to locate a specific point on Earth. Do you know where India is in terms of latitude and longitude on the globe? India's latitude position Mid-northern and continental Extension: 8°4′

to 37°6′ Longitude 68°7′-97°25′ east Located in the center of longitude East.





Do and learn –

Find the latitude and longitude of your state and write them.

Name of the state = ...

Latitude position = ...

Longitude position = ...

Exercise 6.1

1. Select the correct option for the following multiple-choice questions.

(a) In which quadrant is the coordinate point (- 8, 6) located?

(I) First (II) Second (III) third (IV) fourth

(b) What is the point of intersection of two axes called?

(I) Coordinate (II) Origin

(III) Coordinate axis (IV) None of these

(c) The coordinates of the origin are …………..?

(I) (1 ,1) (II) (0, 1) (III) (0, 0) (IV) (1, 0)

(d) The ordinate of the coordinate point (6, 10) is …………

(I) 6 (II) 10 (III) – 4 (IV) 16

(e) What form are the coordinates of a point on the y- axis?

(I) (0, 0) (II) (1 ,1) (III) (x, 0) (IV) (0, y)

(f) What is the distance of a point from the y- axis called?

(I) x - Coordinate (abscissa) (II) y - coordinates (abscissa)

(III) y- axis (IV) Ordinate

(g) In which of the following quadrants will the abscissa of a coordinate point be positive?

(I) First quadrant (II) Fourth quadrant

(III) First and fourth quadrant (IV) Only second quadrant

(h) What is the y- coordinate of a point on the x- axis ?

(I) 1 (II) 0 (III) -1 (IV) None of these

2. Write the quadrant in which the following point lie.

A) (2, 4) B) (−3, 4) C) (− 2, − 2) D) (3,−4)

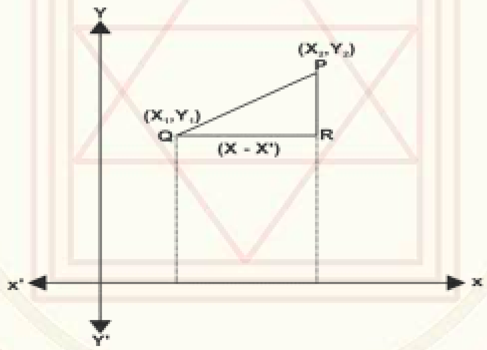
3. Plot the points (2,4), (3, 7), (−4, −2) and (4, −2) in the rectangular coordinate system.

4. In the first quadrant, plot the points (1, 2), (1, 3), (1, 4) and (- 2, - 4).

5. In the second quadrant, plot the points (-1, 2), (-1, 3), and (-1, 4).

6. Plot the points on a graph paper (3, 4), (1, 2) and (−1, 2).

Distance between two points:



Hence, from the right angled triangle POR, from Boudhayan's formula-

Two points: $P(x_1,y_1)$ and $Q(x_2,y_2)$

Distance between two points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Or

$$\text{Distance} = \sqrt{\begin{pmatrix}\textbf{The difference between the}\\ \textbf{coordinates of the x axis}\end{pmatrix}^2 + \begin{pmatrix}\textbf{The difference between the}\\ \textbf{coordinates of the y axis}\end{pmatrix}^2}$$

The formula for calculating distance between two points is:

Example : Find the distance between the points (2, 3) and (5, 6).

Solution : Let the points (2, 3) and (5, 6) be PQ respectively. Hence the distance between them

P (2, 3) and Q (5, 6)

x_1 y_1 x_2 y_2

$$\mathbf{PQ} = \sqrt{(\boldsymbol{x_2} - \boldsymbol{x_1})^2 + (\boldsymbol{y_2} - \boldsymbol{y_1})^2}$$

$$= \sqrt{(5-2)^2 + (6-3)^2}$$

$$= \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

Example : Find the distance between the points (−2, 3) and (−4, 5).

Solution: Consider point P (−2, 3) and Q (−4, 5),hence the distance between them is -

$$\mathbf{PQ} = \sqrt{(\boldsymbol{x_2} - \boldsymbol{x_1})^2 + (\boldsymbol{y_2} - \boldsymbol{y_1})^2}$$

$$= \sqrt{\left(-4-(-2)\right)^2 + (5-3)^2}$$

$$= \sqrt{(-2)^2 + (2)^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2}$$

Exercise 6.2

1. Select the correct option for the following multiple-choice questions.

(a) What is the distance of a point from the x- axis called?

(I) Abscissa (II) Axis (III) Graph (IV) Ordinate

(b) What will be the distance of point (3,5) from y- axis?

(I) 5 (II) 2 (III) 3 (IV) 4

(c) The distance between the points (8, 3) and (4,0) will be-

(I) 5 unit (II) 2 unit (III) 3 unit (IV) 4 unit

(d) The point, which is located at a distance of 4 units on the y-axis in the negative direction of the y-axis, will be

(I) (0, 4) (II) (0, - 4) (III) (- 4, 0) (IV) (4, 0)

(e) The point, which is located at a distance of 3 units on the x-axis in the y-positive direction of the axis,will be:

(I) (0, 3) (II) (0, - 3) (III) (- 3, 0) (IV) (3, 0)

(f) The point that is situated at a distance of 6 units on the x-axis in the negative direction of the x-axis will be:

(I) (0, 6) (II) (0, - 6) (III) (- 6, 0) (IV) (6, 0)

(g) The point which is situated at a distance of 6 units on the x- axis in the positive direction of the x -axis will be -

(I) (0, 6) (II) (0, - 6) (III) (- 6, 0) (IV) (6, 0)

2. Find the distance between the following two points.

a) A (3, 4), B (5, 6)

b) P (3, 4), Q (6, 7)

c) P (1, 2), Q (3, 4)

d) P (- 3, 4), Q (5,-7)

e) P (5,-2), Q (- 3, 4)

We learned -

a) The plane is called the Cartesian or coordinate plane, and the coordinates of the lines are called the axes.

b) A rectangular coordinate system is divided into four quadrants. Quadrant coordinate symbol :

In the first quadrant as (+, +),

In the second quadrant as (−, +),

In the third quadrant as (−,−)

and in the fourth quadrant they are in the form (+,−).

The signs show the signs of coordinates

c) In the coordinates of a point, the distance between the point and the y-axis is its x coordinates (abscissa) and the distance between the x-axis and the point is its y coordinates (ordinate).

$$\underset{\text{abscissa}}{\overset{\uparrow}{\text{P (x,}}}\ \underset{\text{ordinate}}{\overset{\uparrow}{\text{y)}}}$$

d) **Distance between two points:**

If the points are $P(x_1, y_1)$ and $Q(x_2, y_2)$ then, distance :

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Chapter 7

Vedic Mathematics

Preface -

Dear Vedic students! Vedic mathematics is an ancient Indian mathematical system. Its main source is the Vedic scriptures. Vedic mathematics provides quick solutions to various mathematical problems, such as addition, subtraction, multiplication, and division. This problem can be solved in a few simple steps. Vedic mathematics is not limited to basic mathematics but is also used in difficult concepts such as algebra, geometry, and calculus. Vedic mathematics provides a holistic approach to mathematical problem solving and promotes verbal computation, thereby developing the concentration and memory of students.

Importance of Vedic mathematics:

By using the formulas of Vedic mathematics, mathematical calculations become very easy. By continuously practicing these formulas, calculations can be done verbally. Oral calculation develops children's concentration, increases memory power and also helps students think logically. The simplicity and interesting nature of Vedic mathematics increase student's engrossment in mathematics, which becomes the basis for the development of their confidence and reasoning skills.

Practice of basic mathematical operations:

1) Addition operation:

In our earlier classes, we practiced finding the sum of numbers using the Ekadhikena Purvan method. Let us find the sum of the measurements (currency, distance, weight, capacity) by using the Ekadhikena Purvan formula.

Example : Find the sum of 112. kg and 65 grams, 360kg and 78 grams and 289kg and 872 grams.

Solution: Kg. Gram

$$
\begin{array}{rccc}
 & 1\,1\,2 & . & 0\,6\,5 \\
 & 3\,6\,0 & . & \dot{0}\,\dot{7}\,8 \\
+ & \dot{2}\,\dot{8}\,\dot{9} & . & \dot{8}\,7\,2 \\
\hline
 & \boxed{7\,6\,2 \quad . \quad 0\,1\,5} & &
\end{array}
$$

Hint:

1) 65 grams is written as 065 and 85grams is written as 085 grams.

2) Addition starts from the top of the unit column.

3) 5 + 8 = 13, hence the sign of Ekaadhik on the first digit of 8, 7, remains 13 - 10 = 3.

4) Write the remainder 3 below in the space above the addition. ($\dot{7}$= 8)

Continue the process in the same manner.

Remember: after writing the number in columns, add them using the formula like the addition of whole numbers.

Example : Find the following sum. 13km and78 m, 72 km and 28 m and 39 km. 12 m.

Solution: km. Meter

	km.		Meter
	1 3	.	7 8
	$7\dot{2}$	.	$\dot{2}8$
+	$\dot{3}9$	.	1 2
	1 2 5	**.**	**1 8**

Subtraction (Difference):

Earlier classes we had practiced with two Vedic formulas of subtraction.

1) Method based on EkaadhikenPurvena+ ParamamitraAnk
2) Method based on Ekaanunen Purvena + Paramamitra Ank

Subtraction using the above formulas do you remember that process?

Any two numbers are called paramamitra numbers if their sum is 10.

In subtraction –

$$\begin{array}{rl} 5 & \rightarrow \text{Separable} \\ \underline{-\,2} & \rightarrow \text{separator} \\ 3 & \rightarrow \text{remnant} \end{array}$$

Example: subtract (difference) using Vedic method:

872 – 189

Solution:

$$
\begin{array}{r}
8\ 7\ 2 \\
-\dot{1}\ \dot{8}\ 9 \\
\hline
\boxed{6\ 8\ 3}
\end{array}
$$

Hint;

1) 9 does not decrease from 2. Therefore, the paramamitranumber 9 is1 whichis added to 2. The sum 3 is written below in place of the answer. Mark Ekadhiken sign on the previous digit of the subtra hand,
2) 9 ($\dot{8}$= 9) is not subtracted from 7, hence on adding Paramamitra of 9 is1, adding to 7, we get 8 whichis written in the answer and put a Ekaadhiken sign on the digit 1 before $\dot{8}$.
3) Subtract 2($\dot{1}$ = 2) from 8.

Example : Subtract by Vedic method.

37km. 128m 25 cm – 18 km. 271 m 46 cm

Solution: **Km. metercm.**

$$
\begin{array}{rrrr}
 & 3\,7 & 1\,2\,8 & 2\,5 \\
- & \dot{1}\,8 & \dot{2}\,7\,\dot{1} & \dot{4}\,6 \\
\hline
 & 1\,8 & 8\,5\,6 & 7\,9
\end{array}
$$

Multiplication operation:

We studied the multiplication operation in earlier classes. Let us repeat the previous experience and learn to perform the multiplication operation using the following formulas: By practicing the formulas of Vedic mathematics well, you can do the multiplication quickly.

Example : Multiply. 588 ×512

Select the formula that gives the easiest product for multiplying the given numbers.

Solution :

$$\begin{array}{r} 5\ 8\ 8 \\ \times\ \ 5\ 1\ 2 \\ \hline \boxed{} \end{array}$$

3. 'Antyorshatkepi'Method:

1) This formula is used in multiplication when the sum of the units and tens digits of the multiplier and multiplicand is 100 and the remaining digits are the same.

2) By multiplying the units and tens digits of the multiplier and multiplicand, the product is converted into four digits and written on the right side of the answer.

3) Multiply the left side by its ekadhicane and write it on the left side of the answer.

Remember : on the right sideif a four-digit number is not formed then we put zero in front of the number.

Multiply - 588 ×512

$$\begin{array}{r} 5\,8\,8 \\ \times\quad 5\,1\,2 \\ \hline \end{array}$$

(100 written above the circled digits 88 and 12)

5 × 6 / 88 × 12

30 / 1056

= 301056

> **Hint -**
>
> 1) The sum of digits of one's tens is
>
> 88 + 12 = 100. Further digits are same.
>
> Then,
>
> 2) Left side = 5 × (multiple of 5)
>
> = 5 × 6 = 30
>
> 3) Right side = 88 × 12 = 1056 (four digits)
>
> left side / right side
>
> (So 5 × 6 / 88 × 12 = 301056.)

1. Multiplication by 'Anurupayen' formula :

Multiply - 588×512

588 + 8 8

× 512 + 1 2

(588 + 12) / 88 ×12

600× 5/1056

3000/1056

+10

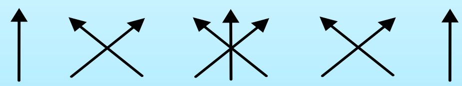
= 301056

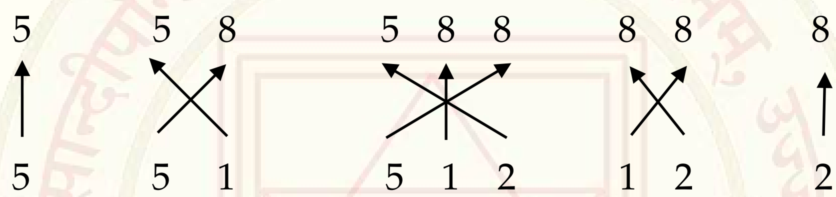
> **Hint -**
>
> base = 100
>
> Sub - base = 100 × 5 = 500
>
> Base number = 5
>
> deviation +88 and +12
>
> Two digits are kept on the right side. When the base is 100.

Note: Ekkunnen Purven This formula is ineffective because both numbers do not have the number 9.

2. Multiplication by 'Urdhvatiryagbhayam' formula

  5 8 8
×<u>5 1 2</u>

> **Hint –**
>
> Dot stick for multiplying three digits by three digits
>
> 
>
> V IV III II I



5×5 / 5×1+5×8 / 5×8+5×2+1×8 / 8×2+8×1 / 8×2

25 / 5+40 / 40+10+8 / 16+8 / 16

25 / 45 / 58 / 24 / 16

+6 +2 +1

= 301056

New option:

Example: find the product of 742 ×758 .

In this we use the formula Ekadhikena Purvena in the beginning and the formula Urdhvatiryagbhayam later.

$$
\begin{array}{r}
7 \quad 4 \quad 2 \\
\times \quad 7 \quad 5 \quad 8 \\
\hline
7\times 8 \ / \ 42 \times 58 \\
56 \ / \ 2436 \\
= 562436
\end{array}
$$

Hint-

Urdhvatiryagbhyam -

$$
\begin{array}{r}
42 \\
\times 58 \\
\hline
2436
\end{array}
\qquad
\begin{array}{ccc}
4 & 4\ \ 2 & 2 \\
5 & 5\ \ 8 & 8 \\
\text{III} & \text{II} & \text{I}
\end{array}
$$

Division operation:

We have learnt three methods of division in our previouse classes.

1) Nikhilam Sutra (when the divisor is less than the base number)
2) Paravartya Yojana formula (when the divisor is greater than the base number)
3) Urdhvatiryagbhayam formula (general method of this division)

In 'Urdhvatiryagbhayam formula while dividing the order of writing the numbers is:

Any question involving huge numbers of division operations can be solved with minimum calculations, comprehensively using the Urdhvatiryagbhayam formula. For convenience, it is split into two parts. Main number and dhavjank number.

(i) **Dhavjank number:** The unit digit of the divisor or several digits containing units that are written in place of the are called dhavjanks.

(ii) **Main number:** the remaining digits of the divisor, which are written at the base place. The one who completes the operation is called the main number.

Method:

(1) The parts divide the place of operation into three sections.

(2) The main number is written at the base place in the first section, and the dhavjank number is written at the exponent place.

(3) The number of digits in the dhavjank number corresponds to the number of digits on the third side of the unit side of the dividend. should be kept in the section, and the rest in the middle section.

Dhavjank Method:

The Dhwajaank method is a method of dividing using the Urdhvatiryagbhayam formula. The selection of the main number and dhavjank number from the divisor is critical in this method. Dhavjank and main numbers can have any number of digits. Divide the modified dividend by the Dhavjank number when using this method.

Remember: The number of digits in the divisor divides the number of digits in the dividend, which should be written in the third section, and the remaining digits in the dividend should be written in the middle section.

Now, let us learn from the following examples.

Example: Find the quotient. (dhavjank method) 98765 ÷ 87

Solution:

7	9	8	7	6	5
8		1	3	6	5
	1	1	3	5	55 – 5 ×7
					= 2

Hint –

1) denominator= 87 or 8^{7}

   8 = main number,7 = dhavjank number

2) One digit of dividend in third section = 5

3) 9 ÷8 Quotient First digit = 1, Remainder = 1

4) New dividend= 18, modified part

5) 11 ÷8 Quotient, second digit = 1, remainder = 3

6) New dividend= 37 modified dividend

7) 30 ÷ 8 quotient third digit = 3, remainder = 6

8) New dividend= 66 modified dividend

   45 ÷ 8 Quotient Fourth digit = 5 Remainder = 5

9) New dividend= 55

modified dividend or final remainder

Quotient = 1135, remainder = 20

Example : Find the quotient of 529 ÷ 23.

Solution:

$$
\begin{array}{c|cc|c}
3 & 5 & 2 & 9 \\
2 & & 1 & 0 \\
\hline
 & 2 & 3 & 0
\end{array}
$$

Quotient = 23, remainder = 0

Example : Find the quotient of 601325 ÷ 76.

Solution:

$$
\begin{array}{c|cccc|c}
6 & 60 & 1 & 3 & 2 & 5 \\
7 & & 1 & 6 & 2 & 2 \\
\hline
 & 7 & 9 & 1 & 2 & 13
\end{array}
$$

Quotient = 7912, remainder = 13

Exercise 7.1

1. Find the products of the following using Antyayoshatkepi formula.

   a) 5 9 0 ×5 1 0 = ...

   b) 5 9 6 ×5 0 4 = ...

   c) 8 8 8 ×8 1 5 = ...

   d) 9 9 9 ×9 0 1 = ...

   e) 7 5 5 ×7 4 5 = ...

   f) 5 6 4 ×5 3 6 = ...

   g) 1 1 8 0 ×1 1 2 0 = ...

   h) 1 0 3 2 ×1 0 6 8 = ...

2. Find the sum. by (Ekaadhiken by formula)

a)		b)	
	3 4 5 6 7		3 4 5 6
	1 3 5 6 7		1 3 4 2
	8 2 1 3 4		8 1 3 4
+	3 4 2 1 2	+	3 4 1 2
	$\boxed{}$		$\boxed{}$

3. Subtract(difference) using the Vedic method.

a)		b)	**kg.**	**Gram**
	9 8 3 7 3		1 3 7 .	0 6 5
−	3 1 9 8 7	−	1 1 2 .	3 8 5
	$\boxed{}$		$\boxed{}$	

4. Multiply:

By Akadhikena Purvena formula

a) $\begin{array}{r} 64 \\ \times\ 64 \\ \hline \boxed{} \end{array}$ b) $\begin{array}{r} 108 \\ \times\ 102 \\ \hline \boxed{} \end{array}$ c) $\begin{array}{r} 73 \\ \times\ 77 \\ \hline \boxed{} \end{array}$

By Ekkunnen Purvena formula

d) 23×99 e) 136×9999

By Urdhvatiyagbhayam formula

f) 362×123 g) 341×214

5. Divide:

By Nikhilam formula:

a) $11034 \div 889$ b) $1994 \div 98$

By Paravartya yojayeta formula:

c) $13385 \div 121$ d) $1358 \div 113$

By dhavjank method:

e) $37941 \div 79$ f) $4096 \div 64$

Cube:

Important methods of finding the cube of a number in Vedic mathematics -

1) Nikhil formula
2) Akadhikena Purvena formula
3) Anuroopyen formula

Base Method:

In this method, the multiplication operation is divided into three parts. The deviation is written twice on the left side. The square of the deviation is written in the middle section, multiplied by three, and the cube of the deviation is written in the third section.

For example : if the base is 10, then one digit each or if the base is 100, then two digits are kept in all three sections.

Cube = Number + (Deviation $\times$ 2)/ 3$\times$(deviation)^2/ (deviation)^3

{Where, **deviation = number - Base** }

6. (When base is 10)

Example : Find the cube of the number 15.

Solution : 15^3

base = 10

15 – 10 = +5 (deviation)

Number = 15, Deviation = 5

Then,

Cube = Number + (Deviation$\times$2)/ 3$\times$(deviation)^2/ (deviation)^3

$= 15 + (5\times2) / 3\times(5)^2 / (5)^3$

$= 15 + 10 / 3 \times 25 / 5 \times 5 \times 5$

$= 25 / 75 / 125$

or $= 25 / {}_{7}5 / {}_{12}5$

$= 3375$

7. (When base is 100)

Example : Find the cube of the number 104.

Solution :$(104)^3$

deviation = number - base

$= 104 - 100 = 4$

Then,

Cube = Number + (Deviation×2)/ 3×(deviation)^2/ (deviation)^3

cube = $104 + 4 \times 2 \;/\; 3 \times (4)^2 \;/\; 4^3$

$= 104 + 8 \;/\; 3 \times 16 \;/\; 4 \times 4 \times 4$

$= 112/\underline{48}/\underline{64}$ **(Note:** Two digits will be kept here because the base is100.)

$= 1124864$

cube root:

cube of 4

Cube of 4 = $4^3 = 4 \times 4 \times 4 = 64$

$4 = \sqrt[3]{4 \times 4 \times 4} = \sqrt[3]{64}$

cube root of 64

Here cube of 4 is 64 and cube root of 64 is 4.

Remember: The opposite operation to the operation of a cube is a cube root.

For Example: the cube of 4 is 64,then the cube root of 64 is 4.

mathematical symbol of cube root = '$\sqrt[3]{\;\;}$' Or $(\;\;)^{\frac{1}{3}}$ are called mathematical symbols of Cube roots.

Such as: $(64)^{\frac{1}{3}} = \sqrt[3]{64}$= 64 cube roots = 4

Identification of perfact cubes:

1) The beejank of perfact cube numbers is 1, 8, 9, or 0.

   Such as: The beejaak of 625 is 6 + 2 + 5 = 13 = 1 + 3 = 4, hence, 625 is a perfect cube. the number is. The number with the above-mentioned beejank is a perfect cube.

2) If the unit digit number of an integer is 1, 4, 5, 6, 9, or 0, then the unit digit of its cube root is also the same.

3) If the unit digit number of a perfect cube number is 2, 8, 3, or 7 then the unit digit of its cube root is the permmitra ank of this given number.

4) The last digit of the cube root can be found in the last group of numbers.

Example : Is the number 1729 a perfect cube?

Solution: Digit sum of 1729 1 + 7 + 2 + 9 = 1

Hence, it can be a perfect cube number.

Example : Is the number 9528127 a perfect cube?

Solution: beejank 9528127

9 + 5 + 2 + 8 + 1 + 2 + 7 = 7

Hence, it is not a perfect cube number.

We know **that the beejank of perfect cube numbers is 1, 8, 0 and 9.**

Before finding the cube root, look at the following table carefully, it will help you in determining the cube root.

Table - 1

If the last digit of the cube is –	The unit digit of the cube root will be.
0	0
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

Above **Table – 1** Can be understoodobserving.

1. 1, 4, 5, 6, 9 and zero (0) repeat themselves at the end of the cube.
2. The unit digit of cube root of 2, 3, 7 and 8 is the parammitra ank of the given number.

Table 2

leftmost pair of cube roots	nearest cube root
1 – 7	1
8 – 26	2
27 – 63	3
64 – 124	4
125 – 215	5
216 – 342	6
343 – 511	7
512 – 728	8
729 – 999	9

Through the above table 1-2, the left and right digits of the cube root of any 7 digit number can be easily obtained.

Vedic method of finding cube root (Upasutra Vilokanam):

The literal translation of vilokanamsutra is inspection, to examine. This formula is used when the cube root of a number must be less than 7 digits. Then, using this formula, you can easily determine the units and tens of digits of its complete cube root.

Rule : The Vedic Vilokanam sutra is used to find the cube root of a number. For that, create two groups of three digits each from the right.

Let us find the units and ten digits of the cube root using the above tables 1 and 2. Let us illustrate with an example.

Example: Find the cube root of 97336.

Solution : 1) Make a group of three digits each from the right.

$\overline{97}\ \overline{336}$

2) The last digit of the cube is 6, the unit digit of the cube root will be 6. (See Table 1 for reference)

3) The tenth digit of cube root is 4,because -

$4^3 < 97 < 5^3$

$\sqrt[3]{97336} = 46$

Hence, cube root of 97336 is 46.

Example : Find the cube root of 941192.

Solution: 1) Make a group of three digits each from the right.

$\overline{941}\quad \overline{192}$

2) Since there are two groups the cube root will have two digits.

3) Since the units digit is 2. Therefore the unit digit of the cube root will be 8.

4) For tens digit, see Table-2

$9^3 < 941 < 10^3$

Ten's digit = 9

$\sqrt[3]{941132} = 98$

Hence,cube root of 941132 is 98.

Exercise 7.2

1. Use beejank to determine whether the following numbers are perfect cube numbers or not.

a)	68921	b)	1279	c)	256
d)	216	e)	625	f)	1725

2. Find the perfect cube root by Vedic formula (vilokanam).

a)	68921	b)	1279	c)	256
d)	216	e)	625	f)	1725

We learned -

1) Basic operations (+,−, ×, ÷) of mathematical calculations used in practical life.Learned to solve using the formulas of Vedic mathematics.

2) Discuss about cube and cube root -

cube of 5

Cube of 5 = $5^3 = 5 \times 5 \times 5 = 125$

$$5 = \sqrt[3]{5 \times 5 \times 5} = \sqrt[3]{125}$$

cube root of 125

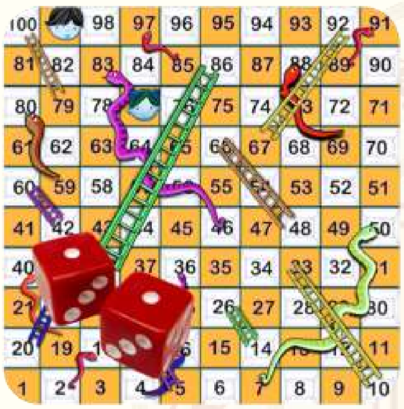
In this, cube of 5 is 125 and cube root of 125 is 5.

3) The beejank of a perfect cube number is 1, 8, 9 or 0.

Chapter - 8

Arithmetic series

Dear Students! In what situations do you use digits and numbers in your daily life? We must use numbers, whether on the playground



or in our studies. Consider the game of snake and ladder, in which we must rely on numbers or check the time on a clock. Or, when looking at a calendar, one must rely on numbers to figure out the date. Aside from that, numbers are used on coins, notes, and rupees to represent the amount of money. We can see from the preceding discussion how important digits and numbers are in our lives, but sometimes these numbers are in the same order and sometimes they are not. In this chapter, we will study numbers that are in sequence (arithmetic progression) with definite order.

Look carefully at the numbers in the following group and fill in the blanks.

1) 1, 2, 3,....., 5, 6, ..., 8,.....
2) 5, 10, 15, 20,……

What will be the next digit in the set of numbers? Yes, the next issue will be 25.

Thus

Utasav: In this the next number will be 60.

3) 10, 20, 30, 40, 50,......

What will be the number next to it?

4) 3, 6, 9, 12, ….., 18

5) 4, 8, 12, 16, ….., 24, 28,

You must have understood by looking at the above number sequences that all the example number sequences here are written in a certain order.

Example : 5 , 10 , 15 , 20 , 25

In the above sequence, the difference between any two successive numbers is 5.

Look carefully at the numbers in the following group and fill in the blanks.

First term = 5

Second term = first term+ difference = 5 + 5 = 10

Third term = Second term + difference = 10 + 5 = 15

fourth term = third term+ difference = 15 + 5= 20

fifth term = fourth term+ difference = 20 + 5 = 25

Hence, to get 25, we have to add 5 to 20 which is the fourth term.

Thus, there is a same difference between any two successive numbers in other examples also.

3) In (10 , 20 , 30 , 40 , 50 , **60**) common a difference of 10.

4) In (3 , 6 , 9 , 12 , **15** , 18) commone a difference of 3.

5) (4 , 8 , 12 , 16 , **20** , 24 , 28) comman a difference of 4.

Hence it is clear that those numbers which are written in an order form a sequence.

Sequence:

The function of numbers that follow a certain order is called a sequence. Every number in that series is called a term. The position of any term in the sequence is depicted by the latter n, where n is a natural number.

Then the term is called the general term. In other words, the arrangement of numbers in a certain order is called **a sequence.**

Example :2, 4, 6, 8, 10

- 10, -9, -8, -7, - 6

terms of a sequence are represented by $a_{1,}$ $a_{2,}$ $a_{3,}$…..,a_n.

Common difference:

The number that has to be added or subtracted from the given number to obtain its successor or predecessor is called the common difference.

In other words, the difference between any term and its preceding or succeeding term is called the **common difference**. The common difference is represented by 'd'.

If the sequence is a_1, a_2, a_3,..., an, the common difference is the difference between any two successive terms.

$$d = a_2 - a_1 \text{ or } a_3 - a_2 \text{ or } a_4 - a_3$$

In any sequence (**such as** 2, 4, 6, 8, or 10), to get the next term, add the common difference to the term; thus, we see that to write the numbers of a sequence, every term can be represented as. Let us understand with an example.

Example : If the general term of a sequence is given $a_n = n + 2$, write the first five terms.

Solution : n^{th} term of the sequence $a_n = n + 2$

Where n = natural number then

By substituting n = 1, 2, 3, 4 and 5 respectively

By keeping $n = 1$, $a_1 = 1 + 2 = 3$

when $n = 2$, $a_2 = 2 + 2 = 4$

when $n = 3$, $a_3 = 3 + 2 = 5$

when $n = 4$, $a_4 = 4 + 2 = 6$

when $n = 5$, $a_5 = 5 + 2 = 7$

The first five terms of the sequence obtained are : 3, 4, 5, 6, 7.

We can observe that, to obtained any term of the sequece we substitute the corresponding value of n in the expression for the general term a_n.

Do and learn:

Given that the general term of a sequence is $a_n = 22 - n$, find the first three terms of the sequence.

where n = naturalnumber

Example: For the given expression of a_n, find the 12th term.

1) $a_n = 2n - 1$ 2) $a_n = 3n - 2$

Solution :1) If n^{th}term$a_n = 2n - 1$

To find the 12th term of the sequence, substitute$n = 12$ in general form-

$a_{12} = 2 \times 12 - 1$

$= 24 - 1 = 23$

the 12th term of the given general form $a_n = 2n - 1$ is 23.

Solution: 2) If nth term $a_n = 3n - 2$

To find the 12th term of the sequence, substitute$n = 12$ in general form-

$a_n = 3 \times 12 - 2$

$= 36 - 2$

$= 34$

The 12th term of the given general form $a_n = 3n - 2$ is 34.

Exercise 8.1

1. Fill in the following blank.

    1) 1, 3, 5, ……, 7, 11

    2) 7, ……, 21, 28, 35

    3) – 10, – 9,.….,– 7, – 6

    4) A, B, ….., D, E, F

    5) 14, 15, 16, ….., 18, 19

    6) 11, 13, 15, ….., 19

    7) 100, 95, ….., 85, 80

2. What do you understand by sequence?

Series:

Expressing the terms of a sequence with an addition or a subtraction sign is called the series.

Example: 1 + 2 + 3 + 4 + 5 + 6 +.........

2 – 5 – 11 – 14 –...............

Airthmetic series:

An arithmetic series is a list of numbers in which each term (except the first term) is obtained by adding or subtracting a certain number from the previous term. The common difference in arithmetic progression is denoted by 'd, and the first term is represented by ‘a'. In other words, a series in which there is the same difference between any two successive terms is called an **arithmetic series.** It is abbreviated as 'A.P.' (arithmetic progression).

Example: 2, 4, 6, 8, 10

Every element of arithmetic progression is called a 'Term. The second term in this series is obtained by adding 2 to the first term, the third term by adding 2 to the second term, and the fourth term is obtained by adding 2 to the third term. Hence the given series is an arithmetic progression.

Any progression is **an arithmetic progression**. If:

$a_{n+1} = a_n + d$, where d= common difference and all n∈N (NaturalNumber)

Example :2, 5, 8, 11, 14, ...

This is an arithmetic progression because:

5 = 2 + 3

8 = 5 + 3

11 = 8 + 3

14 = 11 + 3

The difference between any two successive terms is three, which means, the common difference d = 3.For the given progression, 2, 5, 8, 11, 14 ,................. The first term is 2 and the common difference is 3.

first term = 2

Second term =5 =2 + 3 $=a+d$

third term = 8 = 5 + 3 = $(a+d)+d = a+2d$

fourth term = 11 = 8 + 3 $=(a+2d)+d = a+3d$

It can also be written in the following way.

A, a +d, a + 2d, a + 3d, ...

first term $a_1 = a$

Second term $a_2 = a + d = a + 1 \times d$

third term $a_3 = a + 2d = a + 2 \times d$

fourth term $a_4 = a + 3d = a + 3 \times d$

Where a_n = nth term

n = no. of terms

d = difference

n^{th}Term $a_n = a + (n-1)d$

Or general term $a_n = a + (n-1)d$

Example: Is the progression 26, 21, 16, 11, 6, 1...... an arithmetic progression?

Solution: first post term $a_1 = 26$, $a_2 = 21$, $a_3 = 16$, $a_4 = 11$, $a_5 = 6$

$a_2 - a_1 = 21 - 26 = -5$

$a_3 - a_2 = 16 - 21 = -5$

$a_6 - a_5 = 1 - 6 = \quad -5$

Observe that the difference between any two successive terms is (-5), Hence, the common difference is (– 5). We know that when a series has the same common difference then it is called an arithmetic series. Hence, the given series is an arithmetic progression.

Example: In arithmetic progression 4, 8, 12, 16, 20, find the 10th term of the series.

Solution : Series 4, 8, 12, 16, 20

$a = 4$

$d = a_2 - a_1 = 8 - 4 = 4$

We know that the nth term of the arithmetic progression is

$$\mathbf{a_n = a + (n - 1)\,d}$$

By substituting n = 10

$$a_{10} = 4 + (10 - 1)\,4$$
$$= 4 + (9) \times 4$$
$$= 4 + 36 = 40$$

Hence, 10th term of arithmetic progression is 40.

Example : Find the first four terms of the given arithmetic progression whose first term is a and common difference is d.when

$a = 10, \qquad d = 3$

Solution : We know that **$a_n = a + (n-1)d$**

where an = nth term

n = number of posts

d = common difference

First term $a_1 = 10$

second term $a_2 = 10 + (2-1)\,3$

$= 10 + 1 \times 3$

$= 10 + 3$

$= 13$

third term $a_3 = 10 + (3-1)\,3$

$= 10 + 2 \times 3$

$= 10 + 6$

$= 16$

fourth term $a_4 = 10 + (4-1)\,3$

$= 10 + 3 \times 3$

$= 10 + 9$

$= 19$

Hence, the first four terms of the given arithmetic progression are 10, 13, 16 and 19.

Arithmetic mean (middle term of arithmetic progression):

If a,b,c are in AP,

$b = \frac{(a+c)}{2}$ is called the arithmetic mean of a and c.

Example: Find the middle term of arithmetic progression 5 and 15.

Solution: We know the middle term of the arithmetic progression

$$b=\frac{a+c}{2}$$

Then, $b=\frac{5+15}{2} = \frac{20}{2} = 10$

Hence, the middle term is 10.

Exercise 8.2

1. Select the correct option for the following multiple-choice questions.

(a) In the arithmetic series 6, 3, 0, -3…., what is the common difference?

(I) 3 (II) – 3 (III) – 2 (IV) 1

(b) Which of the following is an arithmetic progression?

(I) 2, 4, 8, 16, … (II) 1, 3, 9, 27, …

(III) 1,2,3,4,5 … (IV) 0,5,25,10, …

(c) If the general term of the arithmetic progression is 3n + 5 its common difference will be:

(I) 1 (II) 3 (III) 2 (IV) 5

(d) The value of the 18th term of the arithmetic progression 1, 4, 7, 10,…… is –

(I) 50 (II) 45 (III) 52 (IV) 35

(e) If the first term of the arithmetic progression is a and the common difference is d, the nth term will be –

(I) a + nd (II) a + (n – 1) d

(III) a + (n + 1) d (IV) (2a + (n – 1)

(f) The 30th term of the arithmetic progression 10, 7, 4, … is equal to?

(I) -55 (II) 66 (III) -77 (IV) 81

(g) What will be the 10th term of the arithmetic progression 2, 7, 12, …?

(I) 45 (II) 52 (III) 60 (IV) 47

2. Write the first three terms of the arithmetic progression, where the first term a and common difference d are as follows:

a) a = 4, d = 2 b) a = 10, d = 10

c) a = 5, d = 10 d) a = 20, d = 5

3. The given series 25, 27, 29, 31, 33 is an arithmetic progression; find the 10th term.

4. In the arithmetic progression 3, 5, 7, 9, 11, Find the 12th term.

5. Is the progression 1, 5, 10, 2, 3, 1 an arithmetic progression?

6. Is the progression 8, 16, 24, 32, and 40 an arithmetic progression?

7. Fill in the boxes for the following arithmetic progressions.

a) 2, $\square$, 26

b) 20, $\square$ 60

c) $\square$, 3, $\square$

8. Find the 78th term in 3, 8, 13, 18,...

9. The 17th term of an arithmetic progression is 7 more than its 10th term; find its common difference.

We learned -

1) **Sequence:** The arrangement of numbers in a certain order is called a sequence.

   Example: 1, 2, 3, 4, 5, ...

2) **Series:** If the terms of a sequence are expressed with plus or minus signs, then the expressions obtained are called series.

   Example: 1 + 2 + 3 + 4 + 5 + 6 +.........

3) **Arithmetic Series:** A series in which the difference between any two successiveterms isequal, then such series is called arithmetic series, which is abbreviated as A. P.

   General form of an arithmetic progression:

   $a, a + d, a + 2d, a + 3d$

   where a= first term and d= common difference.

   common difference between terms d, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

   nth term of arithmetic progression:

   $$\mathbf{a_n = a + (n - 1)\, d}$$

   where a = first term, d = common difference, n = the number of the term

4) **Arithmetic** mean **(middle term of arithmetic progression):**

   If a,b,c are in arithmetic progression, arithmetic mean $\mathbf{b = \frac{a+c}{2}}$

Chapter - 9

Constructions

Dear students! You are familiar with many types of shapes in geometry (angle, triangle, quadrilateral, square, rectangle, circle, etc.). In earlier classes, you learned to construct angles with the help of a protractor. In this chapter, we will learn to construct angles and triangles with the help of compasses.

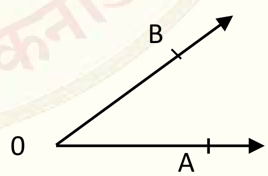
You will remember:

Angle: An angle is formed by two rays having a common initial point.

Two rays, OA and OB, make an angle, AOB. (It can be written as ∠BOA)

Triangle – A simple closed shape made of three line segments is called a triangle. It has three vertices, three sides, and three angles.

In the following figure (ΔABC):

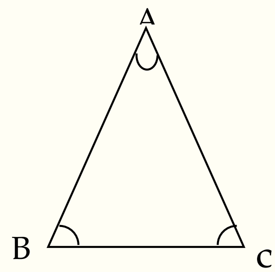


sides: $\overline{AB}, \overline{BC}, \overline{CA}$

angle: ∠BAC, ∠ABC, ∠BCA

Vertex: A, B, C

side opposite to vertex A is $\overline{BC}$. Name the angle opposite to the side $\overline{CA}$



Classification of triangles: Can you classify the triangles on the basis of their sides and angles?

(1) **On the basis of sides:** scalene,isosceles and equilateral triangles

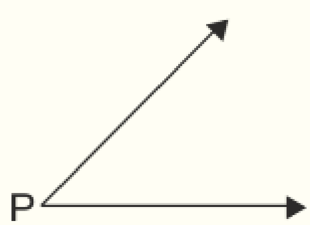
(2) **On the basis of angles:** acute angle,obtuse angle and right angle triangle

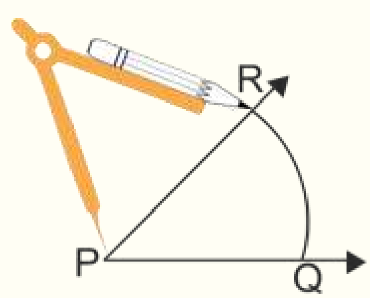
Procedure: Prepare the paper cuttings of the shapes of traingles mentioned above, compare the cuttings with those of your friends, and discuss.

Bisection of a given angle with the help of compasses:

∠P is given in the picture.

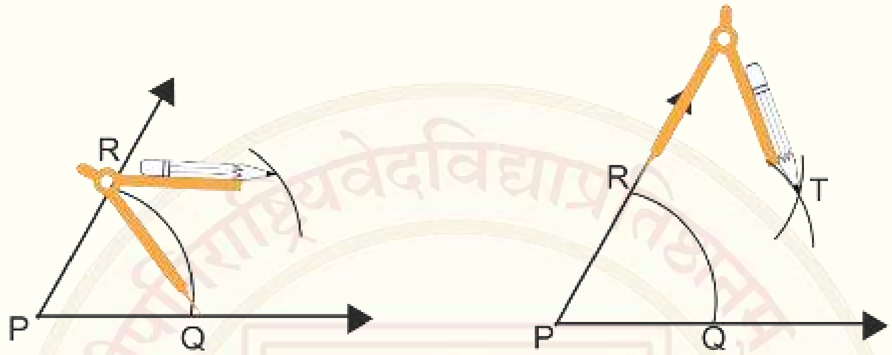
Step1) Considering point P as the center, draw an arc of any radius with the help of compasses, which cut the arms of angle P at Q and R, respectively.



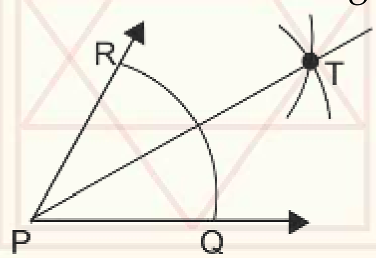


Step 2) Considering Q as the center, draw an arc of the same radius between the two arms of the angle P.

Step 3) Similarly, considering R as the center, draw an arc of the same radius to intersect the previously drawn arc at T.



Step 4) Join T,and P. TP is the bisector of angle P.

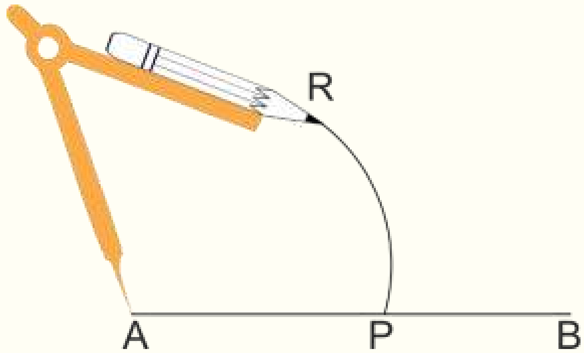


Construction of angles with the help of compasses:

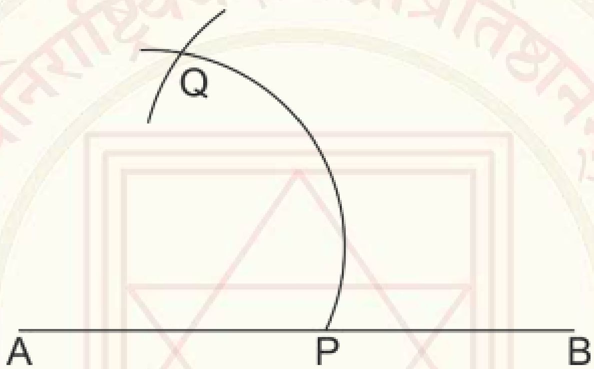
(1) Construct the angles 60^0 $120^0$and 180^0

(A) Construction of angle 60^0 :

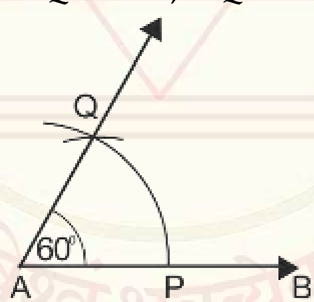
Step1: Draw a line segment AB of any length. Keeping the pointed end of the compass at a, draw a semicircle of radius to cut the line segment AB at P, as shown in the figure.



Step 2: Cut an arc of the same radius with P as the center. Name the point of intersection as Q.

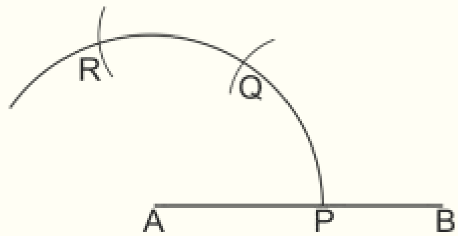


Step3: Join the points A and Q. Now, $\angle QAB = 60^0$ is formed.

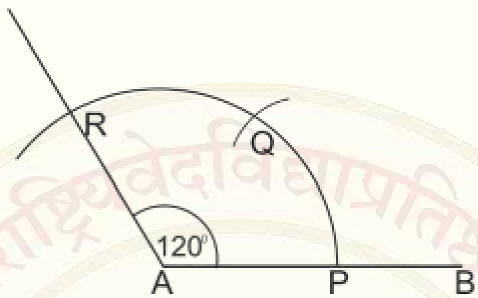


(B) Construction of 120^0

Step 1: In the previous picture, draw an arc of the same radius, keeping Q as the center, to intersect the semicircle at R.



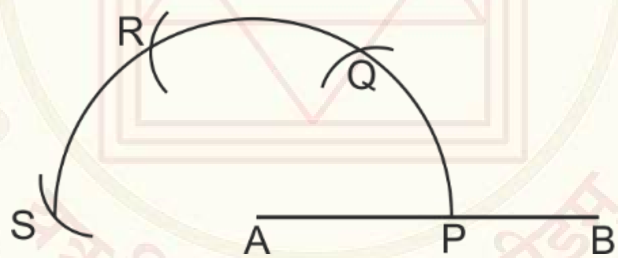
Step 2: Join the pointsAR. Now, ∠RAB = 120 ^{0}is formed.



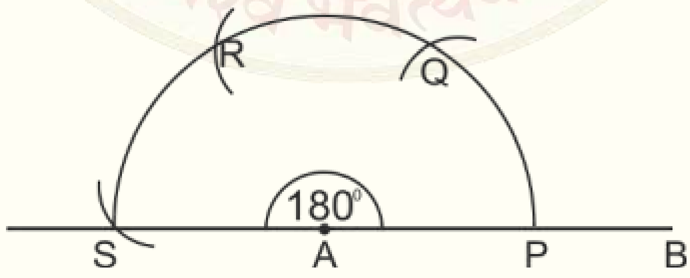
(C) Construction of an angle of 180^0

By proceeding further in the above picture,

Step 1: Cut an arc of the same radius with R as its center. Name the point of intersection of the arc and the Sami circle as S.



Step 2: Join points A and S. Now, ∠SAB=180 ^{0} is formed.



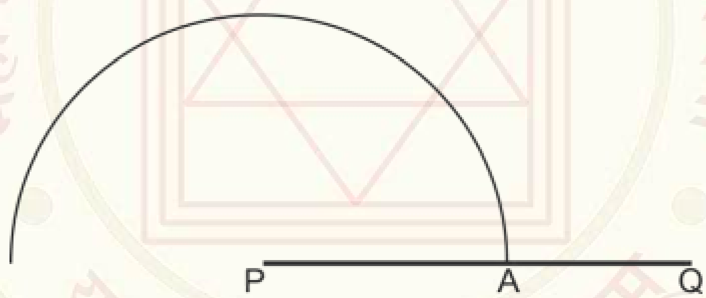
(ii) Construction of angles 30^0, 45^0, 90^0

We know that a half of 60° is 30°. Hence, by drawing the bisector of ∠60°, we get ∠30°. Similarly, 90° is half of 180°. Thus, by drawing an angle bysector in the construction of 180°, we get 90°.

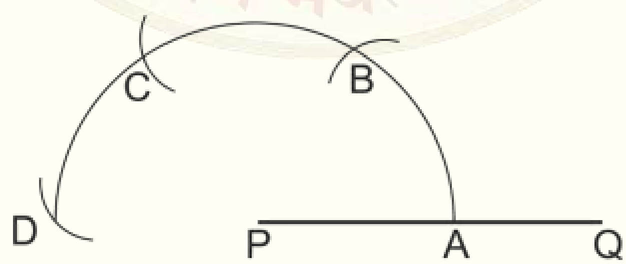
In the same way, by drawing an angle bisector in the construction of 90°, we get 45°, as half of 90 are 45.

(A) Construction of an angle of 30^{0}

Step 1: Draw a line segment (PQ) of any length. Draw a sami circle of any radius with a center P. Name the point of intersection of line segment PQ and the semi-circle as A.

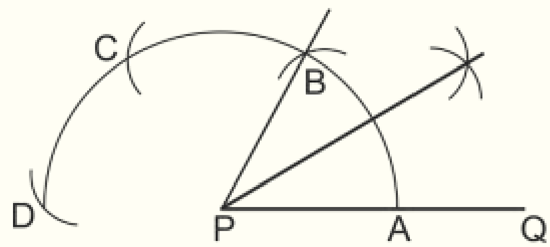


Step 2: Draw an arc of the same radius, with A as the center, to cut the semi-circle at B.



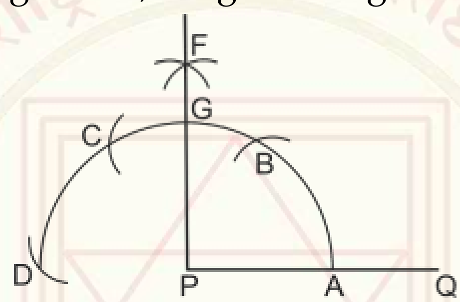
Step 3: Join P and B. Now, ∠BPA = 60° is formed.

Bisecting the**∠BPA, we get,** ∠BPM = ∠MPQ = 30^0



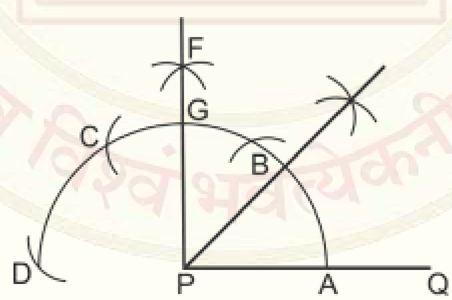
(B) Construction of an angle of 90^0

By bisecting 180^0, we can get 90°.Similarly, in the construction of 120°, bisecting angle BPC, we get an angle of 90^0. Angle FPQ = $90^{0.}$



(C) Construction of angle of 45°

We can construct 45° by bisecting 90°.In this ∠GPB = ∠APB = 45°.



Exercise 9.1

1. Using the Compass andruler (scale) construct the following angles.

   a) 90^0 b) 45^0 s) 120^0 d) 60^0

2. Draw any three angles of your choice and measure them using a protractor.

Construction of triangles:

To construct a triangle, it is not necessary to know the measurements of six elements (3 sides and 3 angles). If we are given any one of the measurement groups given below, then we can construct the required triangle. The group of elements required to construct a unique triangle is as follows:

Three sides

1. two sides and the angle between them.
2. two angles and the side between them.
3. hypotenuse and another side of a right triangle

Remember:

In the construction of a triangle:

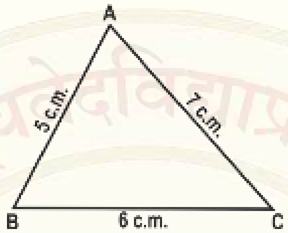
The sum of the measurements of two sides of a triangle is always greater than the measurement of the third side.

1. **Construction of a triangle when all three sides are given.**

Example: Construct a triangle ABC, in which AB = 5 cm, BC = 6 cm, and AC = 7 cm.

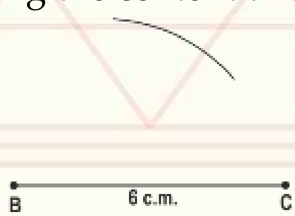
Solution:

Step 1: First of all, we draw a rough sketch of the given measurements.

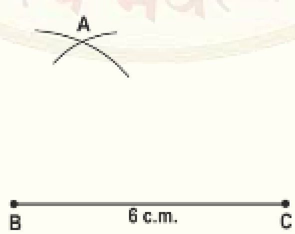


Step 2: Draw a line segment of length BC = 6 cm.

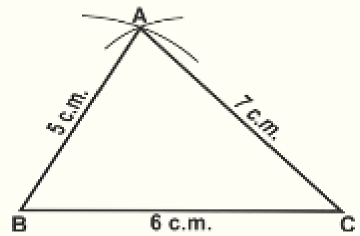
Step 3: The distance between B and A is 5 cm. Hence, draw an arc of radius 5 cm. Having the center at hand.



Step 4: AC = 7 cm. Hence, having C as the center and 7cm. Radius draw and arc.



Step 5: The point of intersection of both arcs is A. Join the points A, B, and C, which give triangle ABC.



Do and learn –

Construct triangle ABC, in which AB = 4 cm., BC = 6 cm. And CA= 5 cm.

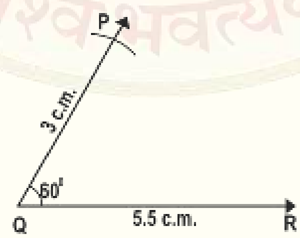
2. **Construction of the triangle when its two sides and the angle between them are given:**

When we are given two sides and the angle between them, first of all, we draw a rough diagram. Let us understand with the following example:

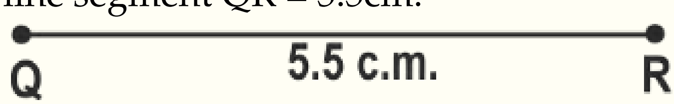
Example : Construct a triangle ΔPQR, given that PQ = 3 cm, QR = 5.5 cm. and ∠PQR = 60 ^{0}.

Solution:

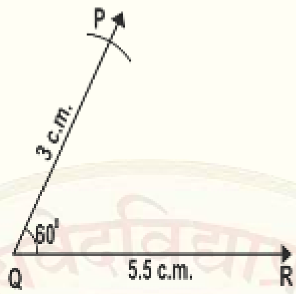
Step1: First of all, we draw a rough sketch, which will help us construct the required triangle.



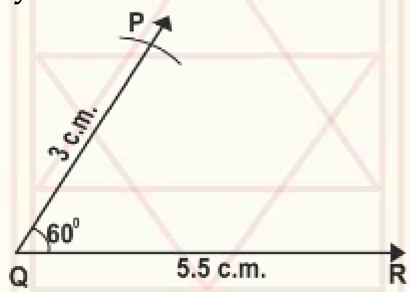
Step2: Draw a line segment QR = 5.5cm.



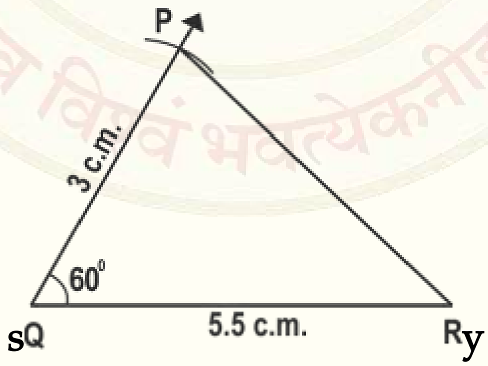
Step3: Draw a ray $\overrightarrow{QX}$ at Q such that ∠XQR = 60°. P will be on the ray.$\overrightarrow{QX}$



Step 4: To determine point P, we are given the distance QP. With Q as the center and a 3cm radius, cut an arc on the ray. To intersect the ray at P.



Step 5: Join P and R.Thus, ΔPQR is obtained.



Do and learn –

Construct ΔPQR,whenPQ = 5 cm., PR = 3 cm. And ∠RPQ = $90^{0.}$

3. **Construction of triangle when two angles and the sideincluding them are given:**

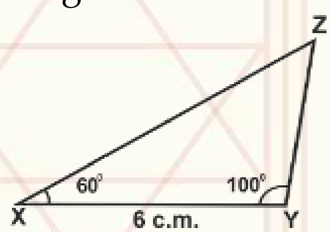
First of all, draw a rough sketch. Now draw the given line segment. Draw an angle at both endpoints.

Let us consider an example to understand the method of construction of a triangle when two angles and the side including them are given.

Example : Construct ΔXYZ if XY= 6.m., ∠ZXY = 30 ^{0}and ∠XYZ = 100 ^{0}

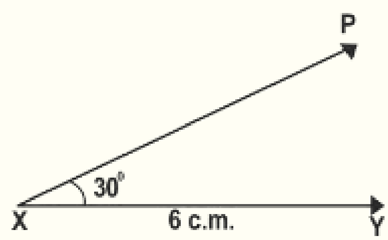
Solution:

Step1: First of all, we draw a rough sketch according to the given measurements. (This gives an idea of how to construct.)



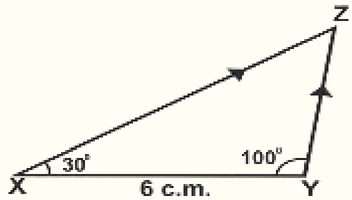
Step 2: Draw a line segment of length XY=6 cm.

Step3: At X, draw a ray XP such that angle PXY = 30°. According to the problem, the point Z should lie on this ray.

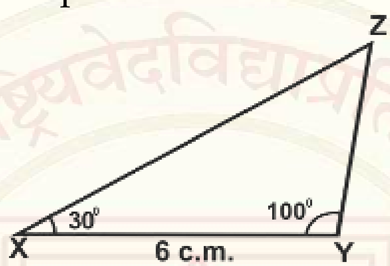


Step 4: At Y draw a ray YQ which makes an angle of $100°$ with XY.

Given According to the condition the point Z should lie on this ray.



Step 5: The point Z is the point of intersection of the rays XP and YQ.



Do and learn:

Construct Δxyz, When $\angle x = 60^{0}$, $\angle y = 30^{0}$ and xy = 5 cm.

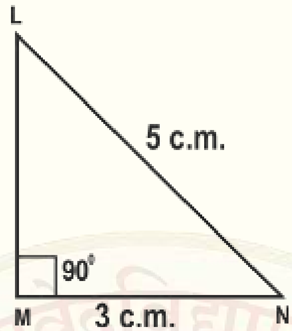
4. **Construction of the triangle when the hypotenuse and one side are given:**

Draw a line segment according to the given side and make a right angle at one of its endpoints. Use a compass to draw the side and hypotenuse of a triangle of the given length. Complete the triangle. Consider the following example:

Example: ConstructΔ LMN, when $\angle LMN$ = right angle, LN = 5 cm. and MN = 3cm.

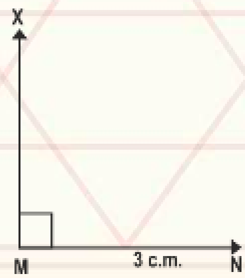
Solution:

Step1: Draw a rough sketch of the triangle, not forgetting to mark the measurements including the right angle.

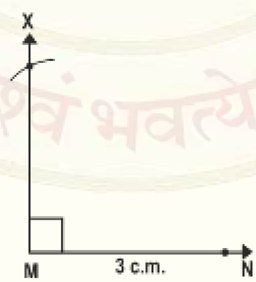


Step2: Draw a line segment MN = 3cm.

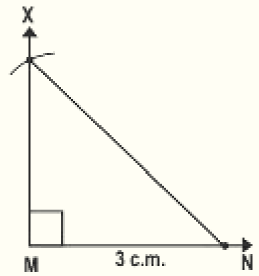
step 3: Draw MX⊥ MN. To draw this, construct a right angle (90°) at M.



Step 4: Draw an arc with center N and radius 5 cm. To cut MX at the L



Step 5: Join L and N. The triangle LMN is obtained.



Do and learn –

1. Construct a ∠right angled triangle PQR where $\angle Q = 90^{0}$, QR = 8 cm., PR = 10 cm.

Exercise 9.2

1. Write the required measurements to construct a triangle.
2. Construct ΔABC when AB = 4 cm., BC = 3 cm., CA = 5cm.
3. Construct ΔPQR when PR = 6 cm., PQ = 4.5 cm.$\angle P = 90^0$,
4. Construct Δxyz when xy = 4 cm.,$\angle x = 45^0$ and $\angle y = 50^{0.}$
5. Construct ΔABC where $\angle B = 90^0$, AC = 5 cm., BC = 3 cm.

We learned:

1. In this chapter we studied the construction of angles with the help of compasses.,
   (1) construction of $60^{0,}$ $120^0$and 180^0
   (2) construction of angles of $30^{0,}$ $45^{0,}$ 90 (by bisection)
2. I studied the methods of constructing triangles with the help of scales and compasses. We can construct the required triangle when the following measurements are given:
   1. Three sides
   2. two sides and the angle between them.
   3. two angles and the side between them.
   4. hypotenuse and another side of a right triangle.

Chapter - 10

Areas related to Circles

Dear students! In previous classes, we have studied various definitions and properties related to circles. You will remember that the circle is the path of a point. It is a simple but specific geometric shape in which there is no corner or side. Every point on the circle is at a constant distance from the given point, called its center. In this chapter, we will revise the concepts of a circle, learn to find the area of a sector of a circle, and learn to find the length of an arc.

Revision of concepts on circle:

Circumference of circle:

The following verse is found in the book named 'Trishatika' (Patiganita) written by 'Shridharacharya's, which mentions the circumference of a circle.

'वृत्तव्यासस्य कृतेर्मूलं परिधिर्भवति दशगुणायाः'

(त्रिशतिका क्षेत्र. श्लो. 45)

Meaning, Circumference = $\sqrt{10 \times Diameter}$

$$\text{Hence } \pi = \frac{\text{circumference}}{\text{Diameter}}$$

The distance covered in a single walk along a circle is its perimeter, which is often called the circumference. The circumference of a circle has a constant ratio to its diameter. This constant ratio is expressed by the Greek letter π (read as pi).

$$\pi = \frac{\text{circumference}}{\textbf{Diameter}}$$

$$\textbf{Circumference} = \boldsymbol{\pi} \times \textbf{Diameter}$$

$$\textbf{Circumference} = \boldsymbol{\pi} \times 2 \times \textbf{Radius}$$

Or circumference of circle= $2\,\pi r$

The reference about the value of $\boldsymbol{\pi}$ is found in the book **'Aryabhatiyam'** written by Aryabhatta, which is given below.

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।

अयुतद्वयविष्कम्भस्यासन्नौ वृत्त परिणाहः ॥

(आर्यभट्टीयम् , गणितापाद श्लोक 10)

Aryabhatta (476–550) A.D., a great Indian mathematician. Gave an approximate value of π. He said that $\pi = \frac{62{,}832}{20{,}000}$ which is equal to 3.1416. Also note that a great and talented **Indian mathematician 'Srinivasa Ramanujan' (1887–1920) found** an identity by which mathematicians are able to calculate the value of π correct to millions of decimal places. For practical purposes, this value is usually taken as $\frac{22}{7}$ or 3.14.

Example: Find the radius of the circle whose circumference is 44 cm.

Solution: Given: circumference of circle = 44 cm.

We know,

Circumference of circle = 2 π r

Or $2\pi r = 44$ cm.

$$r = \frac{44}{2\pi}$$

$$r = \frac{44}{2 \times \frac{22}{7}}$$

$$r = \frac{44 \times 7}{2 \times 22}$$

$$r = 7 \text{ cm}$$

Hence Radius of circle = 7 cm.

Area of circle:

An interesting method of finding the area of a circle is given by Bhaskaracharya ji in his book Lilavati Mathematic, which is given below.

व्यासस्य वर्गे भनवाग्निनिघ्ने सूक्ष्मं फलं पञ्चसहस्त्रभक्ते ।

रुद्राहते शक्रहृतेऽथवा स्यात् स्थूलं फलं तद्व्यवहारयोग्यम् ॥

(लीलावती गणित पृ.284)

According to the above verse, the area of a circle is found by multiplying 3927 by the square of its diameter and dividing the product by 5000.

For example:

If the diameter of a circle is 7 cm, the area of the circle will be?

Solution:

Diameter of the circle = 7 cm. Multiplying the square of the diameter (49) with 3927

$$3927 \times 49 = 192423$$

Dividing the product by 5000

$$\frac{192423}{5000} = 38.84$$

Hence, area of the circle is 38.84cm^2.

Activity: Draw a circle on some graph paper and count small squares inside the circle. The approximate area of the circle can be found.

The area of a circle can be expressed using the following formula:

$$\frac{\textbf{Area of circle}}{(\text{radius})^2} = \boldsymbol{\pi}$$

If the area of the circle is A and radius is r

$$\frac{A}{r^2} = \pi$$

$$\textbf{Area of circle } (\boldsymbol{A}) = \boldsymbol{\pi r^2}$$

Example: Find the circumference and area of a circle whose radius is 7 cm.

Solution: Given: radius of circle(r) = 7 cm.

We know that

Circumference of circle = $\mathbf{2\pi r}$

$$= 2\times \frac{22}{7} \times 7$$

$$= 2\times 22$$

$$= 44 \text{ cm.}$$

Area of circle$(A) = \boldsymbol{\pi r^2}$

$$= \frac{22}{7} \times (7)^2$$

$$= \frac{22}{7} \times 7 \times 7$$

$$= 22 \times 7$$

$$= 154 \text{ (cm)}^2$$

Hence the circumference of a circle = 44 cm. and area of circle= 154 $(\text{cm})^2$.

Exercise 10.1

1. Select the correct option for the following multiple-choice questions.

(a) The radius of a circle with a circumference of 44 meters will be –

(I) 14 meters (II) 7 Meter (III) 5 meter (IV) 4 4 meters

(b) If the ratio of the areas of two circles is 4:1, the ratio of their radii is

(I) 4: 1 (II) 2: 1 (III) 1: 2 (IV) 1: 4

(c) If the circumferences of two circles are in the ratio 2:3, then the ratio of their radii will be:

(I) 4: 9 (II) 2: 3 (III) 3: 2 (IV) 1: 4

(d) $\pi =$ ....

(I) Perimeter (II) $\frac{\text{Circumferences}}{\text{Diameter}}$

(III) $\frac{\text{Diameter}}{\text{Circumferences}}$ (IV) None of these

(e) The area of a circle is 64 π cm^2. What will be its diameter?

(I) 15 cm (II) 1 6 cm (III) 20 cm (IV) 1 7 cm

2. Find the radius of the circle whose circumference is 66 cm.

3. If the circumference of the circle is 88 cm., find the value of the radius of the circle.

4. If the radius of the circle is 14 cm., find the circumference and area of the circle.

5. Find the area of the circle whose radius is 7 cm.

6. The radius of the circle is 2 cm; then find the circumference and area of the circle.
7. From a circular sheet of radius 21cm, a circular part of radius 17.5 cm is cut out. Find the perimeter and area of the remaining sheet.

Area of a sector of a circle:

The angle enclosed by a circle is 360°. The reference to this is given in the Rigveda, which is given below.

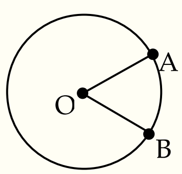
द्वादश प्रधयश्चक्रमेकं त्रीणि नभ्यानि क उ तच्चिकेत ।

तस्मिन्त्साकं त्रिशता न शंकवोऽर्पिता षष्टिर्न चलाचलास:।

(ऋग्वेद : 1/164/48)

Meaning, there is a circle that is surrounded by twelve (twelve) rays. Its three divisions (navels) Only a scholar (a mathematician) understands it. Three hundred sixty (360) movable nails are attached to it. The above-mentioned verse (Satyavachan) describes a circular wheel, which is connected to the axis by twelve spokes. It has three equal divisions. It is attached to the center by three hundred and sixty (360) moving spokes. Each division of this wheel creates an angle of one hundred and twenty (120) degrees at the center.

Enclosed by two radii, an arc of a circle is called a sector of the circle.



The area enclosed by the sector of a circle is called the **area of the** sector.

$$\text{Area of sector} = \pi r^2 \times \frac{\theta}{360}$$

The following formula is used to find the length of the arc AB of radius –

$$\text{Arc length} = 2\pi r \times \frac{\theta}{360}$$

Example: If the area of the circle is 180 (cm) 2 and the angle of the sector is 90°. then find the area of the minor sector made by the arc.

Solution: Given: area of circle= 180 $(cm)^2$

Angle of sector= 90^0

We know,

Area of sector $= \pi r^2 \times \frac{\theta}{360}$

$= 180^0 \times \frac{90}{360}$

$= \frac{90}{2}$

$= 45\ (cm)^2$

Example: If the circumference of the circle is 90 cm is and the angle subtended by the arc at the center is 80^0 find the length of the arc.

Solution: Given: circumference of circle ($2\pi r$) =90 cm

angle subtended by arc = 80^0

Then we know,

The length of Arc = $2\pi r \times \frac{\theta}{360}$

$= 90 \times \frac{80}{360}$

$= 20$ cm

Example: Find the area of the sector of a circle whose area A and the angle subtended at the center of the sector are as follows.

Area (πr^2) = 36 $(cm)^2$ angle subtended at the center (θ)= 50^0

Solution: We know

Area of sector $= \boldsymbol{\pi r^2} \times \frac{\boldsymbol{\theta}}{\mathbf{360}}$

$= 36 \times \frac{50}{360}$

$= \frac{\mathbf{50}}{\mathbf{10}}$

$= 5\ (\text{cm})^2$

Hence, the area of the sector is 5 $(\text{cm})^{2.}$

Example: If the circumference of the circle and the angle subtended by one of its arcs are 60 cm, and 54°, respectively, find the length of the arc.

Circumference (2 π r) = 60 cm., angle subtended at center(θ) = 54^{0}

Solution– We know-

Arc length of sector $= \mathbf{2\pi r} \times \frac{\boldsymbol{\theta}}{\mathbf{360}}$

$= 60 \times \frac{54}{360}$

$= \frac{\mathbf{54}}{\mathbf{6}}$

$= 9$ cm

Exercise 10.2

1. Fill in the blanks by writing the following formulas,

   (a) Area of circle =……………………..........

   (b) Circumference of circle =……………………..........

   (c) Area of the segment = …………………………..

   (d) length of the arc of the sector =…………………………..

2. Find the length of the arc of the sector, if the circumference of the circle (2 π r), subtended by the sector the angle (θ) is as follows.

   a) Circumference (2 π r) = 120 cm., θ=180 ^{0}

   b) Circumference (2 π r) = 240 cm., θ=150 ^{0}

3. Find the area of the sector, if the area of the circle ($\boldsymbol{\pi r^2}$) and the angle between their radii (θ) are as follows.

   a) Area of circle ($\boldsymbol{\pi\, r^2}$) = 90 $(cm)^2$, θ= 120 ^{0}

   b) area of the circle ($\boldsymbol{\pi\, r^2}$) = 36 $(cm)^2$, θ= 15 ^{0}

4. If the area of the circle is 40 $(cm)^2$ and the angle between two radii is 180^0, Find the area of the sector formed by the radii of the circle.

5. If the circumference of the circle is 36 cm. and the angle subtended by one of its arcs is 60^0 find the length of the arc of the circle.

We learned -

1) **Circumference of a circle of** radius (r) **= 2πr**
2) **Area of circle with** radius(r)**=** $\boldsymbol{\pi r^2}$
3) A sector of a circle of radius (r) whose internal angle is in degrees, corresponding to

   The length of arc = 2 π r × $\frac{\theta}{360}$.

4) The sector of a circle has a radius (r), and the internal angle is in degrees.

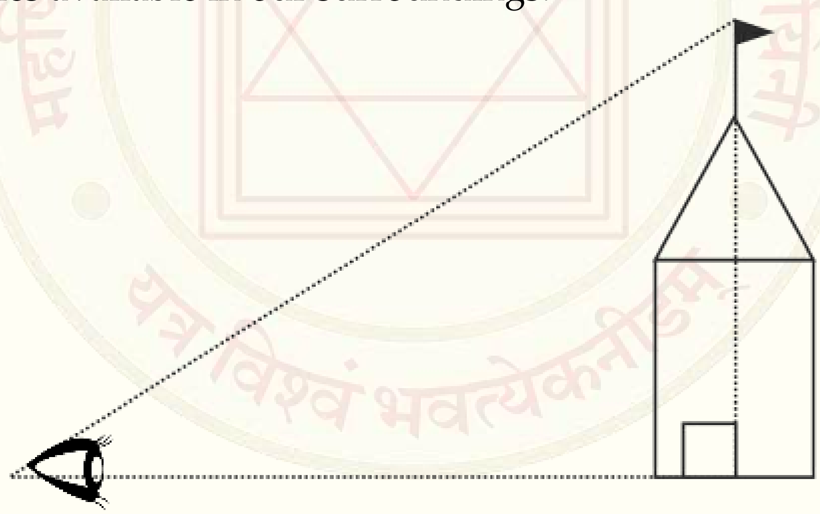
   Area of sector = $\boldsymbol{\pi r^2} \times \frac{\theta}{360}$

Chapter - 11

Introduction to Trigonometry

Dear students! Trigonometry is an important branch of mathematics. It combines the concepts of shape and space with other mathematical concepts such as proportion, deduction, and mathematical proof. It also provides scope to connect what is seen in real life with the world of mathematics. You have studied in detail about right triangles under 'Baudhayana Theorem' in earlier classes.

Understand the idea of the formation of a right-angled triangle with the examples available in our surroundings.



Consider the following:

A student of Gurukul is looking at the flag on the top of the temple, so a right-angled triangle can be imagined to be formed, as shown in the above figure. In the above-mentioned situation, distances or heights can be found using some mathematical techniques that fall under a branch of mathematics called trigonometry.

Trigonometry:

Indian mathematician Bhaskaracharya has given the following definition of trigonometry in 'Simple Trigonometry' in the form of a sholka.

त्रिकोणस्य मानमेव त्रिकोणमिति शब्दार्थः, त्रिभुजस्य भुजानां कोणानां च परिमापनं तथा पारस्परिक सम्बन्धज्ञानमेवास्य मुख्यलक्ष्यमस्ति । भारतीयगणितशास्त्रे ज्योत्पत्त्या क्षेत्र – फलाध्यायेन च मूलरूपेण यद्गणितं तदेव विस्तरेण त्रिकोणमिति नाम्ना व्यवहृतम् ।

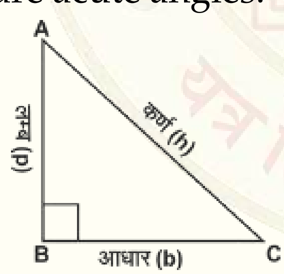
(सरलत्रिकोणमितिः, परिभाषा)

Scholars consider the word trigonometry to be derived from the Sanskrit word 'Tri', from the word angle + gonia from the metal 'm'(meaning) +metron. Almost on hearing, the word trigonometry is exactly the same as the Sanskrit word 'trigonometry'. Trigonometry studies the relationships between the sides and angles of a triangle. English word 'Trigonometry' It is made up of three words (Tri + gon + metron), which means the word trigonometry stands for measurement of the three sides of a triangle. In ancient times, trigonometric knowledge was used to calculate the distance of planets and stars from the Earth. Knowledge of trigonometry is used in making maps and finding the position of an island with respect to longitude and latitude.

In this chapter, we will study the ratio of the sides of a right-angled triangle with respect to the acute angle, which are called trigonometric ratios of the angles.

Right angle Triangle:

A triangle whose one of the angles is 90^0 is called **a right-angled triangle.** In this triangle, one angle is a right angle, and two other angles are acute angles.



In this $\angle B$ = right angle (90^0)

Hypotenuse:

In a right triangle, the side opposite the right angle is called **the hypotenuse.** This is the largest side of the triangle; it is denoted 'h'.

Base:

The horizontal side adjacent to an acute-angle triangle is called the **base,** which is represented by 'b'.

Perpendicular:

In a right triangle, the vertical side adjacent to the acute angle is called the **perpendicular**. Which is represented by 'P'.

Baudhayana theorem (Pythagoras theorem):

The following shloka of the Baudhayana theorem is found in the Vedic literature: Manavashulbasutram'.

Sutra – **आयाममायामगुणं विस्तारं विस्तरेण तु ।**

समस्य वर्गमूलं यत्तत्कर्णं तद्विदो विदुः ॥

(मानवशुल्बसूत्रम् / 10.10)

The square root of the sum of the squares of the perpendicular and square of the base gives the length of the hypotenuse.

Apart from Manavshulabsutra (10.10), the Baudhayan theorem is also mentioned in Baudhayan Shulbsutra (1.48) and Lilavati Ganita (Kshetravyavahar verse 2).

Boudhayan's theorem shows the relationship between the sides of a right triangle.

Baudhayana (Pythagoras theorem): in a right triangle

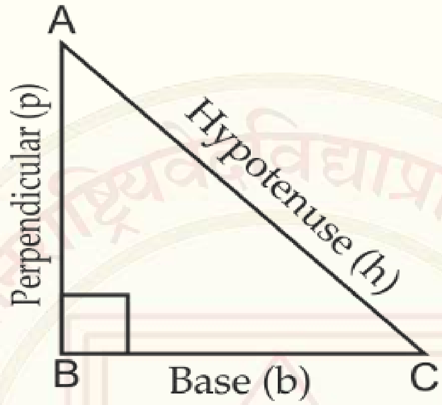
$$(\text{hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$

$$(H)^2 = (P)^2 + (B)^2$$

Where H = Hypotenuse, P = Perpendicular and B = Base.

Trigonometric Ratio:

In a right triangle, the ratios between the sides expressed in relation to the acute angles are called **trigonometric ratios of the angles.**



Suppose∠C is an acute angle, then the side AB opposite to the acute angle will be perpendicular, the adjacent side BC is the base and the side AC opposite to the right angle ∠B is the hypotenuse (h). The sine of C is written in abbreviated form as" $\sin C$".

Note : Here" $\sin C$" does not mean"sin" times C. Rather its stands for sine of the angle C.

$$\text{sine } C = \frac{\text{Perpendicular}(p)}{\text{Hypotenuse}(h)}, \qquad \text{cosecant } C = \frac{\text{Hypotenuse}(h)}{\text{Perpendicular }(p)} = \frac{1}{\sin C}$$

$$\text{cosine } C = \frac{\text{base}(b)}{\text{hypotenuse}(h)}, \qquad \text{Secant } C = \frac{\text{Hypotenuse}(h)}{\text{Base(b)}} = \frac{1}{\cos C}$$

$$\text{Tangent } C = \frac{\text{Perpendicular }(p)}{\text{Base}(b)} = \frac{\sin C}{\cos C}$$

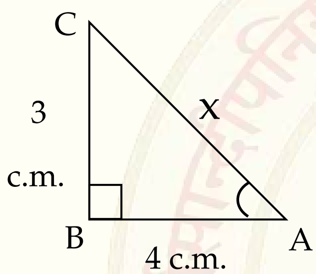
$$\text{Cotangent } C = \frac{\text{Base(b)}}{\text{Perpendicular }(p)} = \frac{1}{\text{Tan } C} = \frac{\text{Cos } C}{\text{Sin } C}$$

The three trigonometric ratios formed between the sides of a right triangle are written as sin C, cos C and tan C. The other three ratios are written as cosec C, sec C and cot C. Therefore, there are **six trigonometric ratios relative to the acute angles of a right triangle.**

Example: In a right-angled triangle, the perpendicular is 3 cm and the base is 4 cm. Find the measurement of the hypotenuse.

Solution: Given: perpendicular of right-angled triangle = 3 cm and base = 4 cm we know that - by, Baudhayan'stheorem:

$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$



$(\text{Hypotenuse})^2 = (3)^2 + (4)^2$

$(\text{Hypotenuse})^2 = 9 + 16$

$(\text{Hypotenuse})^2 = 25$

$\text{Hypotenuse} = \sqrt{25}$

$= 5$ cm.

Example: If $\tan A = \frac{3}{4}$, find the values of $\cos A$ andsin A.

Solution: Given:$\tan A = \frac{3}{4}$

We know that

$$\tan A = \frac{3}{4} = \frac{Perpendicular}{Base}$$

By bhodhayan theorem

$$(\text{Hpotenuse})^2 = (\text{Perpendicular})^2 + (\text{ase})^2$$
$$(\text{hypotenuse})^2 = (3)^2 + (4)^2$$
$$(\text{hypotenuse})^2 = 9 + 16$$
$$(\text{hypotenuse})^2 = 25$$
$$\text{hypotenuse} = \sqrt{25} = 5\text{cm}.$$

Then, $\cos A = \frac{\text{base}}{\text{hypotenuse}} = \frac{4}{5}$ $\sin A = \frac{\textbf{perpendicular}}{\text{hypotenuse}} = \frac{3}{5}$

Baudhayana (Pythagorean) triplets

In the above example3, 4 and 5 are Baudhayana (Pythagorean) triplets. You will remember: **in a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.**

From Baudhayana's theorem,

$$\textbf{(hypotenuse)}^2 = \textbf{(perpendicular)}^2 + \textbf{(base)}^2$$

It means that all three sides are related in a special manner. To calculate fast in **Vedic mathematics, you** will have to memorize Baudhayana triplets (Pythagorean triplets).

तासां त्रिक चतुष्कयोः द्वादशिक पंचिकयोः पञ्चदशिकाष्टिकयोः सप्तिक चतुर्विंशिकयोः

द्वादशिक पञ्चत्रिंशिकयाः पञ्चदशिक षट्त्रिंशिकयोः इत्येतासु उपलब्धिः।

(बौधायन शुल्बसूत्र 1.38)

Following is the table of Baudhayana triplets for your help according to the verses of the above-mentioned Baudhayana Shulbasutra.

A	B	C
3	4	5
12	5	13
15	8	17
7	24	25
12	35	37
15	36	39
6	8	10
8	15	25
9	40	41
9	12	15
12	15	18
….	…	…
3n	**4n**	**5n**

From the above table, suppose the two sides of a right-angled triangle are 3 and 4, and you have to find its largest side, which means hypotenuse. In the traditional method, you would use Pythagoras (Baudhayana theorem) for this, as you have used in the example. On the other hand, if you are familiar with the said triplets, then in less than a moment you will be able to find through the method of Vedic mathematics that the third side is 5.

करणानि ततोऽस्याःकारयेत्त्रिचतुःपञ्चत्रिरभिपर्यस्य यच्चुभयम्

चयनेषु विधिः पुरातनैर्ऋषिभिर्योऽभिहितश्च नित्यशः।

(बौधायन शुल्बसूत्र 11. 17)

The above formula, the general formula for the square of a right-angled triangle has been stated from Baudhayan's theorem.

Which is as follows:

$$(5n)^2 = (3\ n)^2 + (4\ n)^2$$

Where n is a non-zero number. If n = 2

$$(10)^2 = (6)^2 + (8)^2$$

Let's learn to calculate Pythagorean triplets.

Case 1 : When the number is odd.

We know that the square of an odd number is also an odd number. This square is the sum of two consecutive numbers. If you know the value of one of the triplets, then you can easily find the value of the remaining two.

Look carefully at the following example.

$3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

$9^2 = 81 = 40 + 41$

If you look at the above example, you will know that any square of an odd number can be written as the sum of two consecutive numbers, and from them, a triplet is formed.

$3^2 = 9 = 4 + 5$ forms the triple (3, 4, 5).

In this way, (5, 12, 13), (9, 40, 41), and Baudhayana **(Pythagoras)** triplets can be found.

Pratyush: How do I get consecutive pairs of numbers?

Guruji: It is very easy. Divide the square by 2 and round it.

In the first example the square of 3 is 9, which means $3^2 = 9$. Dividing it by 2 gives 4.5. Converting 4.5 to an integer gives the next number 5 and the previous number 4. Thus, **(3, 4, and 5)** Together, they form a triplet. where the square of the larger number is equal to the sum of the squares of the other numbers.

In the second example, the square of 5 is 25. When 25 is divided by 2, we get 12.5. The corresponding integers will be 12 and 13. **The interesting thing is that (5, 12, 13) form a triplet.**

Case 2: When the number is even.

Rule: Divide the number by 2, 4, 8, etc. So that you get an odd number. Apply the rules described in **Case 1** to getting an odd number. If you have divided the number by an even number such as 2, 4, 8, etc., then the quotient of the final answer should be multiplied by the same number, which will give a triplet.

Let us understand with an example.

Example: If one of the values of the triplet is 6, find the other two values.

Solution: Divide the number 6 by 2 to get an odd number. For 3 as mentioned above:

Triplet $3^2 = 9 = 4 + 5$

So the triplets are 3, 4 and 5, since we divided by 2,

by multiplying all the values by 2, we get

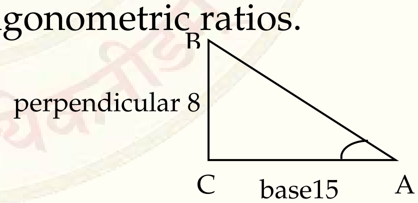
$2 \times 3, 2\times 4, 2 \times 5$

Hence, the triplet formed by 6 is 6, 8, and 10.

Calculation of Trigonometric ratio -

Example: if $\tan A = \frac{8}{15}$ then find other trigonometric ratios.

Solution: $\tan A = \frac{\text{perpendicular}(p)}{\text{base }(b)} = \frac{8}{15}$



Frome the table of triplets of Vedic mathematics we know

Perpendicular= 8, Base= 15, Hypotenuse= 17

Hence,

$\sin A = \frac{\textbf{perpendicular}(p)}{\text{Hypotenuse}(h)} = \frac{8}{17}$ $\quad \text{cosec } A = \frac{\text{Hypotenuse}(h)}{\textbf{perpendicular}(p)} = \frac{17}{8}$

$$\cos A = \frac{\text{base}(b)}{\text{Hypotenuse}(h)} = \frac{15}{17} \qquad \sec A = \frac{\text{Hypotenuse}(h)}{\text{base(b)}} = \frac{17}{15}$$

$$\cot A = \frac{\text{base(b)}}{\textbf{perpendicular}(p)} = \frac{15}{8}$$

After easily finding the values of three ratios, you can also find other ratios. Because $\operatorname{cosec} A$ and $\sin A$ are inversely proportional to each other. Similarly, $\sec A$ and $\cos A$ are inversely proportional $\tan A$ and $\cot A$ are inversely proportional.

Exercise11.1

1. Select the correct option for the following multiple-choice questions.

(a) If sin A = 3/4, then the value of cosec A will be

(I) 4/3 (II) 3/4 (III) 3/5 (IV) 5/ 4

(b) If sec A = 5/3, then what will be the value of tan A ?

(I) 4/3 (II) 3/4 (III) 3/5 (IV) 5/ 4

(c) If sin A = 3 / 4, then what will be the value of cos A ?

(I) $\frac{4}{3}$ (II) $\frac{\sqrt{3}}{4}$ (III) $\frac{\sqrt{4}}{3}$ (IV) $\frac{\sqrt{7}}{4}$

(d) Which of the following is $\cot\theta$ equal to.

(I) $\frac{\sin\theta}{\cos\theta}$ (II) $\frac{\cos\theta}{\sin\theta}$ (III) $\frac{1}{\sec\theta}$ (IV) $\frac{1}{\sin\theta}$

2. Fill in the blanks.

($\frac{\text{hypotenuse}}{\text{base}}$, $\frac{\text{perpendicular}}{\text{base}}$, $\operatorname{cosec} A$, hypotenuse, $(\text{perpendicular})^{2}$, perpendicular, hypotenuse, 6)

a) In a right angled triangle, the side opposite to the right angle is called …………..

b) In a triangle, the side opposite to the acute angle is called………….

c) Baudhayan's theorem: $(\text{hypotenuse})^2$=................ + $(\text{base})^2$

d) The number of trigonometric ratios is…………..

e) The largest side of a right angled triangle is…………..

f) $\tan A$= ________________.

g) $\sec A$= ________________.

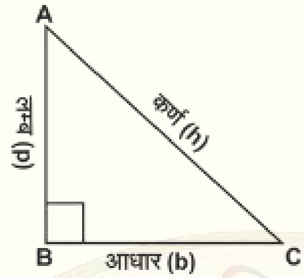
3. if $\tan A = \frac{3}{4}$ what is the value of $cotA$?

4. If $\sin A = \frac{3}{5}$ Find the value of $tanA$ and $cotA$.

5. If$\cos A = \frac{5}{13}$ find other trigonometric ratios.

We learned -

1) ABC is a right angled triangle with angle B at right angle(90^0),



$\sin C = \frac{\text{perpendicular}(p)}{\text{hypotenuse}(h)}$, $\text{cosec}\, C = \frac{\text{hypotenuse}(h)}{\text{perpendicular}\ (p)}$

$\cos C = \frac{\text{base}(b)}{\text{hypotenuse}(h)}$, $\sec C = \frac{\text{hypotenuse}(h)}{\text{base(b)}}$

$\tan C = \frac{\text{perpendicular}\ (p)}{\text{base}(b)}$, $\cot C = \frac{\text{base(b)}}{\text{perpendicular}\ (p)}$

2) Trigonometric ratios $cosec\ A$ and $\sin A$ are inversely proportional. $\sec A$ and $\cos A$ are inversely proportional, $\tan A$ and $\cot A$ are inversely proportional.

3) Studied the Vedic method of making Pythagorean triplets.

Chapter - 12

Statistics

Dear Students! We have discussed some concepts of statistics in previous classes. You will remember the collection of data, classification, tabulation and representation in the form of graphs to assess the data in their grouped forms. In simple language, the word statistics is used for data or numerical information.

In statistics, various unclassified data are summarized and their values studied. Central measures are found to understand the collection of data comprehensively.

To represent unclassified data, we calculate certain values called measures of central tendency. Generally, the arithmetic mean (average), median, and mode are used to measure central tendency.

Calculation of Mean:

Calculation of mean of ungrouped data:

The arithmetic mean of ungrouped data is obtained by dividing the sum of all the given values of the variable by the total number of variables. Let the total number of values n and the considered variable is x have the values $x_1, x_2, x_3 \ldots\ldots\ldots x_n$ in unclassified data, the definition of mean is as follows.

Definition:

The arithmetic mean (mean) is the number obtained by dividing the sum of all the terms by the number of terms. The mean is represented by ($\bar{x}$).

Hence, mean = $\frac{\text{(Sum of all the terms)}}{\text{(Total number of terms)}}$

$$\bar{\boldsymbol{x}} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Note: Sign $\sum_{i=1}^{n} X_i$ stands for the sum of all the values of x. '$\sum$' is a Greek letter, and it is read as Sigma.

Example: The weight of 10 apples taken out of a basket are as follows.

150 grams, 200 grams, 175 grams, 170 grams, 250 grams, 215 grams, 220 grams, 260 grams, 270 grams, and 190 grams. Find the average weight of one apple.

Solution:

Mean weight of apples in the basket=$\frac{\text{sum of weight of 10 apples}}{\text{total no. of apples}}$

$= \frac{150+200+175+250+215+220+260+270+190}{10}$

$= \frac{2100}{10}$ gram

$=$ 210 grams

We observe that the weight of all the apples is around the mean weight of the apples. **The mean is between the lowest and highest weights.**

Exercise 12.1

1. Select the correct option for the following multiple-choice questions.

(a) If the mean of x, 3, 4 and 5 is4, then the value of x will be –

(I) 4 (II) 60 (III) 10 (IV) 5

(b) The mean of the first five whole numbers is –

(I) 2 (II) 3 (III) 2.5 (IV) 4

(c) The mean of the first five natural numbers will be –

(I) 2 (II) 3 (III) 1 (IV) 4

(d) If the mean of 1, 4, x, 5, 12 is 7, then x is equal to –

(I) 15 (II) 13 (III) 14 (IV) 16

(e) Which of the following will be the mathematical mean of the numbers a, b and c –

(I) abc (II) $\frac{a+b+c}{3}$ (III) b (IV) $\frac{a+c}{3}$

(f) The mean of the first five odd natural numbers will be –

(I) 6 (II) 5 (III) 8 (IV) 4

2. 10^{th} grade students in the 5^{th} year of Vedbhushan scored the following marks in a mathematics test: Find the mean and average marks of the students.

7, 8, 5, 7, 7, 8, 9, 6, 7, 6

3. A cricket player scored 60, 62, 56, 64, 0, 57, 32, 27, 9 and 71 runs in 10 innings. Find the average number of runs scored in these innings.

4. The speed of 5 Motorcyclist's speed (in km/hour) was recorded as follows.

   Find the mean of 55, 60, 30, 45, and 40.

Median:

Median of ungrouped data: The median of ungrouped data is the data that is exactly at the middle (center) when it is arranged in either ascending or descending order.

Let the total number of given data b n', which can be either an odd or even number. To find the median of unclassified data, first arrange the data in either ascending or descending order. If the number of data points in a collection is odd, the median will be unique, which is equal to $\left(\frac{n+1}{2}\right)^{th}$ data in the arrangement of the datain their ascending or descending order.

If the value of n is even, the median of data set is the average of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ data in the arrangement of data in their ascending or descending order.

Important point:

1) If n is odd then median = the value of $\left(\frac{n+1}{2}\right)^{th}$ data

2) If n is even

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th}\text{ term} + \left(\frac{n}{2}+1\right)^{th}\text{ term}}{2}$$

1. When n is odd

Example: Find the median of the following values.

15, 35, 18, 26, 19

Solution: On arranging in ascending order,

15, 18, 19, 26, 35

In this n = 5 (an odd number)

Median = $\left(\frac{n+1}{2}\right)^{th}$ value of the term

= $\frac{5+1}{2}$ value of the term

= $\frac{6}{2}$ = 3 value of the term

Hence, 3 Value of the term= 19.

2. When n is even

Example: Find the median of the following data.

17, 15, 8, 1, 6, 5, 11, 3

Solution: Arrange the given data in ascending order

1, 3, 5, 6, 8, 11, 15, 17

In this total number of terms (n) = 8 (even number)

Hence, **Median** $= \dfrac{\text{value of } \frac{n}{2} \text{ th term} + \text{value of } \left(\frac{n}{2}+1\right) \text{ th term}}{2}$

$= \dfrac{\text{value of } \frac{8}{2} \text{ th term} + \text{value of } \left(\frac{8}{2}+1\right) \text{ th term}}{2}$

$= \dfrac{\text{value of 4 th term} + \text{value of 5 term}}{2}$

$= \dfrac{6+8}{2}$

$= \dfrac{14}{2} = 7$

Hence given Median of data= 7.

Exercise 12.2

1. Select the correct option for the following multiple-choice questions.

(a) The median of 18, 13, 17, 12, 16 and 19 is –

(I) 15 (II) 16.5 (III) 1 9 (IV) 17.5

(b) What will be the median of the variable values 1, 2, 3, 4, 5 and 6 ?

(I) 4 (II) 5 (III) 3.5 (IV) 6

(c) When variables are placed in ascending or descending order, the value of the middle variable is called.

(I) Median (II) Mean (III) Mode (IV) None of these

(d) What will be the median of Variables 7, 8, 10, 9 and 6?

(I) 6 (II) 7 (III) 8 (IV) 10

(e) What will be the median of 5, 3, 7, 6, 4, 2, 1?

(I) 5 (II) 4 (III) 3 (IV) 2

2. Find the median of the following data. (when n = even number)

a) 21, 18, 12, 14, 3, 4

b) 1, 8, 15, 21, 34, 12, 3, 6

c) 20, 50, 30, 40, 10, 70

3. Find the median of the following data. (When n = odd number)

a) 18, 35, 12, 15, 8

b) 1, 12, 17, 8, 5, 21, 34, 42, 3

c) 2, 4, 5, 6, 17, 11, 0

Mode:

Mode is the value of the term that appears most often in the given data, which means the term that has the highest frequency; in other words, the **value with the maximum frequency is mode.** Let us understand the solution from the following example:

Example: Find the mode of the following data.

6, 9, 2, 3, 4, 2, 12, 9, 7, 8, 9, 5, 9

Solution: We know that mode is the value of the term that has the highest frequency.

Following is the frequency table for the given quantities.

scale	2	3	4	5	6	7	8	9	12
frequency	2	1	1	1	1	1	1	4	1

The most frequent number here is 9. (which means. 9 has occurred four times)

Hence, the mode of the above data is 9.

Example: Find the mode of the following observations (data).

8, 1, 3, 2, 3, 3, 4, 5, 3, 3, 6, 2, 2

Solution: Frequency table of the above data

scale	1	2	3	4	5	6
frequency	1	3	5	1	1	1

In this case of observation 3 is more frequent than 5.

Hence, mode of the above data is 3.

Exercise12.3

1. Select the correct option for the following multiple-choice questions.

(a) Mode of any frequency is –

(I) Least frequency value (II) Mean frequency value (III) Maximum frequency value (IV) None of these

(b) Which of the following is the mode of 3, 5, 0, 3, 5, 5, 4, 5 ?

(I) 3 (II) 2 (III) 5 (IV) 4

(c) 4, 2, 5, 2, 0, 7, 7, 7 is the mode of –

(I) 7 (II) 2 (III) 5 (IV) 4

2. Find the mode of the following data.

a) 10, 8, 15, 10, 12, 10, 8, 15, 10, 12, 13

b) 1, 5, 4, 5, 3, 5, 2, 1, 5, 5, 2, 3

c) 1, 1, 1, 2, 3, 5, 3, 3, 4, 4, 5, 5, 5, 5

We learned:

1) **Mean– The arithmetic** mean is the number obtained by dividing the sum of all the data points by the total number of points. It is represented by '$\overline{x}$'.

Now, **mean ($\overline{x}$)** $= \frac{\text{(Sum of terms)}}{\text{(Total number of terms)}} = \frac{\sum_{i=1}^{n} x_i}{n}$

2) **Median:** When ungrouped data is arranged in ascending or descending order, the exact middle value is called the **median.**

3) When number of terms n = even number:

$$\text{Median} = \frac{\text{value of } \frac{n}{2} \text{ th term} + \text{value of } \left(\frac{n}{2}+1\right) \text{ th term}}{2}$$

4) When number of terms n = odd number–

Median = $\frac{n+1}{2}$respect for that position

5) **Mode:** The mode of ungrouped data is the data that has the highest frequency.

Chapter-13

Probability

My dear students! In previous class we have read about the topic of probability. You will remember that in probability theory, we calculate the numerical value of the probable occurrence of events. The probability theory is used in various fields (statistics, mathematics, science, and philosophy). In this chapter, we will study in detail probability.

Infect, there are some questions that are based on uncertainties. For which the result may be favorable or unfavorable. Consider the following simple experiments:

a) To record the result of heads or tails on tossing a coin.

B) Getting an even number of results on throwing a die.

Head Tail dice

In these experiments, we cannot reach any conclusion with certainty about whether the outcome will be favorable or unfavorable. It depends on the probabilities of the occurrence of events.

In the Vedas, the word Avyaya '**Nunun**' has been used in the sense of possibility or doubt. In this context, the words are –

न नूनमस्ति नो श्वं कस्तद् वेद यदद्भुतम् ।

अन्स्य चित्तमभि संचरेण्य मुताधीतं वि नश्यति ॥

(ऋग्वेद : 1/170/1)

Meaning, what is not for me today, I will get tomorrow. This cannot be said to be a possibility because fickle people are suspicious.

Therefore, probability is a numerical representation of the possibilities of an event occurring. Probability is the concept through which the possibilities of events happening or not happening **can be expressed numerically.**

Definition of probability:

I fan experiment has a total of n outcomes and among them m outcomes are favorable to a particular event A, the ratio of $\frac{m}{n}$, is expressed by the symbol P (A) **(probability of occurrence of A**).

Hence,

$$P(A) = \frac{No.of \text{ favorable outcome}}{\text{Total no.of outcome}}$$

$$= \frac{m}{n}$$

If the occurrence of an event in an experiment is certain, then (n = m) and

$$P(A) = \frac{n}{n} = 1$$

If the occurrence of an event A is impossible, then m = 0 and

$$P(A) = \frac{0}{n} = 0$$

For any event A

$$0 \leq P(A) \leq 1$$

Which means, the probability of an event cannot be less than 0 and more than 1, and **the range of probability is from 0 to 1.** The probability of the event not occurring is represented by P($\bar{A}$).

$$P(\bar{A}) = \frac{\text{No.of Unfavorable outcomes of event A}}{Total\ number\ of\ outcomes\ of}$$

$$= \frac{n-m}{n}$$

$$= 1 - \frac{m}{n}$$

Or $\mathbf{P(\bar{A}) = 1 - P(A)}$

$\mathbf{P(\bar{A}) + P(A) = 1}$

Hint:

1) P(A) = Probability of occurrence of event(A)

2) P($\bar{A}$) = Probability of event (A) not occurring

Let us understand using example:

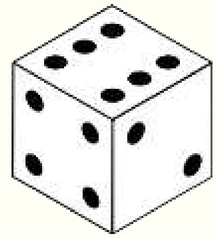
Example: Find the probability of getting an even number when a dice is thrown.

Solution: On throwing a dice, there can be 6 different outcomes (1, 2, 3, 4, 5, and 6).

Hence, total number of outcomes = 6

Number of favorable outcomes (2, 4, 6) = 3

Then **probability** = $\frac{No.of \text{ favorable outcome}}{\text{Total no.of outcome}}$



$= \frac{3}{6}$

Or $= \frac{1}{2}$

Hence, the probability of getting an even number when throwing a dice is $= \frac{1}{2}$

Example: If two coins are tossed together, find the probability of getting heads on both the coins.

Solution: when two coins are tossed together the total number of possible outcomes =

(H, H), (T, H), (H, T), (T, T)

Hence, Total number of outcomes = 4

Out of these, favorable outcome of the event = 1 (H, H)

Hence, **P (A) =** $\frac{no.of \text{ favorable outcome}}{\text{Total no.of outcome}}$

$= \frac{1}{4}$

Hence, the probability of getting heads on both coins = $\frac{1}{4}$.

Example: Find the probability of getting the number 9 on its face when a dice is thrown.

Solution: The total number of outcomes of the event is 6, which includes 1, 2, 3, 4, 5, and 6. The number 9 is not included in these total results; hence, we will not get the number 9, which means it is impossible to get the number 9. In this, the **number of favorable results is 0, and the total number of results is 6.**

Hence, **P (getting number 9) =** $\frac{no.of\ \text{favorable outcome}}{\text{Total no.of outcome}}$

$$= \frac{0}{6}$$

$$= 0$$

Hence The probability of getting the number 9 on the face of the dice is 0. Which is an impossible probability.

Exercise 13.1

1. Select the correct option for the following multiple-choice questions.

(a) The probability of getting the number 5 on throwing a dice is –

(I) $\frac{1}{2}$ (II) $\frac{1}{3}$ (III) $\frac{1}{5}$ (IV) $\frac{1}{6}$

(b) The probability of getting a head on tossing a coin will be–

(I) $\frac{1}{2}$ (II) $\frac{2}{3}$ (III) $\frac{2}{2}$ (IV) $\frac{1}{4}$

(c) The probability of getting an even number when throwing a dice is–

(I) $\frac{1}{2}$ (II) $\frac{2}{3}$ (III) $\frac{1}{3}$ (IV) $\frac{1}{6}$

(d) The probability of a certain event has its maximum value.

(I) 0 (II) 1 (III) $\frac{1}{4}$ (IV) 2

(e) What is the sum of the probabilities of all the events of an experiment?

(I) 0 (II) 1 (III) -1 (IV) 4

2. Fill in the blanks.

(0 and 1, 1, 0, 1, P (A), 6)

1) The value of probability lies between………… and………….

2) The probability of a certain event occurring is…………..

3) If the occurrence of an event (e) is impossible then the probability will be…………..

4) The total number of outcomes when a dice is thrown is …………..

5) $P(\bar{A}) + P(A) =$ ______

6) $P(\bar{A}) = 1 -$ ________

3. Find the probability of getting a number greater than 4 when a dice is thrown.

4. Find the probability of getting an odd number on its face when a dice is thrown.

5. If two coins are tossed together, find the probability of getting tails on both coins.

6. On throwing a die find the probability of getting the number 8 on its face.

7. If $P(A) = 0.05$, then find $P(\bar{A})$.

8. A die is thrown once. Find the probability of getting the following:

   (1) be a prime number

   (2) Any number lying between 2 and 6

   (3) is an odd number

9. There are 4 red and 6 black pills in a bag. The probability that a pill will be black when taken out is –

10. If a number is chosen from the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 the probability that it will be an even number, is.

We learned -

1) Probability is a concept in which the probability of events happening or not happening can be expressed numerically.

2. Probability of the occurrence of event A

$$P\,(A) = \frac{no.of \text{ favorable outcome}}{\text{Total no.of outcome}} = \frac{m}{n}$$

3. Event A is not occurring

$$P\,(\bar{A}) = \frac{\textbf{No.of unfavorable outcomes of event A}}{\textbf{Total number of outcomes}}$$

$$= \frac{n-m}{n}$$

Or $$= 1 - \frac{m}{n}$$

Or $$P\,(\bar{A}) = 1 - P\,(A)$$

Or $$P\,(\bar{A}) + P\,(A) = 1$$

2) Range of probability -

$$0 \leq P\,(A) \leq 1$$

Chapter -14

Quantitative and Reasoning Aptitude

Dear students! In earlier classes, we learned concepts related to the applicational and theoretical aspects of mathematics. The study of mathematics helps the students improve their logical ability and intelligence. In competitive exams, various types of questions are asked to test the reasoning power of the candidate. Have you ever thought about the questions related to reasoning and aptitude being asked in various competitive examinations?

Its objective is to test the intelligence and reasoning ability of the participant. In this chapter, we will briefly study the common questions asked in competitive exams.

Series:

The series is based on a combination of alphabets, numbers, or mixed (characters, numbers, symbols) series.

Alphabet series: Only characters are available in this series, which follow a certain sequence. The participant has to figure out this sequence and is required to answer the question.

Position of Characters:

Left to right – correct order ⟶

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	I	J	K	L	M	n	Oh	P	Q	R	S	T	u	v	w	X	Y	Z
26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Right to left (reverse order) ⟵

Worth remembering the points:

1. From the left the series starts and ends on the right.
2. If you want to find the letter at the left position of a certain letter, counting from your left, then you should use the subtraction method.
3. If you want to find the letter to the right of a certain letter, counting from your right, then you should use the addition method.

Example question:

Q.1 Study the positions of the following letters carefully and answer the questions.

A B C D E F G H I J K L M N

O P Q R S T U V W X Y Z

(a) In the above statement find the number of vowels such that the letters before and after the vowel are consonants.

(a) one b) two s)three d)Four

Answer: **D)** four

Explanation:	**Consonant**	**Vowel**	**Consonant**
	D	E	F
	H	I	J
	N	O	P
	T	U	V

(b) What should come in place of the question mark (?)

A C F J ?

a) Q b) O c) U d) None of these

Exercise 14.1

Q.1 Study the following number series carefully and answer the questions.

12653218965321231

In the above mentioned arrangement How many such 1's are there in which it is immediately followed by 2?

a) one b) two c) three d) four

Q.2 Study the following series carefully and answer the questions.

F @ 5 3 R $ J P E 1 H μ L 8 4

B 8 A W 2 U G 6 * 9 δ Z N M V

1. Which of the following element is 13^{th} from the left?

a) % b) L c) Q d) 1

2. In the above arrangement which of the following letters is at the 10th position on the left of 18th letter from the left ?

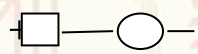
a) J b) E c) A d) P

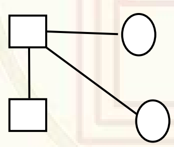
Blood related:

In this section the participant has to solve the questions on the basis of blood relations. Questions based on blood relations can be of many types.

Note: It is very important to have information about relationships.

Main points:

1. There should be a classification of men and women.
2. Like husband-wife relationship 
3. like the relationship between a son and a daughter.



4. The only son of my father means –I (myself)
5. Meaning of the only daughter of in - laws of Ritu's husband is, Ritu (herself)

Now let's consider the following questions.

1. What will be my father's sister's father's son's daughter's relationship with me?

a) Aunt b) Sister c) Daughter d) Niece

Answer: **B)** Sister

Explanation: Meaning of my father's Sister's Father is my Grandfather. Grandfather's Son's Daughter is my Sister.

2. Preeti has a son named Arun, Ram is Preeti's brother and Neeta also has a daughter named Reema. Neeta is Ram's sister. How is Arun related to Reema?

a) Brother b) nephew c) cousin d) uncle

Answer: c) Cousin

Preeti Brother ↔ Ram Sister ↔ Neeta

Arun ↔ Cousin ↔ Reema

Exercise 14.2

1. Seeing a picture, the son smiled and said that she is the only sister of my maternal uncle. How is a boy related to her?

   a) Mother b)Cousin c) Yourself d) Sister

2. A is B's mother. How B related to A?

   a) Brother b) Daughter c) Daughter-in-law d) Cousin

3. A woman, while introducing a man said, "He is my mother's, brother's, father's, son's, son. what is the relation between the man and the women?

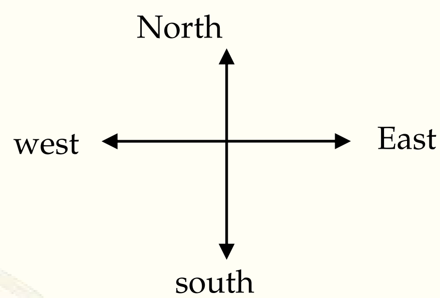
   a) Father b) Brother c) Son d) Cousin

4. One Man said to a woman My mother is the sister of your mother's father. How are the man and the woman related?

   a) Cousin b) son c) grandfather d) brother

Direction and distance:

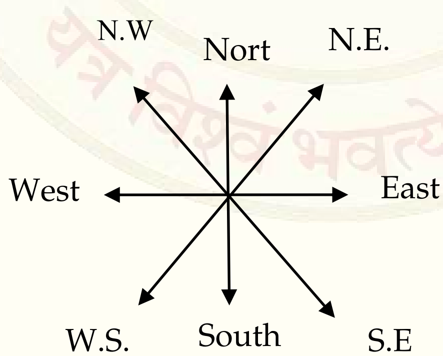
A question is asked in the form of a puzzle related to direction and distance, where the participant is asked to determine the complete and final direction between two points by following the direction in a sequential manner. The objective of this question is to check whether the participant is moving in the sequential order or not, as given on the question.



There are four main directions:

North, East, South, West

There are four directions-North-East (N–E), South-East(S–E), South-West(S–W) and North-West (N–W)



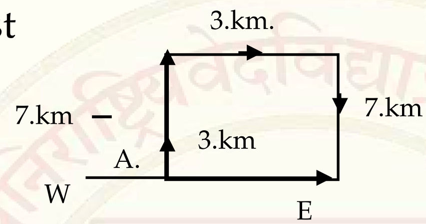
These sub-directions are between the main directions. Let us now consider the following example questions.

Example: Shyam, he travels 7 Km. towards the North then turns to his right and walks 3 km. He then turns to his right and walks 7 km. Now in which direction he is moving forward from his starting point?

A) North B) South C) East D) West

Answer:C) East

Explanation:



A = initial, the dot

Exercise14.3

Q.1 Gaurav starts walking towards the north and walks 5 km. Turns left and walks 10 km. Turns left and walks 5 km. What distance does he cover and how far is he now from his starting point?

A) 5 km. B) 20 km. C) 10 km. d) none of these

Q.2 Hemant walks 10 km towards the north. He turned to his left and walked 4 km. He walks then turns to his right and walks 5 km. He again turns right and walks 4 km, now how far is he from the starting point?

A) 10 km. B) 15 km. C) 5 km. D) 25 km.

Q.3 A car driver travels 10 km north. He turns left and moves for 5km to meet his friend. He further turns right and travels

10km. In total he travels 25km. In which direction will he be facing now?

A) North B) East C) West D) South

Q.4 A girl goes 30 meters north-west from her house, then 30 meters south-west and 30 meters south-east, and finally she turns towards her house. Now, in which direction is she facing?

a) South-East b) South- West

c) North-East d) North- West

Q.5 Anuradha goes 3 meters north, turns left, and walks 2 meters. Turns left and walks 3 meters. Then she turns to her right and walks in a straight line. In which direction is she walking now?

a) North b) Eastc) South d) West

Q.6 Ram walks 10 m towards south from his house. He turns left and walks 25 meters, then turns left and walks 40 meters. He then turns to the right and walks 5 meters to reach his school. In which direction does his school say it is from his house?

a) North b) North- east c) South-west d) East

Sequencing:

In order to determine, generally the position of a person is specified from both left and right or from top and bottom; sometimes the total number of people is asked. At times, the question is based on their entering new places.

Important points:

1) Total number of people in a class or row =
(Position of one person to the left or top) + (Position of that person to the bottom or right)

2) (Total number of persons + 1) – (Position from left or top) = Position from right or bottom

3) (Total number of persons+ 1) – (Position from right or bottom) = Position from left or top

Let us consider the following examples.

Example: Rahul is 14th from the right in a row of 40 boys. What is his position from the left end?

a) 24^{th} b) 25^{th} c) 26^{th} d) 27^{th}

Explanation: Rahul is at number 14 among 40 boys from the right his position from the left is = (40 – 14) = 26.

Hence Rahul is at 27 positionfrom the left.

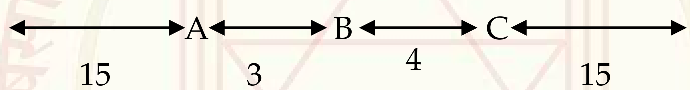
Example: A the position of A, among boys facing north is 16^{th} and C is 16^{th} from the right. B is fourth from the A on the right and fifth from C on the left. Find the total number of students in the row.

a) 39 b) 40

c) 41 d) 42

Explanation: Obviously, as per the given conditions, the number of students to the left of A is 15 and to the right of C is 15.

The number of students between A and B is 3, between B and C is 4.



Hence, number of boys in the row

= (15 + 1 + 3 + 1 + 4 + 1 + 15) = 40

Exercise 14.4

1. In a row, Ravi is 18^{th} from the left and 19th from the right, then how many students are there in the row?

   a) 3 6 b) 37 c) 38 d) 35

2. Among 50 students in a row, Renu is 15th from the left and Raju is 18th from the right. How many students are there between them?

   A)17 B) 16 C) 7 d) 8

3. Students are standing in a row; Ashish is 15th from the left and Sachin is 7th from the right. If their positions are changed, Sachin will be in the 15th position from the right. Find the total number of students in the raw.

   a) 21 b) 22 c) 29 d) 30

4. Manoj is 24th from the right in the row. There are 40 children in the row. What is Manoj's position from the left?

   a) 18 b) 16 c) 17 d) cannot be determined

5. Ashok is 20th from the top and 25th from the bottom; find the total number of children.

   a) 40 b) 44 c) 35 d) cannot be determined

Clocks:

In this chapter on "clock," participants are expected to find the angle between the two readings (between the hands of the clock) carefully.

Important points:

(1) Minute hand:

1 hour = 1 full rotation

Time duration	Angle of rotation
1 hour = 60 minutes	360°
1 minute	$\frac{360}{60} = 6^{\circ}$

(2) Hours Hand: 12 hours = 1 full rotation

We Know, that in 1 hour there are 60 minutes.

Time duration	= Angle of rotation
12 × 60 minutes	= 360°
720 minutes($720^{'}$)	= 360°
$1^{'}$	$= \frac{360}{720} = \left(\frac{1}{2}\right)^{0}$

(3) Seconds Hand: 1 minute =1 full rotation = 360°

60 seconds ($1^{'}$) = 360°

$60^{''} = 360^{o}$

$1^{''} = \frac{360}{60} = 6^{o}$

The second hand makes an angle of 6 ° (degrees) in 1 second.

Formula to find angle:

$$\theta = \frac{11}{2} \times \text{Minutes} - 30^\circ \times \text{ hours}$$

There is no difference in the positive or negative sign of the angle 'θ' obtained. The sign of the angle gives information about whether the minute's hand has crossed the hour's hand; if it is positive, the minute's hand is ahead of the hour's hand and lags behind if it is negative.

If they are crossed, the angle will be positive. If not crossed, the angle will be negative.

For example:

Let us consider the following questions.

Example: When the time on the clock is 7.20, then find the angle between the hour hand and the minute hand.

Solution: Time 7.20 = 7 hours and 20 minutes

$$= (7 + \frac{20}{60}) \text{ hours}$$

$$= (7 + \frac{1}{3}) \text{ hours} \quad = \frac{22}{3} \text{hours}$$

$\because$ The angle made by the hour hand in 12 hours = 360°

$\therefore$The angle made $\frac{360}{12} \times \frac{22}{3}$ by the hour hand during the hour $= \frac{22}{3} = 220^\circ$

$\because$The angle made by the minute hand in 60 minutes = 360°

$\therefore$Angle made by the minute hand in 20 minutes $= \frac{360}{60} \times 20 = 120^\circ$

Required angle = $(220^\circ - 120^\circ) = 100^\circ$

Trick:

$$\theta = \frac{11}{2} \times \text{Minutes} - 30^{\circ} \times \text{hours}$$

$$\theta = \frac{11}{2} \times 20 - 30^{\circ} \times 7$$

$$\theta = 110^{\circ} - 210^{\circ}$$

$$\theta = -100 \qquad \text{(which means } 100^{\circ}\text{)}$$

Another angle, $\theta = 360^{\circ} - 100 = 260^{\circ}$

Example: When the time in the clock is 2:40, find the angle between the hour and minute hands.

Solution: Time 2:40 = 2 hours and 4 0 minutes

$= (2+\frac{2}{3})$ hours

$= (2 +\frac{2}{3})$ hours $= \frac{8}{3}$hours

∵The angle made by the hour hand in 12 hours = 360 °

∴The angle made by the hour hand during the $\frac{8}{3}$ hour $= \frac{360}{12} \times \frac{8}{3} = 80^{\circ}$

∵ The angle made by the minute hand in 60 minutes = 360 °

∴ Angle made by the minute hand in 40 minutes $= \frac{360}{60} \times 40^{\circ} = 240^{\circ}$

Required angle $= (240^{\circ} - 80^{\circ}) = 160^{\circ}$

Trick:

$$\theta = \frac{11}{2} \times \text{Minutes} - 30 \times \text{hours}$$

$$= \frac{11}{2} \times 40 - 30 \times 2$$

$$= 220^{\circ} - 60^{\circ}$$

Then required angle = 160 °

And other angles $= 360^{\circ} - 160^{\circ} = 200^{\circ}$

Exercise 14.5

1. How many degrees of angle will be formed in 5 minutes?

   (a)30 ^{o} (b) 45 ^{o} (c) 15 ^{o} (d) 5 ^{o}

2. How many degrees of angle does the second hand make in 3 seconds?

   (a)12 ^{o} (b) 15 ^{o} (c) 18 ^{o} (d) 24 ^{o}

3. What is the angle between the hour and minute hands at 3:35 p.m.?

   (a)$107\frac{1^{o}}{2}$ (b) 210 ^{o} (c) $102\frac{1^{o}}{2}$ (d)None of these

4. What is the angle between the hour hand and minute hands at 8:15?

   (a)90 ^{o} (b) $157\frac{1^{o}}{2}$ (c) $247\frac{1^{o}}{2}$ (d)None of these

5. What is the angle between the hour hand and minute hand at 5:12?

   (a)156 ^{o} (b) 72 ^{o} (c) 84 ^{o} (d) 78 ^{o}

6. What will be the angle between the hour hand and minute hand at 11 o'clock?

   (a)15 ^{o} (b) $22\frac{1^{o}}{2}$ ^{o} (c) 30 ^{o} (d) 36 ^{o}

Calendar:

In the chapter of Calendar, participants have to carefully read the questions and answers. The participant should have basic knowledge about the calendar.

Important points: -

1. **Extra days:** The number of days in a given interval that are more Then the number of full weeks is called extra days.
2. The year (not being a century) that is exactly divisible by 4 and Every century that is divisible by 400 will be a leap year.
   (1) Years 1620, 1860, 1940, 1984, 1996, 2004, 2008 etc. are all leap years.
   (2) Years 400, 800, 1200, 1600, 2000, 2400 etc. are leap years.
   (3) Years 1726, 1982, 2021, 2035 etc. are not leap years.
3. 1 ordinary year = 365 days and 1 leap year = 366 days.
4. Calculating the extra days
   (i) 1 ordinary year = 365 days = (52 weeks) + (1 day)
   Hence, in an ordinary year there are 52 weeks and 1 extra day
   (ii) 1 leap year = 366 days = (52 weeks) + (2 days)
   So, there are 2 extra days in a leap year.
   (iii)100 years= 76 ordinary years+ 24 leap years
   = 76 extra days+ (24×2) extra days
   = 124 extra Days

= (17 weeks+ 5 days)

= 5 extra Day

5. To find the number of extra days, divide the number of days by 7 and find the remainder.

Trick to find the day:

$$\frac{\text{day} + \text{Month} + \text{Year} + \text{Leap Year} + \text{Century}}{7}$$

Code for months

Month	January	February	march	April	May	June	July	august	September	October	November	December
code	1	4	4	0	2	5	0	3	6	1	4	6

1. Consider the last two digits of the given year.

2. For a leap year, the last two digits of the year are divided by 4, and the quotient is written.

2. Century code:

-----	1600	1700	1800	1900	2000	2100	2200	2300	-----
	6	4	2	0	6	4	2	0	

3. For the days

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2	3	4	5	6	0

Let's take an example Let us consider the following question.

Example: India became independent on August 15, 1947. What day of the week was that?

Solution:

15 August 1947 = (1946 year +1 January 1947 to 15 August 1947)

Extra days in 1600 years = 0

Extra days in 300 years = 15 days = (7 ×2 +1) day = 1 day.

46 years =(11 leap years + 35 ordinary years) = (22 + 35) extra days

=57 additional days = (8 × 7+ 1) extra day

(31 +28+31+30+31+30+31+15) = 227 days

= (7×32+3) days

= 3 extra days

Total number of additional days = (0+1+1+3) =5

Hence, required day = Friday

Trick: day +month+Year+leap Year+Century

= 15+03+47+11+00

= 76

Dividing the extra days by 7

$= \frac{76}{7} = 6$ (remainder)

6 remainder means the day will be Friday.

Exercise 14.7

1. Which day will be 31st May 2020?
2. What day of the week was 26 January 1950?

   (a)Thursday (b) Saturday (c)Tuesday (d) Monday
3. What day was October 19, 2000?

   (a)Tuesday (b) Thursday (c) Friday (d) Saturday
4. How many weeks are there in a year?

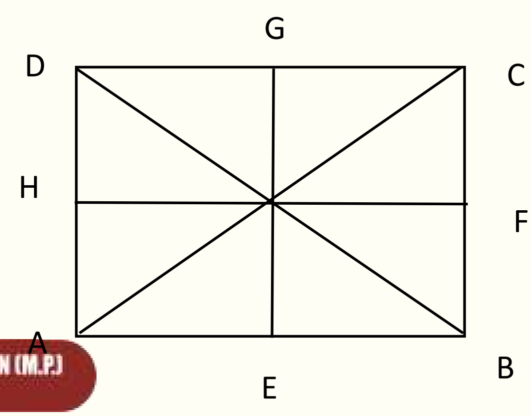
   (a)50 (b) 52 (c) 51 (d) 58
5. How many days are there in the month of February in a leap year?

   (a)30 (b) 28 (c) 31 (d) 29

Counting of shapes: In the questions covered under the topic of differentiation, a figure will be given. From this, the geometric shapes (such as circles, triangles, squares, rectangles, etc.) have to be identified, and their total number has to be found.

Example: how many straight lines are there in the following question:

a) 5 b) 8 c) 10 d) 15

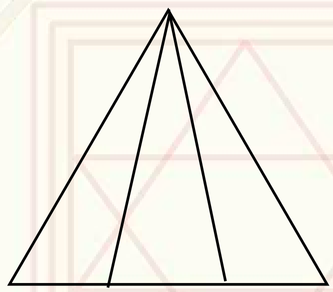


Solution: b) 8

After carefully observing the figure given in the question, it was found that there are three horizontal lines (AB, HF, and DC), three vertical lines (AD, EG, and BC), and two oblique lines (AC and BD).

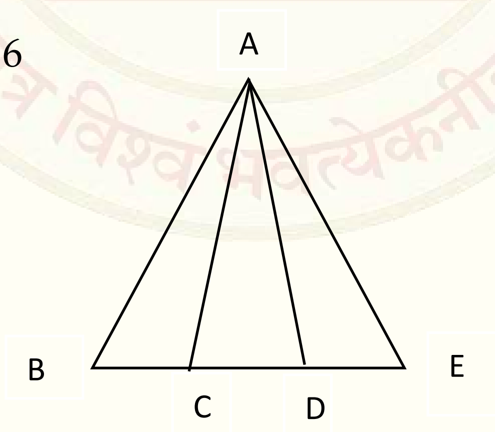
Example:

How many triangles are there in the following figure?



a) 3 b) 4 c) 5 d) 6

Solution :- d) 6

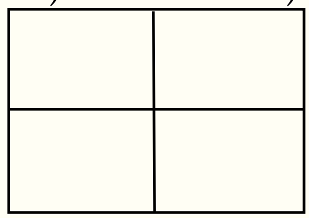


In the above-mentioned shape, the number of triangles is 6.

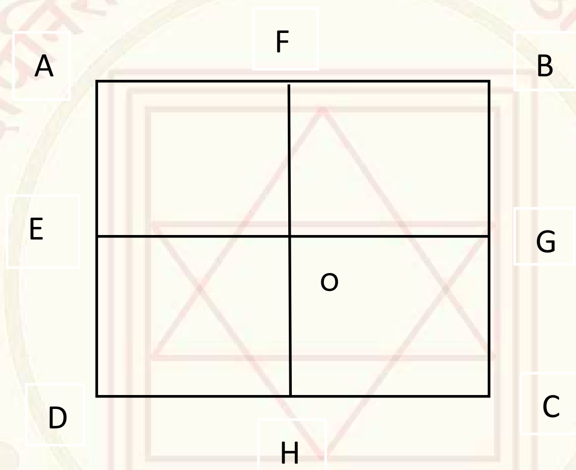
The triangles areΔABC, ΔABD,Δ ABE, Δ ACD, Δ ACE, Δ ADE.

Example: How many squares are there in the following figure?

a) 5 b) 8 c) 9 d) 10



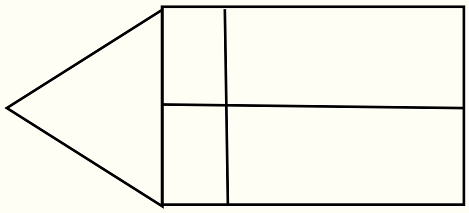
Solution:-a) 5



There are 5 squares in the above-mentioned shape. The squares are AEOF, FOGB, OGCH, EODH and ABCD.

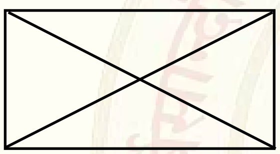
Exercise 14.8

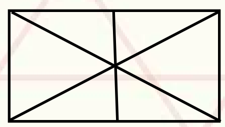
1. How many rectangles are there in the figure given below?

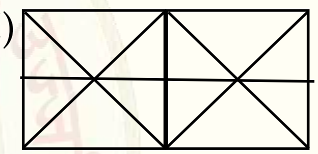


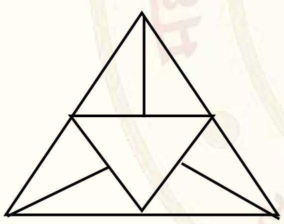
a) 7 b) 8 c) 9 d) 12

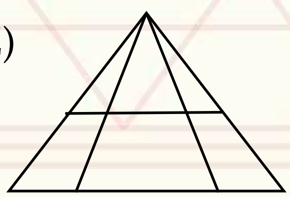
2. How many triangular shapes are there in the figure given below?

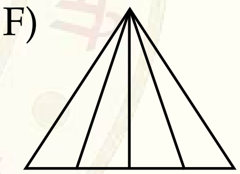
A) 

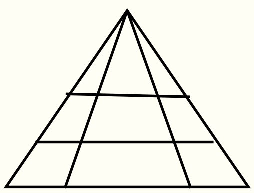
B) 

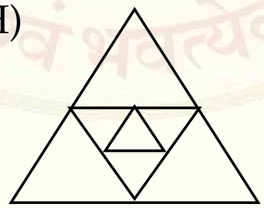
C) 

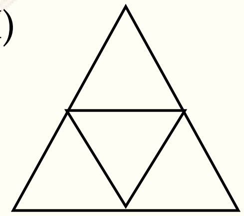
D) 

E) 

F) 

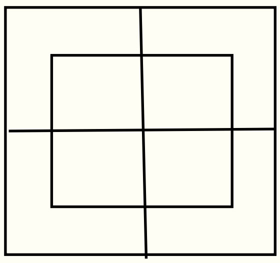
G) 

H) 

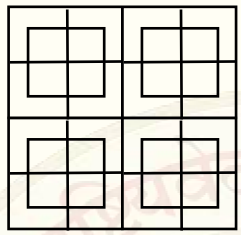
I) 

3. Find the number of squares in the following figures.

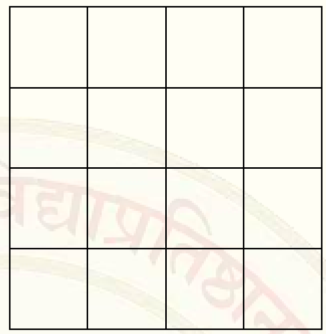
(a)



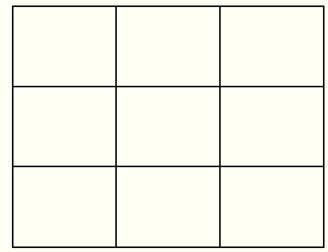
(b)



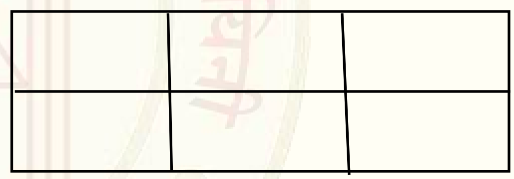
(c)



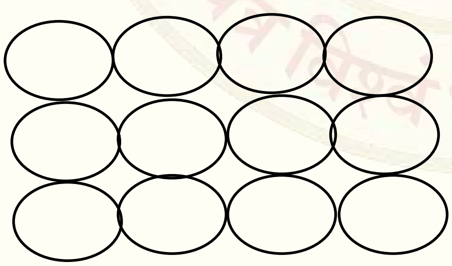
(d)

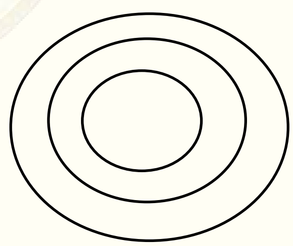


4. Find the number of rectangles in the following figures.



5. How many circles are there in the following figures?





Introduction of Indian mathematicians and their contribution

❖ **Shridharacharya (Shridhar acharya)**

Sridharacharyawa is a famous mathematician. His birth is believed to have been around 750 AD. His father's name was Baldevacharya, and his mother's name was Achkok. His father was a great scholar of Kannada and Sanskrit literature. He received education in literature, Sanskrit, and Kannada from his father and later became famous as a great thinker and philosopher. Shridharacharya ji explained zero and quadratic equations. I proposed the formula for solving it. His famous work, Ganitasar, which is called Trishti' or also known as 'Trishatika', Shridharacharya himself has written in the first verse of his famous book that this book is the essence of his Patiganitha. A description of his other works, Patiganit and Algebra (which are accessible), is also available. He made many important discoveries in algebra. The rules invented by him for solving quadratic equations by making them perfect squares are still popular by the name of' Shridhar's rule' or Hindu rule.

Based on Shridharacharya's famous work 'Trishatika', Bhaskaracharya wrote his famous book 'Lilavati'.

Shridharacharya, in his book 'Ganitasar', has discussed the following topics: integral multiplier, division, square, square root, cube, cube root, fraction, compound part caste, division part caste, part relation, part caste, triarashik, saptarashik, navarashik, bhand-pratibhand, mixer (व्यवहार) behavior, and emotional behavior. The incorporation and description of the monopolization formula, gold mathematics, projective mathematics, simultaneous purchase and sale formula, category behavior, area behavior, account behavior, mind behavior, wood behavior, number behavior, shadow behavior, etc. are presented.

Explanation of zero: Compared to other mathematicians, the explanation of zero presented by Shridharacharya is the clearest– his explanation is that if zero is added to any number, then the sum is equal to that number. If zero is subtracted from any number then the result is equal to that number. If zero is multiplied by any number then the product will be zero.

- To divide a number by a fraction, he said that the number should be multiplied by the reciprocal of that fraction.

Shridharacharya has explained the method of solving quadratic **equation** –

$$ax^2 + bx + c = 0$$

(Bhaskaracharya quoted the solution to the same equation in his beejganitam. Which is given below?)

चतुराहतवर्गसमैःरूपैःपक्षद्वयं गुणयेत् ।अव्यक्तवर्गरूपैः उक्तौ पक्षौ ततो मूलम् ॥

The method mentioned in the above verse is used in modern mathematics to solve second-degree equations.

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

❖ Sudyumna acharya

Sudyumna acharya He is a famous grammarian. He completed his M.A. from Allahabad University and was awarded Ashtasvarna Padak, D.Phil., worked for 34 years as a professor, and is currently working as the Director of Ved Vani Vitan at the Oriental Studies Research Institute, Satna. He is the author of various books such as philosophy, physics, grammar, linguistics, ancient and modern, etc. He has also written books on 'Thinking Stream of Development of Science in Indian Mathematics' and 'Indian Tradition of Development of Mathematics'. His book is written in such a way that it can be taught directly as a textbook. Apart from this, he has also translated Sridhar's Trishatika.

Amulya Kumar Bagh (b. 1937)

He was associated with S. N. Sen. during the establishment of the INSA Commission. Has been associated with the Indian Journal of History of Science and has contributed significantly to the success and growth of the journal. Bagh has published an overview of mathematics in ancient and medieval India (1991 Banaras) and has also translated the Shulbha Sutra with S. N. Sen.

आदर्श प्रश्नपत्र/ Model Que. Paper: V/23-24/ गणित /
वेदभूषण पञ्चम-वर्ष / Vedabhushan Fifth Year/
कक्षा 10वीं / पूर्व मध्यमा - II / Class 10th / Purv Madhyama - II
वर्ष / Year 2023-24
विषय – गणित/Mathematics

पूर्णांक – 100 **समय – 3 घण्टे**

• सभी प्रश्न हल करना अनिवार्य हैं। • सभी प्रश्न के उत्तर पेपर में यथास्थान पर ही लिखें। • उत्तीर्णता हेतु न्यूनतम 40% अंक निर्धारित हैं। • आदर्श प्रश्न पत्र का छात्रों को लिखित परीक्षा हेतु अभ्यास कराएँ।	• It is mandatory to attempt all the questions. • Write down the answers at the appropriate places provided. • The minimum pass marks are 40%. • The model question paper should be used by the students for written examination practice.

प्रश्न - 01. सही विकल्प के सामने (✓) चिह्न लगाइए – $10 \times 2 = 20$

Question - 01. Mark in front of the correct option (✓) put on.

1. निम्न में से कौन-सा कथन सत्य नहीं है ।
   Which of the following statement is not true?

   (अ) वैदिक वाङ्मय में गणना की दृष्टि से ऋग्वेद का द्वितीय स्थान है ।
   In Vedic literature, rigveda has the second position in terms of calculations.

   (आ) वैदिक वाङ्मय की सबसे बड़ी देन संख्याओं का आविष्कार तथा दाशमिक प्रणाली है।
   The prime contribution of Vedic literature is the invention of numbers and the decimal system.

   (इ) 10^2 = शत / 10^2 = hundred

   (ई) शुल्ब का अर्थ 'धागा अथवा रस्सी' है ।
   The meaning of Shulb is 'thread or rope'.

2. बोधायन शुल्बसूत्र के सन्दर्भ में कौन-सा कथन सत्य नहीं है ।

Which statement is not true in the respect of Bodhayana Shulbasutra?

(अ) बोधायन शुल्बसूत्र में π (पाई) का मान 3 है ।

π The value of is 3 in Bodhayana Shulbasutra.

(आ) बोधायन प्रमेय के मूल रूप को पाइथागोरस प्रमेय के नाम से जानते हैं ।

The original form of Bodhyan's theorem is known as the Pythagorean theorem.

(इ) बोधायन शुल्बसूत्र कृष्ण यजुर्वेदान्तर्गत आता है।

Bhodhayan Shulbasutra comes under Krishna Yajurveda.

(ई) बोधायन शुल्बसूत्र शुक्ल यजुर्वेदान्तर्गत आता है ।

Bodhayan Shulbasutra comes under Shukla Yajurveda.

3. समुच्चय के सन्दर्भ में कौन-सा कथन सत्य नहीं है ।

Which statement is not true about the set?

(अ) वस्तुओं के सुपरिभाषित समूह को समुच्चय कहते है ।

A well-defined collection of objects is called a set.

(आ) ऐसे समुच्चय जिसमें अवयवों की संख्या परिमित हो, परिमित समुच्चय कहलाता हैं ।

Which sets have a finite number of elements are called finite set.

(इ) ऐसे समुच्चय जिसमें अवयवों की संख्या अपरिमित हो, अपरिमित समुच्चय कहलाता हैं।

Which have an infinite number of elements are called a finite set.

(ई) रिक्त समुच्चय के द्वारा तीन उपसमुच्चय बना सकते हैं।

The empty set can form three subsets.

4. बहुपद के सन्दर्भ में कौन-सा कथन सत्य नहीं है ।

Which statement is not true with respect to polynomials?

(अ) किसी बहुपद में चर की घात 1 होने पर उस बहुपद को रैखिक बहुपद कहते हैं।

If the degree of a polynomial is 1, then polynomial is called a linear polynomial.

(आ) किसी बहुपद में चर की घात 2 होने पर उस बहुपद को द्विघातीय बहुपद कहते हैं ।

If the degree of a polynomial is 2, then polynomial is called a quadratic polynomial.

(इ) किसी बहुपद में चर की घात 3 होने पर उस बहुपद को त्रिघातीय बहुपद कहते हैं ।

When the degree of a polynomial is 3, that polynomial is called a cubic polynomial.

(ई) शून्यकों का योगफल $(\alpha + \beta) = \frac{c}{a}$

Sum of zeroes $(\alpha + \beta) = \frac{c}{a}$

5. निम्न में से कौन-सा सत्य है-

Which of the following is true -?

(अ) शून्यकों का योगफल $= {}^{-b}/_{a}$ / Product of zeros $= {}^{-b}/_{a}$

(आ) शून्यकों का गुणनफल $(\alpha\beta) = \frac{c}{a}$ / Product of zeros $(\alpha\beta) = \frac{c}{a}$

(इ) दोनों (अ) एवं (आ) / Both (A) and (aa)

(ई) इनमें से कोई नहीं / None of these

6. श्रीधराचार्य के ग्रन्थ है ।/There are books of Shridharacharya.

(अ) पाटी गणित एवं त्रिशतिका / Pati ganit and Trishatik

(आ) लीलावती गणित एवं त्रिशतिका / Lilavati ganit and Trishatika

(इ) त्रिशतिका / Trishatik

(ई) इनमें से कोई नहीं / None of these

7. समीकरण निकाय $a_1x + b_1y + c_1 = 0$ और $a_2x + b_2y + c_2 = 0$ का अद्वितीय हल होगा, जब

The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ will have a unique solution, when

(अ) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ (आ) $\frac{a_1}{a_2} = \frac{c_1}{c_2}$

(इ) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (ई) $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$

8. $a + (n-1)d$ में d है – / What is d in $a + (n-1)d$

(अ) समान्तर श्रेणी का अन्तिम पद
Last term of arithmetic progression

(आ) समान्तर श्रेणी के पदों की संख्या
No. of terms in arithmetic progression

(इ) समान्तर श्रेणी का सार्व अन्तर
Common difference of arithmetic progression

(ई) समान्तर श्रेणी का प्रथम पद
First term of arithmetic progression

9. निम्नलिखित श्रेणी में से कौन-सी समान्तर श्रेणी है ?

Which of the following series is an arithmetic progression?

(अ) 2 , 4 , 8 , 16 , … (आ) 1 , 3 , 9 , 27 , …

(इ) 1 , 2 , 3, 4, 5… (ई) 0 , 5 ,25 , 10 , …

10. बोधयन प्रमेय किस त्रिभुज से सम्बन्धित है –

Bhodhayan's theorem what triangle from is related to -

(अ)	समबाहु त्रिभुज Equilateral triangle	(आ)	समकोण त्रिभुज Right triangle
(इ)	समद्विबाहु त्रिभुज Isosceles triangle	(ई)	उपर्युक्त तीनों Above all three

प्रश्न - 02. रिक्त स्थानों की पूर्ति कीजिए – **5 × 2 = 10**

Question - 02. Fill in the blanks -

1. $sinA = \frac{3}{4}$ हो, तब $cosecA$ = ... होगा ।

   If $sinA = \frac{3}{4}$, then $cosecA$ ....

2. 'शून्य' शब्द सर्वप्रथम .... वेद के मन्त्र में मिलता है ।

   The word 'zero' is first found in the mantra of .... Veda.

3. घनफल के संक्रिया के विपरीत संक्रिया .... होती है ।

   The opposite of the cube operation is the ....

4. निर्देशांक बिन्दु में x अक्ष के निर्देशांक को .... कहते हैं ।

   The coordinate of the x axis in the coordinate point is called ....

5. निश्चित घटना की प्रायिकता का मान .... होता है ।

   The value of the probability of a certain event is ....

प्रश्न - 03. निम्नलिखित युग्मों के मिलान पर विचार कीजिए – 5 × 2 = 10

Question - 03. Consider matching the following pairs -

1.	आर्यभट्ट Aryabhatta	क.	वैदिक गणित Vedic Mathematics
2.	भास्कराचार्य Bhaskaracharya	ख.	पञ्च सिद्धान्त Panch Siddhaant
3.	ब्रह्मगुप्त Brhmgupta	ग.	आर्यभट्टीयम Aryabhattiyam
4.	वाराहमिहिर Varahamihira	घ.	सिद्धान्त शिरोमणि Siddhanta Shiromani
5.	भारतीय कृष्ण तीर्थ Bhaarateey Krishna Teerth	ङ.	ब्रह्मस्फुट सिद्धान्त Brahmasfut Siddhanta

उपर्युक्त युग्मों के आधार पर सही विकल्प का चयन कीजिए –

Select the correct option based on the above pairs -

(अ) (1) (ग), (2) (घ), (3) (ङ), (4) (ख), (5) (क)

(आ) (1) (घ), (2) (अ), (3) (ङ), (4) (ग), (5) (ख)

(इ) (1) (ग), (2) (घ), (3) (ङ), (4) (क), (5) (ख)

(ई) (1) (ङ), (2) (ख), (3) (क), (4) (ग), (5) (घ)

प्रश्न - 04. सत्य / असत्य कथन पर विचार कीजिए – 5 × 1 = 5

Question - 04. Consider the true / false statement

1. 'शून्य शब्द' का सर्वप्रथम प्रयोग अथर्ववेद में मिलता है ।
   The first use of the word 'Zero' is found in the atharvaveda.
2. वह समुच्चय जिसमें कोई अवयव न हो उसे रिक्त समुच्च्य कहते हैं ।

The set does not contain any element is called the Empty set.

3. 5 का परमित्र अंक 2 है ।

Paramitra digit of 5 is 2.

4. समान्तर श्रेणी के सार्व अन्तर को 'a' से व्यक्त करते हैं ।

Common difference of arithmetic progression to represent From 'a'

5. 'Trigonometry' शब्द को संस्कृत से व्युत्पन्न मानते हैं ।

The 'Trigonometry' word is believed to be derived from Sanskrit.

उपर्युक्त कथनों को पढ़कर सही विकल्प का चयन कीजिए –

Read the above statements and choose the correct option

(अ) (1) सत्य, (2) सत्य, (3) सत्य, (4) सत्य, (5) असत्य
(1) True, (2) True, (3) True, (4) True, (5) False

(आ) (1) सत्य, (2) सत्य, (3) असत्य, (4) असत्य, (5) सत्य
(1) True, (2) True, (3) False, (4) False, (5) True

(इ) (1) सत्य, (2) असत्य, (3) सत्य, (4) असत्य, (5) सत्य
(1) True, (2) False, (3) True, (4) False, (5) True

(ई) (1) असत्य, (2) सत्य, (3) असत्य, (4) असत्य, (5) सत्य
(1) False, (2) True, (3) False, (4) False, (5) True

प्रश्न - 05. अति लघूत्तरीय प्रश्न – **10 × 2 = 20**

Question - 05. Very short answer type questions -

1. समुच्चयों के संघ से आप क्या समझते है ।/What do you understand by union of sets.

----

2. गुणनफल ज्ञात करें /find the product - $(4x + 5)(2x + 3)$

3. एकन्यूनेन पूर्वेण सूत्र से गुणनफल कीजिए।

Find the product by Eknunen Purvena Sutra.

223×99

4. वृत्त का क्षेत्रफल ज्ञात कीजिए जिसकी 7 सेमी. हो।

Find the area of the circle whose side is 7 cm.

----

5. वृत्त के त्रिज्याखण्ड के क्षेत्रफल सूत्र लिखिए।

Write the formula for the area of a sector of a circle.

----

6. बोधायन प्रमेय का कथन लिखिए।

Write the statement of Bodhayan's theorem.

----

7. समान्तर श्रेणी के n वाँ पद ज्ञात करने का सूत्र लिखिए।

Write the formula to find the n^{th} term of an A.P.

----

8. यदि n सम है तो माध्यिका ज्ञात करने का सूत्र लिखिए।

In n is even, then write the formula to find the median.

----

9. बहुलक से आप क्या समझते है उदाहरण से समझाए।

What do you understand by mode, explain with example.

----

10. एक पासे को फेंकने पर उसके फलक पर कितने परिणाम आ सकते हैं।

How many outcomes can occur on its face when a dice is thrown.

प्रश्न - 06. लघुत्तरीय प्रश्न – **$5 \times 3 = 15$**

Question - 06. Short Answer Type questions -

1. $(10\,x + 7)$ के शून्यक ज्ञात कीजिए। Find the zeroes of $(10\,x + 7)$

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2. बिन्दुओं (–2, 3) एवं (–4, 5) के बीच की दूरी ज्ञात कीजिए।

Find the distance between the points (–2, 3) and (–4, 5)

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3. समान्तर श्रेणी 5, ... , 15 का मध्यपद ज्ञात कीजिए ।

Find the middle term of the arithmetic progression 5, ..., 15

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4. उस वृत्त की परिधि एवं क्षेत्रफल ज्ञात कीजिए जिसकी त्रिज्या 7 सेमी. है ।

Find the circumference and area of the circle whose radius is 7 cm.

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5. यदि $tanA = \frac{3}{4}$ हो, तो $cosA$ और $sinA$ का मान ज्ञात कीजिए।

If $tanA = \frac{3}{4}$ then Find the value of $cosA$ and $sinA$.

प्रश्न - 07. दीर्घ उत्तरीय प्रश्न – **4 × 5 = 20**

Question - 07. Long Answer Type Questions -

1. श्रीधराचार्य जी बताए गये सूत्र को लिखकर समीकरण $3x^2 - 5x + 2 = 0$ को हल करें।

Solve the equation $3x^2 - 5x + 2 = 0$ by writing the formula given by Sridharacharya.

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2. ऊर्ध्वतिर्यग्भ्याम् सूत्र द्वारा गुणन ज्ञात कीजिए ।

Find the product by urdhvatiryagbhyam formula.

588×512

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3. समुच्चय से आप क्या समझते हैं? समुच्चय के वर्गीकरण को समझाइए।

What do you understand by set? Explain the classification of sets.

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4. रुद्राष्टाध्यायी के अन्तर्गत आने वाले संख्या से सम्बन्धित वेद मन्त्र को लिखकर व्याख्या कीजिए।
Explain the Veda Mantra related to the number of Rudrashtadhyayi.

Rashtriya Adarsh Veda Vidyalaya Run and Proposed by

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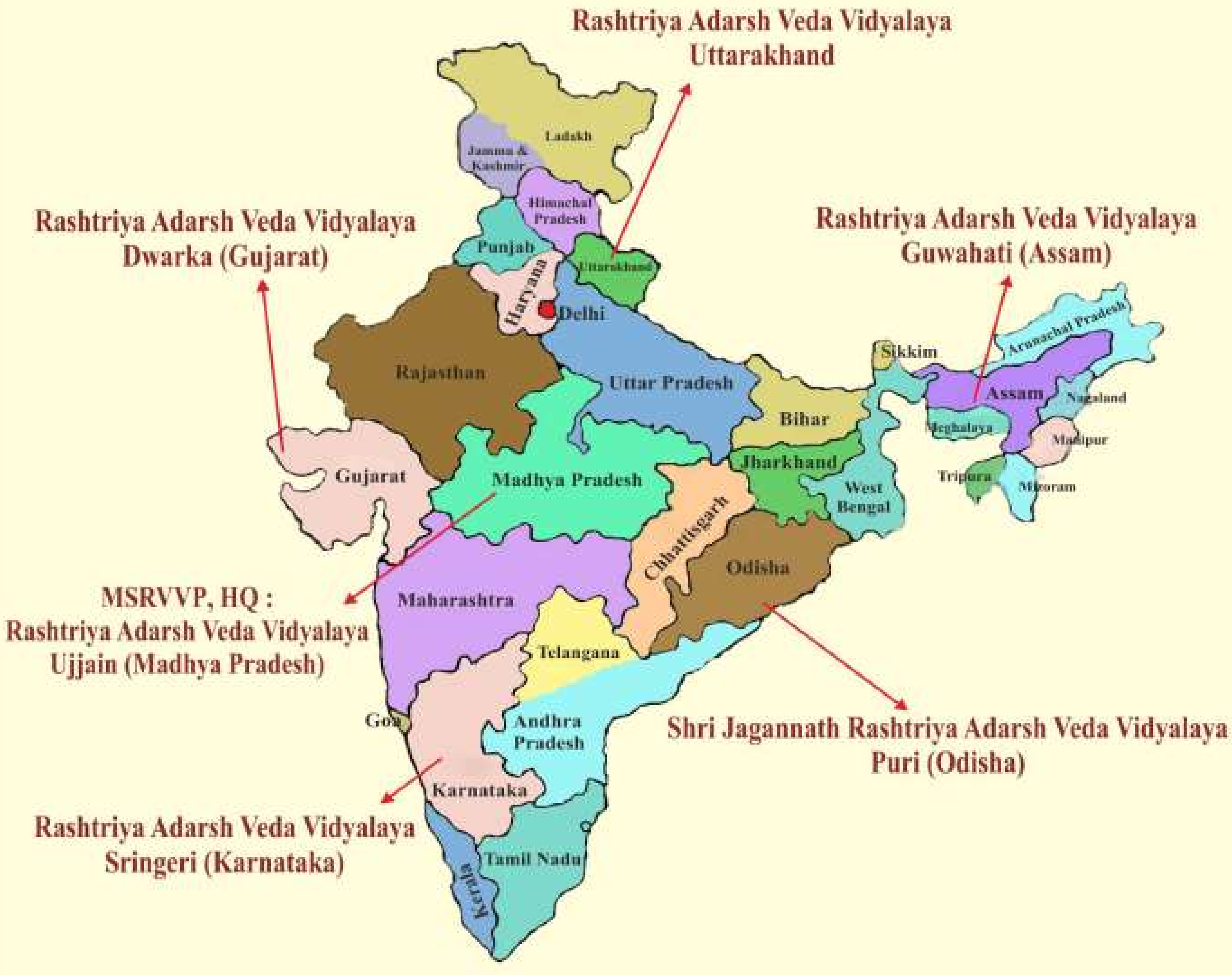

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