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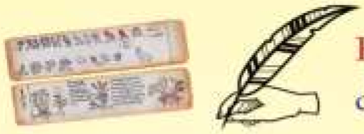

Find the quotient: $4096 \div 64$

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MATHEMATICS TEXTBOOK

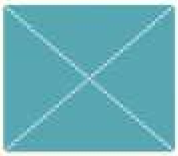
(On the basis of vedic Mathematics and Sanskrit Literature of Mathematics)

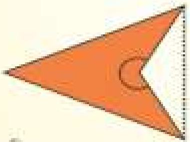
Veda Bhushan III Year / Prathama - III Year / Class VIII

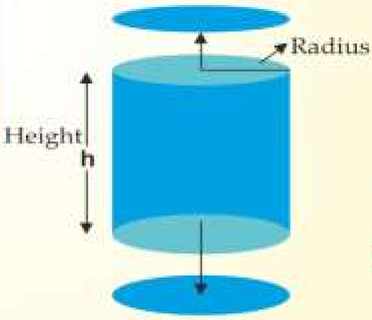
MAHARSHI SANDIPANI RASHTRIYA VEDA SANSKRIT SHIKSHA BOARD

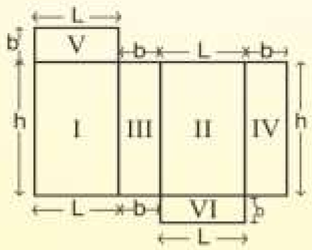
(Established and Recognized by the Ministry of Education, Government of India)

समयोर्धनर्णयोरैक्यं खं शून्यं भवति।
विपरीतच्छेदगुणा राश्योश्छेदांशकाः समच्छेदाः ।
सङ्कलितेंऽशा योज्या व्यवकलितेंऽशान्तरं कार्यम्॥
परिवर्त्य भागहारच्छेदांशौ छेदसंगुणच्छेदः ।
अंशोंऽशगुणो भाज्यस्य भागहारः सवर्णितयोः ॥
समद्विघातः कृतिः उच्यते इति प्रथमः प्रकारः ।
समत्रिघातश्च घनः प्रदिष्टः इति प्रथमः प्रकारः ।
योगोऽन्तरं तेषु समानजात्योर्विभिन्नजात्योस्तु पृथक् स्थितिश्च ।
षष्टि सहस्राश्वस्यायुतासनमुष्ट्रानां विंशति शता।
दश श्यावीनां शता दश त्र्यरुषीणां दश गवां सहस्ता ॥
सदृशद्विवधो वर्गस्त्र्यादिवधस्तद् गतोऽन्यजातिवधः।
अन्योऽन्यवर्णघातो भावितकः पूर्ववच्छेषम्॥
खण्डद्वयस्याभिहतिर्द्विनिघ्नी तत्खण्डवर्गैक्ययुता कृतिर्वा।
इच्छावृद्धौ फले ह्रासो ह्रासे वृद्धिः फलस्य तु।
व्यस्तं त्रैराशिकं तत्र ज्ञेयं गणितकोविदैः ॥









Square of 3

$3^2 = 3 \times 3 = 9$

$3 = \sqrt{3 \times 3} = \sqrt{9}$

Square Root of 9

Qube of 2

$2^3 = 2 \times 2 \times 2 = 8$

$2 = \sqrt{2 \times 2 \times 2} = \sqrt[3]{8}$

Qube Root of 8

Identity

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a-b)^2 = a^2 + b^2 - 2ab$

$a^2 - b^2 = (a+b)(a-b)$

Simple Interest $= \dfrac{\text{Principal} \times \text{Rate of Interest} \times \text{Time}}{100}$

Compound interest $= \text{Principal}\left(1 + \dfrac{\text{Rate of Interest}}{100}\right)^{\text{Time}} - \text{Principal}$

dhvajāṅka: vilokanam ānurūpyeṇa ūrdhvatiryagbhyām nikhilam navataścaramam daśataḥ

MAHARSHI SANDIPANI RASHTRIYA VEDA VIDYA PRATISHTHAN, UJJAIN (M.P.)
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PREFACE

(In the light of NEP 2020)

The Ministry of Education (Department of Higher Education), Government of India established Rashtriya Veda Vidya Pratishthan in Delhi under the Chairmanship of Hon'ble Education Minister (then Minister of Human Resource Development) under the Societies Registration Act, 1860 (XXI of 1860) on 20th January, 1987. The Government of India notified the resolution in the Gazette of India vide no 6-3/85- SKT-IV dated 30-3-1987 for establishment of the Pratishthan for preservation, conservation, propagation and development of oral tradition of Vedic studies (Veda Samhita, Padapatha to Ghanapatha,Vedanga, Veda Bhashya etc), recitation and intonation of Vedas etc and interpretation of Vedas in scientific lines. In the year 1993 the name of the organization was changed to Maharshi Sandipani Rashtriya Veda Vidya Pratishthan (MSRVVP) and it was shifted to Ujjain, Madhya Pradesh.

The National Education Policy of 1986 and Revised Policy Formulations of 1992 and also Programme of Action (PoA) 1992 have mandated Rashtriya Veda Vidya Pratishthan for promoting Vedic education throughout the country. The importance of India's ancient fund of knowledge, oral tradition and employing traditional Guru's for oral education was also emphasized in the PoA.

In accordance with the aspirations of the nation, national consensus and policy in favour of establishing a Board for Veda and Sanskrit Education at national level, the General Body and the Governing Council of MSRVVP under the Chairmanship of Hon'ble Education Minister, Government of India, have set up "Maharshi Sandipani Rashtriya Veda

Sanskrit Shiksha Board" (MSRVSSB) in tune with the mandate of the Pratishthan and its implementation strategies. The Board is necessary for the fulfillment of the objectives of MSRVVP as envisioned in the MoA and Rules. The Board has been approved by the Ministry of Education, Government of India and recognized by the Association of Indian Universities, New Delhi. The bye-laws of the Board have been vetted by Central Board of Secondary Education and curriculum structure have been concurred by the National Council of Educational Research and Training, New Delhi.

It may also be mentioned here that the committee "Vision and Roadmap for the Development of Sanskrit - Ten year perspective Plan", under the Chairmanship of Shri N. Gopalaswamy, former CEC, constituted by the Ministry of Education Govt. of India in 2015 recommended for establishment of a Board of Examination for standardization, affiliation, examination, recognition, authentication of Veda Sanskrit education up to the secondary school level. The committee was of the opinion that the primary level of Vedic and Sanskrit studies should be inspiring, motivating and joyful. It is also desirable to include subjects of modern education into Vedic and Sanskrit Pathashalas in a balanced manner. The course content of these Pathashalas should be designed to suit to the needs of the contemporary society and also for finding solutions to modern problems by reinventing ancient knowledge.

With regard to Veda Pathashala-s it is felt that they need further standardization of recitation skills along with introduction of graded materials of Sanskrit and modern subjects so that the students can ultimately acquire the capabilities of studying Veda bhashya-s and mainstreaming of students is achieved for their further studies. Due emphasis may also be given for the study of Vikriti Patha of Vedas at an appropriate level. The members of the committee have also expressed their concern that the Vedic recitation studies are not uniformly spread all over India; therefore, due steps may be taken to improve the situation without in anyway interfering with regional variations of recitation styles and teaching method of Vedic recitation.

It was also felt that since Veda and Sanskrit are inseparable and complementary to each other and since the recognition and affiliation problems are same for all the Veda Pathashalas and Sanskrit Pathashalas throughout the country, a Board may be constituted for both together. The committee observed that the examinations conducted by the Board should have legally valid recognition enjoying parity with modern Board system of education. The committee observed that the Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain may be given the status of Board of Examinations with the name "Maharshi Sandipani Rashtriya Veda Sanskrita Vidya Parishat with headquarters in Ujjain which will continue all programs and activities which were being conducted hitherto in addition to being a Board of Examinations.

The promotion of Vedic education is for a comprehensive study of India's glorious knowledge tradition and encompasses multi-layered oral tradition of Vedic Studies (Veda Samhita, Padapatha to Ghanapatha,Vedanga, Veda Bhashy aetc), recitation and intonation, and Sanskrit knowledge system content. In view of the policy of mainstreaming of traditional students and on the basis of national consensus among the policy making bodies focusing on Vedic education, the scheme of study of Veda stretching up to seven years in Pratishthan also entails study of various other modern subjects such as Sanskrit, English, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, etc. as per the syllabus and availability of time. In view of NEP 2020, this scheme of study is with appropriate inputs of Vedic knowledge and drawing the parallels of modern knowledge in curriculum content focusing on Indian Knowledge System.

In Veda Pathashala-s, GSP Units and Gurukula-s of MSRVVP, affiliated to the Board transact the curriculum primarily based on oral tradition of a particular complete Veda Shakha with perfect intonation and memorization, with additional subsidiary modern subjects such as English, Sanskrit, Mathematics, Science, Social Science and SUPW. Gradually, the Veda Pathashala-s will also introduce other skill and vocational subjects as per their resources.

It is a well-known fact that there were 1131 shakha-s or recensions of Vedas; namely 21 in Rigveda, 101in Yajurveda, 1000 in Samaveda and 9 in Atharva Veda. In course of time, a large number of these shakhas became extinct and presently only 10 Shakhas, namely, one in Rigveda, 4 in Yajurveda, 3 in Samaveda and 2 in Atharvaveda are existing in recitation form on which Indian Knowledge System is founded now. Even in regard to these 10 Shakhas, there are very few representative Vedapathis who are continuing the oral Vedic tradition/ Veda recitation/Veda knowledge tradition in its pristine and complete form. Unless there is a full focus for Vedic learning as per oral tradition, the system will vanish in near future. These aspects of Oral Vedic studies are neither taught nor included in the syllabus of any modern system of school education, nor do the schools/Boards have the systemic expertise to incorporate and conduct them in the conventional modern schools.

The Vedic students who learn oral tradition/ recitation of Veda are there in their homes in remote villages, in serene and idyllic locations, in Veda Gurukulas, (GSP Units), in Veda Pathashala-s, in Vedic Ashrams etc. and their effort for Veda study stretches to around 1900 – 2100 hours per year; which is double the time of other conventional school Board's learning system. Vedic students have to have complete Veda by-heart and recite verbatim with intonation *(udatta, anudatta, swaritaetc),*on the strength of memory and guru parampara, without looking at any book/pothi. Because of unique ways of chanting the Veda mantras, unbroken oral transmission of Vedas and its practices, this has received the recognition in the UNESCO-World Oral Heritage in the list of Intangible Cultural Heritage of Humanity. Therefore, due emphasis is required to be given to maintain the pristine and complete integrity of the centuries old Vedic Education (oral tradition/ recitation/ Veda knowledge Tradition). Keeping this aspect in view the MSRVVP and the Board have adopted unique type of Veda curriculum with modern subjects like Sanskrit, English, Vernacular language, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture etc. as well as skill and vocational subjects as prescribed by NEP 2020.

As per Vedic philosophy, any person can become happy if he or she learns both *Para-Vidya and Apara-Vidya.* The materialistic knowledge from the Vedas, their auxiliary branches and subjects of material interest were called *Apara-Vidya*. The knowledge of supreme reality, the ultimate quest from Vedas, Upanishads is called *Para-Vidya.* In all the total number of subjects to be studied as part of Veda and its auxiliaries are fourteen. There are fourteen branches of learning or *Vidyas* - four Vedas, Six Vedangas, Mimamsa (Purva Mimamsa and Uttara Mimamsa), Nyaya, Puranas and Dharma shastra. These fourteen along with Ayurveda, Dhanurveda, Gandharvaveda and Arthashastra become eighteen subjects for learning. All curriculum transaction was in Sanskrit language, as Sanskrit was the spoken language for a long time in this sub-continent.

Eighteen Shilpa-s or industrial and technical arts and crafts were mentioned with regard to the Shala at Takshashila. The following 18 skills/Vocational subjects are reported to be subjects of the study– (1) Vocal music (2) Instrumental music (3) Dancing (4) Painting (5) Mathematics (6) Accountancy (7) Engineering (8) Sculpture (9) Cattle breeding (10) Commerce (11) Medicine (12) Agriculture (13) Conveyancing and law (14) Administrative training (15) Archery and Military art (16) Magic (17) Snake charming (18) Art of finding hidden treasures.

For technical education in the above mentioned arts and crafts an apprenticeship system was developed in ancient India. As per the Upanishadic vision, the vidya and avidya make a person perfect to lead contented life here and liberation here-after.

Indian civilization has a strong tradition of learning of shastra-s, science and technology. Ancient India was a land of sages and seers as well as of scholars and scientists. Research has shown that India had been a Vishwa Guru, contributing to the field of learning (vidya-spiritual knowledge and avidya- materialistic knowledge) and learning centers like modern universities were set up. Many science and technology based advancements of that time, learning methodologies, theories and techniques discovered by the ancient sages have created and strengthened the fundamentals of our knowledge on many aspects, may it be on astronomy, physics, chemistry, mathematics, medicine, technology, phonetics, grammar etc. This needs to be essentially understood by every Indian to be proud citizen of this great country!

The idea of India like "Vasudhaiva Kutumbakam" quoted at the entrance of the Parliament of India and many Veda Mantra-s quoted by constitutional authorities on various occasions are understood only on study of the Vedas and true inspiration can be drawn only by pondering over them. The inherent equality of all beings as embodiment of "sat, chit, ananda" has been emphasized in the Vedas and throughout the Vedic literature.

Many scholars have emphasized that Veda-s are also a source of scientific knowledge and we have to look into Vedas and other scriptural sources of India for the solution of modern problems, which the whole world is facing now. Unless students are taught the recitation of Vedas, knowledge content of Vedas and Vedic philosophy as an embodiment of spiritual and scientific knowledge, it is not possible to spread the message of Vedas to fulfill the aspiration of modern India.

The teaching of Veda (Vedic oral tradition/ Veda recitation/ Veda knowledge Tradition) is neither only religious education nor only religious instruction. It will be unreasonable to say that Vedic study is only a religious instruction. Veda-s are not religious texts only and they do not contain only religious tenets; they are the corpus of pure knowledge which are most useful to humanity as whole. Hence, instruction or education in Veda-s cannot be construed as only "religious education/religious instruction."

Terming "teaching of Veda as a religious education" is not in consonance with the judgment of the Hon'ble Supreme Court (AIR 2013: 15 SCC 677), in Civil Appeal no. 6736 of 2004 (Date of judgment-3rd July

2013). The Vedas are not only religious texts, but they also contain the knowledge in the disciplines of mathematics, astronomy, meteorology, chemistry, hydraulics, physics, science and technology, agriculture, philosophy, yoga, education, poetics, grammar, linguistics etc. which has been brought out in the judgment by the Hon'ble Supreme Court of India.

Vedic education through establishment of Board in compliance with NEP-2020

The National Education Policy-2020 firmly recognizes the Indian Knowledge Systems (also known as 'Sanskrit Knowledge Systems'), their importance and their inclusion in the curriculum, and the flexible approach in combining various subjects. Arts' and Humanities' students will also learn science; try to acquire vocational subjects and soft skills. India's special heritage in the arts, sciences and other fields will be helpful in moving towards multi-disciplinary education. The policy has been formulated to combine and draw inspiration from India's rich, ancient and modern culture and knowledge systems and traditions. The importance, relevance and beauty of India's classical languages and literature is also very important for a meaningful understanding the national aspiration. Sanskrit, being an important modern language mentioned in the Eighth Schedule of Indian Constitution, its classical literature that is greater in volume than that of Latin and Greek put together, contains vast treasures of mathematics, philosophy, grammar, music, politics, medicine, architecture, metallurgy, drama, poetry, storytelling, and more (known as 'Sanskrit Knowledge Systems').These rich Sanskrit Knowledge System legacies for world heritage should not only be nurtured and preserved for posterity but also enhanced through research and put in to use in our education system, curriculum and put to new uses. All of these literatures have been composed over thousands of years by people from all walks of life, with a wide range of socio-economic background and vibrant philosophy. Sanskrit will be taught in engaging and experiential as well as contemporary relevant methods. The use of Sanskrit knowledge system is exclusively through listening to sound and pronunciation. Sanskrit textbooks at the Foundation and Middle School level will be available in Simple Standard Sanskrit (SSS) to teach Sanskrit through Sanskrit (STS) and make its study enjoyable. Phonetics and pronunciation prescriptions in NEP 2020 apply to the Vedas, the oral tradition of the Vedas and Vedic education, as they are founded upon phonetics and pronunciation.

There is no clear distinction made between arts and science, between curricular and extra-curricular activities, between vocational and academic streams, etc. The emphasis in NEP 2020 is on the development of a multi-disciplinary and holistic education among the sciences, social sciences, arts, humanities and sports for a multi-disciplinary world to ensure the unity and integrity of all knowledge. Moral, human and constitutional values like empathy, respect for others, cleanliness, courtesy, democratic spirit, spirit of service, respect for public property, scientific temper, freedom, responsibility, pluralism, equality and justice are emphasized.

The NEP-2020 at point no. 4.23 contains instructions on the pedagogic integration of essential subjects, skills and abilities. Students will be given a large amount of flexible options in choosing their individual curriculum; but in today's fast-changing world, all students must learn certain fundamental core subjects, skills and abilities to be a well-grounded, successful, innovative, adaptable and productive individual in modern society. Students must develop scientific temper and evidence based thinking, creativity and innovation, aesthetics and sense of art, oral and written expression and communication, health and nutrition, physical education, fitness, health and sport, collaboration and teamwork, problem solving and logical thinking, vocational exposure and skills, digital literacy, coding and computational thinking, ethics and moral reasoning, knowledge and practice of human and constitutional values, gender sensitivity, fundamental duties, citizenship skills and values, knowledge of India, environmental awareness etc. Knowledge of these skills include conservation, sanitation and hygiene, current affairs and important issues facing local communities, the states, the country and the world, as well as proficiency in multiple languages. In order to enhance the linguistic skills of children and to preserve these rich languages and their artistic treasures, all students in all schools, public or private, shall have the option of learning at least two years in one classical language of India and its related literature.

The NEP-2020 at point no. 4.27 states that -"Knowledge of India" includes knowledge from ancient India and its contributions to modern India and its successes and challenges, and a clear sense of India's future aspirations with regard to education, health, environment, etc. These elements will be incorporated in an accurate and scientific manner throughout the school curriculum wherever relevant; in particular, Indian Knowledge Systems, including tribal knowledge and indigenous and traditional ways of learning, will be covered and included in mathematics, astronomy, philosophy, yoga, architecture, medicine, agriculture, engineering, linguistics, literature, sports, games, as well as in governance, polity, conservation. It will have informative topics on inspirational personalities of ancient and modern India in the fields of medicinal practices, forest management, traditional (organic) crop cultivation, natural farming, indigenous sports, science and other fields.

The NEP-2020 at point no. 11.1 gives directions to move towards holistic and multidisciplinary education. India emphasizes an ancient tradition of learning in a holistic and multidisciplinary manner, including the knowledge of 64 arts such as singing and painting, scientific fields such as chemistry and mathematics, vocational fields such as carpentry, tailoring; professional work such as medicine and engineering, as well as the soft skills of communication, discussion and negotiation etc. which were also taught at ancient universities such as Takshashila and Nalanda. The idea that all branches of creative human endeavour, including mathematics, science, vocational subjects and soft skills, should be considered 'arts', has a predominantly Indian origin. This concept of 'knowledge of the many arts' or what is often called 'liberal arts' in modern times (i.e., a liberal conception of the arts) will be our part of education system.

At point No. 11.3 the NEP-2020 further reiterates that such an education system "would aim to develop all capacities of human beings - intellectual, aesthetic, social, physical, emotional, and moral in an integrated manner. Such an education will help develop well-rounded individuals that possess critical 21st century capacities in fields across the arts, humanities, languages, sciences, social sciences, and professional, technical, and vocational fields; an ethic of social engagement; soft skills, such as communication, discussion and debate; and rigorous specialization in a chosen field or fields. Such a holistic education shall be, in the long term, the approach of all undergraduate programmes, including those in professional, technical, and vocational disciplines."

The NEP-2020 at point no. 22.1 contains instructions for the promotion of Indian languages, art and culture. India is a rich storehouse of culture – which has evolved over thousands of years, and is reflected in its art, literary works, customs, traditions, linguistic expressions, artifacts, historical and cultural heritage sites, etc. Traveling in India, experiencing Indian hospitality, buying beautiful handicrafts and handmade clothes of India, reading ancient literature of India, practicing yoga and meditation, getting inspired by Indian philosophy, participating in festivals, appreciating India's diverse music and art and watching Indian films are some of the ways through which millions of people around the world participate in, enjoy and benefit from this cultural heritage of India every day.

In NEP-2020 at point no. 22.2 there are instructions about Indian arts. Promotion of Indian art and culture is important for India and to all of us. To inculcate in children a sense of our own identity, belonging and an appreciation of other culture and identity, it is necessary to develop in children key abilities such as cultural awareness and expression. unity, positive cultural identity and self-esteem can be built in children only by developing a sense and knowledge of their cultural history, art, language and tradition. Therefore, the contribution of cultural awareness and expression is important for personal and social well-being.

The core Vedic Education (Vedic Oral Tradition / Veda Path / Veda Knowledge Tradition) of Pratishthan along with other essential modern subjects- Sanskrit, English, Mother tongue, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, Indian

Art, Socially useful productive work etc., based on the IKS inputs are the foundations/sources of texts books of Pratishthan and Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board. These inputs are in tune with the NEP 2020. The draft books are made available in pdf form keeping in view the NEP 2020 stipulations, requirements of MSRVVP students and the advice of educational thinkers, authorities and policy of Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain. These books will be updated in line with NCFSE in future and finally will be made available in print form.

The Teachers of Veda, Sanskrit and Modern subjects in Rashtriya Adarsh Veda Vidyalaya, Ujjain and many teachers of Sanskrit and modern subjects in aided Veda Pathshalas of Pratishthan have worked for last two years tirelessly to prepare and present Sanskrit and modern subject text books in this form. I thank all of them from the bottom of my heart. Many eminent experts of the national level Institutes have helped in bringing quality in the textbooks by going through the texts from time to time. I thank all those experts and teachers of the schools. I extend my heartfelt gratitude to all my co-workers who have worked for DTP, drawing the sketches, art work and page setting.

All suggestions including constructive criticism are welcome for the improvement of the quality of the text books.

आपरितोषाद् विदुषां न साधु मन्ये प्रयोगविज्ञानम्।
बलवदपि शिक्षितानाम् आत्मन्यप्रत्ययं चेतः॥

(Abhijnanashakuntalam 1.02)

Until the scholars are fully satisfied about the content, presentation, attainment of objective, I do not consider this effort to be successful, because even the scholars are not fully confident in the presentation *without feedback from the stakeholders.*

Prof. ViroopakshaV Jaddipal

Secretary

Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain

Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board, Ujjain

FOREWORD

There has been a rich tradition of mathematics in India. Indian mystics and mathematicians have done excellent work in this field since the very ancient times of history. In India, mathematics has been given the highest place since the beginning of the Vedic period.

For example, the emphatic statement on the abstract mathematics is given below

बहुभिः प्रलापैः किं त्रैलोक्ये सचराचरे। यत्किंचिद्वस्तु तत् सर्वं गणितेन विना न हि॥

लौकिके वैदिके वापि तथा सामयिकेऽपि यः। व्यापारस्तत्र सर्वत्र संख्यानमुपयुज्यते॥

(गणितसारसंग्रह, संज्ञाधिकार: 8-9)

Meaning, whatever we have in this materialistic universe, the existence of the same cannot be understood without mathematics, because without numbers the behaviors in the cosmic, Vedic and Events in the universe cannot be explained.

By integrating the ancient knowledge, in concurrence with modern mathematics and ancient achievements, this textbook incorporates mathematical concepts available in Sanskrit literature along with Vedic mathematics. An attempt has been made to simplify calculations using the techniques Vedic mathematics.

To harmonize mathematics teaching with the changing environment in the current global scenario and provide proficient learning levels in mathematics to Vedic students all over India, keeping in mind the main principles like discussion, analysis, examples and application as prescribed in the National Education Policy 2020, syllabus and text books have been created according to Vedic students.

The language use in the textbook is very simple and intuitive, which will make it easier for the students to understand. This textbook of Ved Bhushan III (equivalent to Grade 8) year is almost equivalent to the class-8 mathematics syllabus across India.

The textbook contains many Vedic evidences of the Sanskrit literature and references such as Brahmasphut Siddhanta, Shulbasutra, Aryabhava Tiyam, Lilavati and Beeja ganitam etc. By this Vedic student will be able to understand ancient mathematical concepts along with modern mathematics and experience the dignity of their Indian traditions.

Revising the concepts of the previous class, a total of 11 chapters of the textbook have been composed as per the requirements of the Ved Bhushan 3rd year syllabus of Ved Vidyalaya. In Chapter 1, the operations on rational numbers and their properties are presented in detail. Chapter 2 describes the method of solving the linear equations with a variable and the applications of the equations. Chapter 3 deals with the methods of finding the square and square root, chapter 4 explains the methods of finding the cube and cube root of numbers. In Chapter 5, the law of exponents is presented in detail under the mathematical concept of powers and exponents. In Chapter 6, in Vedic mathematics, you will learn how to multiply from the Udharvatriyagyabhayam (vertical) and anurupyen formulas and find the quotient from the Dhwajak sutra. Chapter 7 describes in detail the types of polygons, quadrilateral and their respective properties in 'Understanding of quadrilateral'. Chapter 8 describes algebraic expressions and identities. In Chapter9, the comparison of amounts quantities teaches how to calculate simple interest and compound interest along with finding percentage, of increase/ decrease. The direct and inverse proportions are dealt in detail in Chapter 10. In Chapter 11, the formulae for finding the total surface area of a cube, cuboid and cylinder in the topic of mensuration is presented in detail.

The textbook contains various activities to develop the Vedic students' understanding of mathematics as well as the ability to rediscover facts. Important concepts and learning outcomes are given as "We Learned" at the end of each chapter to enhance the level of understanding and attain proficiency.

To make students appreciate the rich traditions of India and the contribution made by Indian mathematicians in mathematics, at the end of the book, details on “the contribution of Indian mathematicians to mathematics” has also been mentioned.

By understanding the mathematical concepts available in the textbook, Vedic students will be able to prepare for competitive examinations. After studying this book, students should study class VIII mathematics book published by NCERT and books related to specific subjects.

The author will be grateful to you for the positive suggestions sent for correcting the errors in the textbook.

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Chapter 1

Rational number

Dear Student ! We have studied about rational numbers in the previous class. You will remember that rational number is defined as $\frac{p}{q}$. where p, q are integers and q ≠ 0. We have studied in detail about positive and negative rational numbers, equivalent and standard form of rational numbers, their comparison etc. In this chapter, revising the concepts which we have study in the previous class, we will study about operations on rational numbers, additive and multiplicative inverses and how to find the rational numbers between any two rational numbers, in detail.

The following verse related to the form of rational numbers is found in Reaganite.

सच्छेदकराशौ भाज्ये भाजके वा ऋणत्वं चेत् फलमृणमेव,

द्वयोर्ऋणत्वे फलं धनमेव यथा $\frac{-\text{अ}}{\text{व}} = -\frac{\text{अ}}{\text{व}}$, $\frac{\text{अ}}{-\text{व}} = \frac{-\text{अ}}{\text{व}}$ **तथा** $\frac{-\text{अ}}{-\text{व}} = \frac{\text{अ}}{\text{व}}$

(बीजगणितम्, सच्छेदकराशिगणितम्, पृष्ठ 238)

The form of positive and negative rational numbers is explained in the above verse.

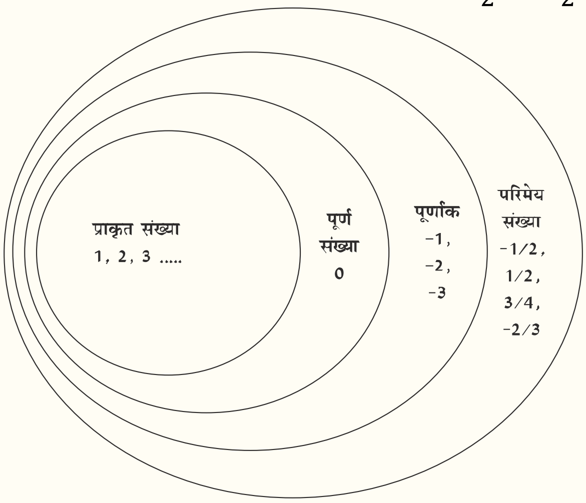
You must be aware of different number systems present in rational number system.

Recall that, rational numbers include the following number system

1. Natural numbers (N) = 1, 2, 3, 4, 5, 6, ...
2. Whole numbers (W) = 0, 1, 2, 3, 4, 5, 6, ...

3. Integer (I) =...... – 3, –2, –1, 0, 1, 2, 3.......

4. Fractions and their negatives (R) =........$\frac{-1}{2}$, 0, $\frac{1}{2}$,.........



From the above we can conclude, all natural numbers(N) are whole numbers and all whole numbers are integers, all integers are rational numbers and all rational numbers are real numbers (irrational and rational numbers).

Properties of rational numbers :

1. **Closed property-** If the sum, difference, product and division of any two rational numbers are also a rational number, the system of rational numbers is said to be closed under the respective operations.

The sum of the rational numbers $\frac{1}{2}$ and $\frac{2}{4}$ is $\frac{3}{4}$ which is also a rational number.

Hence, rational numbers are closed under the operation of addition.

Consider the following examples.

Sum :-(a) $\frac{3}{7} + \frac{1}{2} = \frac{6}{14} + \frac{7}{14} = \frac{13}{14}$ is a rational number.

(b) $\frac{2}{5} + \frac{(-4)}{3} = \frac{6}{15} + \frac{(-20)}{15} = \frac{6-20}{15} = \frac{-14}{15}$ is a rational number.

Subtraction :

(a) $\frac{3}{7} - \frac{5}{3} = \frac{9}{21} - \frac{35}{21} = \frac{9-35}{21} = \frac{-26}{21}$ is a rational number.

(b) $\frac{2}{5} - \frac{2}{5} = \frac{2-2}{5} = \frac{0}{5} = 0$ is a rational number.

Multiplication :-

(a) $\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}$ is a rational number.

(b) $\frac{4}{5} \times \frac{(-3)}{7} = \frac{4\times(-3)}{5\times7} = \frac{(-12)}{35}$ is a rational number.

Division:

(a) $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{2\times7}{3\times5} = \frac{14}{15}$ is a rational number.

(b) $0 \div \frac{2}{3} = 0 \times \frac{3}{2} = \frac{0\times3}{2} = \frac{0}{2} = 0$ is a rational number.

(c) $\frac{3}{5} \div \frac{0}{4} = \frac{3}{5} \times \frac{4}{0} = \frac{3\times4}{5\times0} = \frac{12}{0}$ is not a rational number.

From the above examples, we find that the result of addition, subtraction and multiplication between any two rational numbers is also a rational number. But it is not necessary that the division of any tow rational numbers should always be a rational number. **Hence, rational numbers are closed under addition, subtraction and multiplication, but rational numbers are not closed under division.**

2. Associative properties -

Let's check the associative property on numerical operation (+, -, ×, ÷) between three rational numbers.

Let us examine three rational numbers 1/2, 3/4 and 2/3.

(a) $\frac{1}{2} + \frac{3}{4} + \frac{2}{3}$

$= \frac{1}{2} + \left(\frac{3}{4} + \frac{2}{3}\right)$	$= \left(\frac{1}{2} + \frac{3}{4}\right) + \frac{2}{3}$
$= \frac{1}{2} + \left(\frac{9+8}{12}\right)$	$= \left(\frac{2+3}{4}\right) + \frac{2}{3}$
$= \frac{1}{2} + \frac{17}{12}$	$= \frac{5}{4} + \frac{2}{3}$

$= \frac{6+17}{12}$ | $= \frac{15}{12} + \frac{8}{12}$

$= \frac{23}{12}$ | $= \frac{23}{12}$

for any three rational numbers *a*, *b*, and *c* in general

$$a + (b + c) = (a + b) + c$$

Subtraction:

Checking by taking three rational numbers $\frac{1}{3}, \frac{2}{3}$ and $\frac{(-3)}{4}$-

$= \frac{1}{3} - \left(\frac{2}{3} - \frac{(-3)}{4}\right)$	$= \left(\frac{1}{3} - \frac{2}{3}\right) - \frac{(-3)}{4}$
$= \frac{1}{3} - \left(\frac{8}{12} - \frac{-9}{12}\right)$	$= \left(\frac{3}{6} - \frac{4}{6}\right) - \frac{(-3)}{4}$
$= \frac{1}{3} - \left(\frac{8-(-9)}{12}\right)$	$= \left(\frac{3-4}{6}\right) - \frac{(-3)}{4}$
$= \frac{1}{3} - \left(\frac{8+9}{12}\right)$	$= \frac{-1}{6} - \frac{(-3)}{4}$
$= \frac{1}{3} - \frac{17}{12}$	$= \frac{-2}{12} - \frac{(-9)}{12}$
$= \frac{4}{12} - \frac{17}{12}$	$= \frac{-2-(-9)}{12}$
$= \frac{4-17}{12} = \frac{(-13)}{12}$	$= \frac{-2+9}{12} = \frac{7}{12}$

for rational numbers a, b and c in general

$$a - (b - c) \neq (a - b) - c$$

Multiplication: Check the associative property with respect to multiplication by taking three rational numbers $\frac{1}{2}, \frac{3}{4}$ and $\frac{(-2)}{3}$-

$= \left(\frac{1}{2} \times \frac{3}{4}\right) \times \frac{(-2)}{3}$	$= \frac{1}{2} \times \left(\frac{3}{4} \times \frac{(-2)}{3}\right)$
$= \left(\frac{1\times3}{2\times4}\right) \times \frac{(-2)}{3}$	$= \frac{1}{2} \times \left(\frac{3\times(-2)}{4\times3}\right)$
$= \frac{3}{8} \times \frac{-2}{3}$	$= \frac{1}{2} \times \frac{(-6)}{12}$
$= \frac{3\times(-2)}{8\times3}$	$= \frac{1\times(-6)}{2\times12} = \frac{-6}{24}$
$= \frac{-6}{24}$ or $\frac{-1}{4}$	$= \frac{(-1)}{4}$

for rational numbers a, b and c in general

$$(a \times b) \times c = a \times (b \times c)$$

Division: -

Check the associative property of division by taking three rational numbers $\frac{1}{2}, \frac{2}{3}$ and $\frac{(-1)}{4}$.

$\left(\frac{1}{2} \div \frac{2}{3}\right) \div \frac{(-1)}{4}$	$= \frac{1}{2} \div \left(\frac{2}{3} \div \frac{(-1)}{4}\right)$
$= \left(\frac{1}{2} \times \frac{3}{2}\right) \div \frac{(-1)}{4}$	$= \frac{1}{2} \div \left(\frac{2}{3} \times \frac{4}{(-1)}\right)$
$= \left(\frac{1\times3}{2\times2}\right) \div \frac{(-1)}{4}$	$= \frac{1}{2} \div \frac{(2\times4)}{3\times(-1)}$
$= \frac{3}{4} \div \frac{(-1)}{4}$	$= \frac{1}{2} \div \left(\frac{8}{(-3)}\right)$
$= \frac{3}{4} \times \frac{4}{(-1)}$ or $\frac{3}{4} \times \frac{(-4)}{1}$	$= \frac{1}{2} \times \frac{(-3)}{8}$
$= \frac{3\times(-4)}{4\times1} = \frac{-12}{4}$	$= \frac{1\times(-3)}{2\times8}$
$= \frac{(-3)}{1}$	$= \frac{(-3)}{16}$

for rational numbers a, b and c in general

$$(a \div b) \div c \neq a \div (b \div c)$$

3) Identity property:

'योगे खं क्षेपसमं' (लीलावती गणित, शून्यपरिकर्माष्टकम्)

That is, when zero is added to a number, the sum is equal to that number.

Additive identity and Multiplicative identity

Adding zero to a rational number and multiplying it by 1, the same number is obtained, that is, zero (0) is called additive identity and number one (1) is called multiplicative identity.

(a) $5 + 0 = 5 = 0 + 5$

(c) $(-8) \times 1 = -8 = 1 \times (-8)$

(b) $\frac{3}{2} = 0 + \frac{3}{2} + 0 = \frac{3}{2}$

(d) $\frac{3}{4} = 1 \times \frac{3}{4} = 1 \times \frac{3}{4}$

for a rational number a -

$a + 0 = a = 0 + a$ **$a \times 1 = a = 1 \times a$**

0 is the additive identity. **1 is the multiplicative identity.**

4) Inverse property

In the explanation of the following verse of Brahmasphutasiddhanta, the opposite property has been described.

समयोर्धनर्णयोरैक्यं खं शून्यं भवति।

(ब्राह्मस्फुटसिद्धान्त:,कुट्टकाध्याय:, श्लो.30 (व्याख्या))

That is, the difference between equal positive and negative numbers is zero.

Additive inverse when the sum of two rational numbers is zero (additive identity), then we can say that both the numbers are additive inverse of each other. The additive inverse of a is (– a) and the additive inverse of (– a) is a.

As : $3 + (-3) = 3 - 3 = 0$

$\frac{3}{5} - \frac{3}{5} = 0$

When the product of two rational numbers is a (1) multiplicative inverse (multiplicative identity), then both the numbers are multiplicative inverses of each other. we can say. In general, the multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$ and the multiplicative inverse of $\frac{b}{a}$ is $\frac{a}{b}$.

where $a \neq 0$

Example: $\frac{3}{4} \times \frac{4}{3} = 1$

$\frac{5}{7} \times \frac{7}{5} = 1$

5. Commutative -

(+, −, ×, ÷) on interchanging the orders of two rational numbers while doing an operation, has the same result, we say that commutative law is satisfied under that operation.

Addition: -

Find the sum of two rational numbers $\frac{3}{4}$ and $\frac{2}{5}$.

$\frac{3}{4} + \frac{2}{5}$	$\frac{2}{5} + \frac{3}{4}$
$= \frac{15}{20} + \frac{8}{20}$	$= \frac{8}{20} + \frac{15}{20}$
$= \frac{15+8}{20}$	$= \frac{8+15}{20}$
$= \frac{23}{20}$	$= \frac{23}{20}$

From the above results we can conclude that for any two distinct rational number *a* and *b* commutative law is valid under the operation of addition.

$$a + b = b + a$$

Subtraction: Find the difference between $\frac{7}{3}$ and $\frac{3}{5}$.

$\frac{3}{5} - \frac{7}{3}$	$\frac{7}{3} - \frac{3}{5}$
$= \frac{9}{15} - \frac{35}{15}$	$= \frac{35}{15} - \frac{9}{15}$
$= \frac{9-35}{15}$	$= \frac{35-9}{15}$
$= \frac{-26}{15}$	$= \frac{26}{15}$

From the above we can conclude that, in general for any two distinct rational numbers a and b, do not obey the commutative property with respect to subtraction.

$$\boldsymbol{a - b \neq b - a}$$

Multiplication:- Find the product of two rational numbers $\frac{3}{5}$ and $\frac{4}{7}$.

$\frac{3}{5} \times \frac{4}{7}$	$\frac{4}{7} \times \frac{3}{5}$
$= \frac{3\times4}{5\times7}$	$= \frac{4\times3}{7\times5}$
$= \frac{12}{35}$	$= \frac{12}{35}$

From the above we can conclude that in general for any two distinct rational numbers a and b obey the commutative property with respect to multiplication -

$$\mathbf{a \times b = b \times a}$$

Division:- Find the quotient of two rational numbers $\frac{7}{5}$ and $\frac{2}{3}$.

$\frac{7}{5} \div \frac{2}{3}$	$\frac{2}{3} \div \frac{7}{5}$
$= \frac{7}{5} \times \frac{3}{2}$	$= \frac{2}{3} \times \frac{5}{7}$
$= \frac{7\times3}{5\times2}$	$= \frac{2\times5}{3\times7}$
$= \frac{21}{10}$	$= \frac{10}{21}$

From the above we can conclude that in general for any two distinct rational numbers a and b, do not obey the commutative property with respect to the division –

$$\mathbf{a \div b \neq b \div a}$$

Looking at all of the examples above, we can see that rational numbers follow the commutative rule for addition and multiplication, but not for subtraction and division.

Operations on rational numbers

Students! You are familiar with operations (+,-,×,÷) on integers and fractions Can you do addition, subtraction (difference), multiplication and division of integers and fraction? In Lilavati ganit, the following verse is found to find the addition and subtraction (difference) of rational numbers.

अन्योन्यहाराभिहतौ हरांशौ राश्यो: समच्छेदविधानमेवम् ।

मिथो हराभ्यामपवर्त्तिताभ्यां यद्वा हरांशौ सुधियाऽत्र गुण्यौ ॥

(लीलावती,भिन्नपरिकर्माष्टकम् : 1)

Meaning, find the sum of the product of the denominator of the first quantity and with the numerator of the second quantity, denominator of the second quantity and the numerator of the first quantity, to get the numerator of the result. Further multiply both the denominator to get the denominator of the result. Simplify the rational number obtained.

- **In a similar method**, we find the difference between the two rational numbers.

First method: Find the sum of $\frac{2}{4}$ and $\frac{3}{8}$. (By the first method of cross multiplication)

Solution: $\frac{16}{32} + \frac{12}{32} = \frac{28}{32} = \frac{7}{8}$

Second Method: In this method, Dividing both 4, 8 by their common divisor, we gat 1, 2 respectively.

Now on multiplying both numerator and denominator of the rational numbers in the reverse order (2 with first and 1 with second rational number), to get rational numbers with the same denominator.

Further, it is evaluated.

This method is similar to the first method. $\frac{4}{8} + \frac{3}{8}$ Adding both we get $\frac{7}{8}$. The result is same as the result obtained in method 1. Let us study the operations on rational numbers through examples.

Sum and difference of rational numbers –

- **Sum of two rational numbers having same denominator –**

Example: Find the sum of the rational numbers $\frac{2}{7}$ and $\frac{-5}{7}$.

Solution: $\frac{2}{7} + \frac{(-5)}{7}$

$= \frac{2+(-5)}{7}$

$= \frac{2-5}{7}$

$= \frac{(-3)}{7}$

- **do and learn**

1) $\frac{(-1)}{7} + \frac{3}{7}$ 2) $\frac{2}{5} + \frac{(-3)}{5}$

Let us solve some more examples.

1) $\frac{1}{4} + \frac{5}{4} = \frac{1+5}{4} = \frac{6}{4} = \frac{3}{2}$

2) $\frac{(-13)}{7} + \frac{2}{7} = \frac{(-13)+2}{7} = \frac{(-11)}{7}$,

we observe that to find the sum of rational numbers having the same denominator, we add the numerators keeping the same denominator and simplify.

Dinkar asked Balakrishna how do we add two rational numbers with different denominators? Balakrishna replied, "do you remember, We added two fractions with different denominators".

1) As with fractions, we find the least common multiple of these denominators.

2) After this, find the equivalent rational numbers such that their

denominators are the LCM least common multiple obtained.

3) Add the two rational numbers (which have the same denominator).

Example: Find the sum of the rational numbers $\frac{1}{4} + \frac{(-2)}{5}$.

Solution: $\frac{1}{4} + \frac{(-2)}{5}$

The least common multiple (LCM) of 4 and 5 is 20.

$$\frac{1}{4} = \frac{1\times5}{4\times5} = \frac{5}{20}$$

$$\frac{(-2)}{5} = \frac{(-2)\times4}{5\times4} = \frac{(-8)}{20}$$

then, $$\frac{1}{4} + \frac{(-2)}{5} = \frac{5}{20} + \frac{(-8)}{20} = \frac{5+(-8)}{20} = \frac{5-8}{20} = \frac{(-3)}{20}$$

- **Do and learn**

Find the sum of the following.

1) $\frac{1}{3} + \frac{2}{5}$ 2) $\frac{(-1)}{4} + \frac{2}{3}$

Subtraction (difference) –

The following verse is found in relation to finding the difference of rational numbers in the Ganitadhyay of Brahmasphuta Siddhanta.

विपरीतच्छेदगुणा राश्योश्छेदांशका: समच्छेदाः ।

सङ्कलितेंऽशा योज्या व्यवकलितेंऽशान्तरं कार्यम्॥

(ब्रह्मस्फुटसिद्धान्त,गणिताध्याय : 2)

Meaning, the denominator and numerator of two rational numbers are multiplied by the denominator of the other rational number to make rational number having the same denominator.

For adding, add the numerators to get the numerator of the result and write the common denominator as denominator of the result.

For subtraction, subtract the numerators to get the numerator of the result and write the common denominator as denominator of the result.

- **Subtraction (difference) of two rational numbers with the same denominator -**

Example: Subtract $\frac{(-7)}{8}$ from the rational number $\frac{3}{8}$.

Solution : $\frac{3}{8} - \frac{(-7)}{8}$

$= \frac{3-(-7)}{8} = \frac{3+7}{8} = \frac{10}{8}$

❖ **Do and learn**

Solve the following-

1) $\frac{5}{2} - \frac{3}{2}$ 2) $\frac{5}{9} - \frac{4}{9}$

Let us solve some more examples.

1) $\frac{5}{3} - \frac{8}{3} = \frac{5-8}{3} = \frac{(-3)}{3} = -1$

2) $\frac{(-5)}{4} - \frac{13}{4} = \frac{-5-13}{4} = \frac{(-18)}{4} = \frac{(-9)}{2}$

3) $\frac{2}{11} - \frac{(-3)}{11} = \frac{2-(-3)}{11} = \frac{2+3}{11} = \frac{5}{11}$

we find the differentiate between the numerators to get the numerator of the result and the denominator of the result will be the common denominator of the rational numbers.

Vishnu asked Jagdish, If the denominators of both the rational numbers are different, how will we find their difference ?

Jagdish : Just as we find the sum of rational numbers, in the same way, we find the difference between two rational numbers.

Example: Subtract $\frac{(-1)}{6}$ from the rational number $\frac{5}{4}$.

Solution: $\frac{5}{4} - \frac{(-1)}{6}$

The least common multiple (LCM) of 4 and 6 is 12.

$$\frac{5}{4} = \frac{5\times3}{4\times3} = \frac{15}{12}$$

$$\frac{(-1)}{6} = \frac{(-1)\times2}{6\times2} = \frac{(-2)}{12}$$

Hence, $\frac{5}{4} - \frac{(-1)}{6} = \frac{15}{12} - \frac{(-2)}{12}$

$$= \frac{15+2}{12}$$

$$= \frac{17}{12}$$

❖ **Do and learn**

Solve (subtract) the following.

1) $\frac{2}{3} - \frac{4}{5}$ 2) $\frac{1}{2} - \frac{(-1)}{3}$

Multiplication of rational numbers

Students! We know how to find the product of two numbers.

अंशाहतिश्छेदवधेन भक्ता लब्धं विभिन्ने गुणने फलं स्यात् ॥

(लीलावती गणित, 42)

Meaning, dividing the product of numerators by the product of denominators, we get the product of two rational numbers.

The following verse is found in the Brahmasfuta Siddhanta about finding the product of rational numbers like the multiplication of fractional numbers.

रूपाणिच्छेदगुणान्यंशयुतानि द्वयोर्बहूनां वा ।

प्रत्युत्पन्नो भवतिच्छेदवधेनोद्धृतोंऽशवधः ॥

(ब्रह्मस्फुटसिद्धान्त, गणिताध्याय: 3)

Meaning, for the multiplication of two or more rational numbers, the product of the rational numbers is obtained by dividing the product of the numerators by the product of the denominator.

Example: Let us consider the product of rational numbers $2 \times \frac{(-5)}{3}$.

Solution: Multiplication: $\frac{2}{1} \times \frac{(-5)}{3}$

$$= \frac{2 \times (-5)}{1 \times 3} = \frac{(-10)}{3}$$

Example: Multiply the rational number $\frac{3}{4}$ by $\frac{(-4)}{7}$.

Solution: Multiplication operation: $\frac{3}{4} \times \frac{(-4)}{7}$

$$= \frac{3 \times (-4)}{4 \times 7}$$

$$= \frac{(-12)}{28}$$

❖ Do and learn

Multiply the following -

1) $4 \times \frac{(-2)}{5}$ 2) $\frac{5}{3} \times \frac{(-7)}{4}$

Division of rational numbers –

The method of finding the quotient of rational numbers is like the quotient of a fraction. Do you know how to divide between the two?

छेदं लवं च परिवर्त्य हरस्य शेष: कार्योऽथ भागहरणे गुणनाविधिश्च।

(लीलावती गणित, 42)

Meaning, to find the quotient of a number, the quotient is obtained by interchanging the numerator and denominator of the divisor and multiplying the dividend.

The following verse is found in the Ganitadhyaya of Brahmasfuta Siddhanta on the subject of division of rational numbers like the division of fractional numbers.

परिवर्त्य भागहारच्छेदांशौ छेदसंगुणाच्छेद: ।

अंशोंऽशगुणो भाज्यस्य भागहार: सवर्णितयोः ॥

(ब्राह्मस्फुटसिद्धान्त, गणिताध्याय,5)

Meaning, to find the quotient of a rational number, the quotient is obtained by interchanging the places of the denominator and the numerator (inverse) of the divisor and multiplying with the dividend.

Students! Do you know about reciprocal of fractions ? The reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$. This concept also applies to the reciprocal of rational numbers. Thus, the reciprocal of $\frac{(-5)}{4}$ Is $\frac{(-4)}{5}$ or $\frac{4}{(-5)}$ and the reciprocal of $\frac{(-7)}{9}$ is $\frac{(-9)}{7}$.

we know,

$$2 \times 5 = 10$$

We can write this in two ways.

(1)	(2)
$10 \div 5 = 2$	$10 \div 2 = 5$
$\frac{10}{5} = 2$	$\frac{10}{2} = 5$
$10 \times \frac{1}{5} = 2$	$10 \times \frac{1}{2} = 5$

On finding the product from the reciprocal of the divisor in the division, we get the quotient. In the above example, we have obtained the quotient by converting the operation of division into multiplication.

Example : Solve $\frac{(-21)}{5} \div \frac{2}{3}$.

Solution: $\frac{(-21)}{5} \div \frac{2}{3}$

$= \frac{(-21)}{5} \times \frac{3}{2}$ $\left(\text{Since the reciprocal of } \frac{2}{3} \text{ is } \frac{3}{2}\right)$

$= \frac{(-21) \times 3}{5 \times 2}$

$= \frac{(-63)}{10}$

Example: Solve $\frac{3}{4} \div \frac{(-1)}{5}$.

Solution: $\frac{3}{4} \div \frac{(-1)}{5}$

$= \frac{3}{4} \times \frac{(-5)}{1}$ $\left(\text{Since the reciprocal of } \frac{5}{(-1)} \text{ or } \frac{(-5)}{1}\right)$

$$= \frac{3 \times (-5)}{4}$$

$$= \frac{(-15)}{4}$$

❖ **Do and learn**

Solve the following -

1) $\frac{(-7)}{4} \div \frac{3}{5}$ 2) $\frac{5}{3} \div \frac{(-1)}{4}$

Additive inverse –

If the sum of two rational numbers is zero (0), the numbers are **additive inverse** of each other.

Additive inverse of a = **(- a).**

The additive inverse of 2 is (– 2) and the additive inverse of (– 2) is 2.

Example: Find the additive inverse of the rational number $\frac{3}{4}$.

Solution : Adding $\frac{3}{4}$ to $-\frac{3}{4}$

$$\frac{3}{4} + \frac{(-3)}{4} = \frac{3-3}{4} = \frac{0}{4} = 0$$

Hence, the additive inverse of $\frac{3}{4}$ is $\frac{(-3)}{4}$.

Look at the additive inverse of numbers given in the following example-

Additive inverse of $\frac{(-3)}{5} = \frac{3}{5}$

Additive inverse of $\frac{5}{7} = \frac{(-5)}{7}$

Additive inverse of 1 = – 1

Thus we can say. The additive inverse of a rational number is the same number with the opposite sign.

❖ **Do and learn**

Write the additive inverse of the following numbers.

a) 5   b) $\frac{(-2)}{7}$ c) $\frac{5}{4}$ d) $\frac{(-4)}{3}$

Multiplicative inverse -

When the product of two numbers is 1, the numbers are called the **multiplicative inverses** of each other. The general form of multiplicative inverse is as follows-

$$\frac{a}{b} \times \frac{b}{a} = 1 \text{ where a, b} \neq 0$$

For example: Multiplicative inverse of $\frac{3}{5}$ Is $\frac{5}{3}$ and Multiplicative inverse of $\frac{5}{3}$ is $\frac{3}{5}$.

$\frac{(-2)}{7} \times \frac{(-7)}{2} = 1$ for the rational number $\frac{(-2)}{7}$.

Hence, we can say that the multiplicative inverse of $\frac{(-2)}{7}$ is $\frac{(-7)}{2}$.

❖ **Do and learn**

Write the multiplicative inverse of the following rational numbers.

a) $\frac{3}{7}$ b) $\frac{(-4)}{9}$ c) $\frac{5}{7}$

Number between two rational numbers –

There are infinite number of rational numbers between any two rational numbers.

Let us consider an example.

How many rational numbers are there between the numbers $\frac{(-1)}{20}$ and $\frac{3}{20}$?

Clearly $\frac{0}{20}$, $\frac{1}{20}$, $\frac{2}{20}$ are rational numbers which lie between them.

If we write $\frac{(-1)}{20}$ as $\frac{(-100)}{2000}$ and $\frac{3}{20}$ as $\frac{300}{2000}$ then Number between $\frac{(-1)}{20}$ and $\frac{3}{20}$

We get $\frac{-99}{2000}, \frac{-98}{200}, \ldots\ldots, \frac{299}{2000}$.

Integers:

(..., -3, -2, -1, 0, 1, 2, 3, ...)

Example: Find 5 rational numbers between the rational numbers $\frac{(-3)}{4}$ and $\frac{2}{5}$.

Solution: First of all we consider $\frac{(-3)}{4}$ and $\frac{2}{5}$ as rational numbers with like denominators.

convert into $\frac{(-3)\times 5}{4\times 5} = \frac{(-15)}{20}$ and $\frac{2\times 4}{5\times 4} = \frac{8}{20}$

Similarly we find the following rational numbers between $\frac{-15}{20}$ and $\frac{8}{20}$ $\left(\frac{-15}{20}, \frac{-13}{20}, \ldots, \ldots, \ldots, \ldots, \frac{7}{20}, \frac{8}{20}\right)$ You can take any five of these rational numbers.

$$\frac{-10}{20}, \frac{-7}{20}, \frac{2}{20}, \frac{4}{20}, \frac{5}{20}$$

❖ **Do and learn**

Find two numbers between $\frac{1}{2}$ and $\frac{2}{5}$.

Let us learn to solve problems based on BODMAS rule.

Example : Solve- $\frac{3}{4} \times \frac{2}{5} + \frac{1}{4}$

Solution: $\frac{3\times 2}{4\times 5} + \frac{1}{4}$

$= \frac{6}{20} + \frac{1}{4}$

$\frac{6}{20} = \frac{6\times 1}{20\times 1} = \frac{6}{20}$

$\frac{1}{4} = \frac{1\times 5}{4\times 5} = \frac{5}{20}$

$\frac{6}{20} + \frac{1}{4} = \frac{6}{20} + \frac{1}{4} = \frac{6}{20} + \frac{5}{20}$

$= \frac{6+5}{20} = \frac{11}{20}$

BODMAS rule

B: brackets : ( ), { }, [ ]

O: of (power of) : $(\ )^2$ or $\sqrt{\ }$

D: division: ÷

M: multiplication : ×

A: addition : +

S: subtraction : –

❖ **Do and learn**

Simplify the following (using BODMAS rules) –

1) $\frac{1}{4} + \frac{3}{2} \times \frac{1}{3}$ (2 $\frac{4}{5} \times \frac{3}{2} + \frac{1}{2}$

Exercise 1.1

1. Select the correct option for the following multiple-choice questions.

(a) Which of the following is an additive identity?

(I) +1 (II) -1 (III) 0 (IV) 10

(b) Which of the following is a multiplicative identity?

(I) 1 (II) 0

(iii) Both (a) and (b) (IV) none of these

(c) The smallest prime number is-

(I) 2 (II) 1 (III) 0 (IV) 3

(d) Multiplicative inverse of 7 will be-

(I) $\frac{1}{7}$ (II) 7 (III) -7 (IV) None of these

(v) Which pair is a prime pair?

(I) 15, 17 (II) 17, 19 (III) 23, 25 (IV) 53, 55

(f) The additive inverse of $\frac{5}{6}$ is

(I) $\frac{-5}{6}$ (II) $\frac{5}{6}$ (III) $\frac{6}{5}$ (IV) 0

(b) How many rational numbers will be in between two rational numbers?

(i) 1 (II) 10 (III) 100 (IV) infinite

(m) What is the multiplicative inverse of $\frac{11}{-6}$?

(I) $\frac{-6}{11}$ (II) $\frac{-11}{6}$ (III) $\frac{-11}{-6}$ (IV) $\frac{11}{6}$

2. Find the sum of the following rational numbers.

a) $\frac{2}{5}+\frac{(-3)}{4}$ b) $\frac{(-5)}{3}+\frac{4}{5}$

c) $\frac{1}{4}+\frac{(-3)}{4}$ d) $\frac{4}{7}+\frac{(-8)}{7}$

3. Solve (subtract) the following rational numbers -

a) $\frac{2}{7} - \frac{5}{7}$ b) $\frac{(-5)}{3} - \frac{(-4)}{3}$

c) $\frac{3}{4} - \frac{2}{7}$ d) $\frac{1}{4} - \frac{2}{3}$

4. Find the products of the following rational numbers.

a) $\frac{3}{4} \times \frac{(-4)}{5}$ b) $\frac{(-3)}{4} \times \frac{(-7)}{2}$

c) $\frac{-4}{1} \times \frac{2}{12}$ d) $\frac{2}{5} \times \frac{-3}{7}$

5. Find the value of the following. (Quotient)

a) $\frac{3}{4} \div \frac{2}{5}$ b) $\frac{2}{5} \div \frac{7}{1}$

c) $\frac{3}{4} \div \frac{(-7)}{5}$ d) $\frac{(-4)}{5} \div \frac{7}{2}$

6. Find the value of the following. (Solve by BODMAS rules)

a) $\frac{4}{5} + \frac{2}{3} \times \frac{1}{2}$ b) $\frac{2}{3} + \frac{4}{1} \times \frac{2}{3}$

c) $\frac{4}{1} + \frac{3}{2} \times \frac{3}{5}$ d) $\frac{5}{6} - \frac{1}{2} \times \frac{1}{4}$

7. Write the additive inverse of the following rational numbers.

a) -2 b) $\frac{3}{7}$ c) $\frac{(-8)}{3}$ d) $\frac{3}{5}$

8. Write the multiplicative inverse of the following rational numbers.

a) $\frac{2}{5}$ b) $\frac{(-3)}{7}$ c) $\frac{8}{5}$ d) $\frac{(-2)}{7}$

9. Fill in the blanks.

(Negative one (- 1), infinite, zero, one, positive, zero)

1) The additive inverse of a negative rational number is...........

2) On adding its additive inverse to a rational number, the result is ……………..

3) The result of multiplying a rational number by its multiplicative inverse is …………….

4) Between two rational numbers there or a …………… rational number.

5) On multiplying a rational number by zero, the result is…………..

6) The additive inverse of a number 1 is……………..

❖ We learned -

1) There are infinite number of rational numbers between any two rational numbers. **Properties of rational numbers**.

Operation	Closure	Associative	Identity	Commutative
Addition	Yes	Yes	Zero	Yes
Subtraction	Yes	No	Zero	No
Multiplication	Yes	Yes	1	Yes
Division	No	No	1	No

2) To find the sum and difference of rational numbers having the same denominator, we find the sum and difference between the fractions by keeping the same denominator in place of the denominator.

3) For the sum and difference of rational numbers with unlike denominators, we solve by taking the least common multiple and follow rules addition or subtraction of fraction.

4) To find the products of rational numbers, multiplying the numerator with the numerator and multiplying the denominator with the denominator and written in the following form.

$$\frac{1}{4} \times \frac{(-2)}{3} = \frac{1 \times (-2)}{4 \times 3} = \frac{(-2)}{12}$$

5) In the division of a rational number multiply the dividend by the reciprocal of the divisor.

For example: $\frac{2}{5} \div \frac{1}{4} = \frac{2}{5} \times$ (raciprocal of $\frac{1}{4}$)

$= \frac{2}{5} \times \frac{4}{1} = \frac{2\times4}{5\times1} = \frac{8}{5}$

6) When the sum of two numbers is zero, they are called **additive inverses** of each other.

The additive inverse of a rational number $\frac{a}{b}$ is $\frac{-a}{b}$ and the converse is also true.

Example: The additive inverse of $\frac{3}{8}$ is $\frac{-3}{8}$.

$= \frac{3}{8} - \frac{3}{8} = \frac{3-3}{8} = \frac{0}{8} = 0$

7) When the product of two numbers is 1, then they are called multiplicative inverses of each other. The multiplicative inverse of a rational number $\frac{a}{b}$ is $\frac{b}{a}$ and the converse is also true. where a, b $\neq 0$.

For example: The multiplicative inverse of $\frac{2}{5}$ is $\frac{5}{2}$.

$= \frac{2}{5} \times \frac{5}{2} = \frac{10}{10} = 1$

Chapter 2

Linear equations in one variable

Dear students! You have studied about simple equation in the previous class. Where we discussed about algebraic expressions and equations. You must be remembering the following -

Some examples of algebraic expressions are –

$$2x, \quad 5x, \quad -4, \quad 8y-31, \quad xyz+x, \quad x^2y$$

Some examples of equations are –

$$5x = 25, \; 2x-3 = 9, \; 3x-4 = 10, \; 6z-4 = 25$$

You will remember that '=' sign is always used in equations, which is not there in expressions. In this chapter, we will study about linear equations in one variable.

In relation to a linear equation in one variable, the following verse is found in beeja ganitam

यावत्तावत्कल्प्यमव्यक्तराशेर्मानं

तस्मिन् कुर्वतोद्दिष्टमेव।

तुल्यौ पक्षौ साधनीयौ प्रयत्नात्

त्यक्त्वा क्षिप्त्वा वाऽपि संगुण्य भक्त्वा ॥

(बीजगणितम्, अथैकवर्णसमीकरणम्, 1)

Meaning, such equations in which there is only one variable (a, b, …., x, y, z) and the highest degree of the variable is 1 are called equations in one variable.

We must keep certain points in mind before restart solving linear equations, as mentioned in the following verse taken from beeja ganitam

एकाव्यक्तं शोधयेत् अन्यपक्षात् रूपाण्यन्यस्य इतरस्माच्च पक्षात् ।

शेषाव्यक्तेन उद्धरेत् रूपशेषं व्यक्तं मानं जायते अव्यक्तराशेः ॥

अव्यक्तानां द्व्यादिकानामपीह ,यावत्तावत् द्व्यादिनिघ्नं हृतं वा ।

युक्तोनं वा कल्पयेद् आत्मबुद्ध्या मानं क्वापि व्यक्तमेवं विदित्वा ॥

(बीजगणितम्,एकवर्णसमीकरणम्, 2-3)

The meaning is clear from the lines of the above verse of beejagitam that there is no effect on an equation unless -

Add or subtract the same number to both sides of the equation.

Multiply or divide by the same non-zero number on both sides of the equation.

Ex: $3x - 2 = 5$

↓ ↓

variable equality sign

Here LHS = $3x - 2$ and RHS = 5.

In the previous class you also learned to solve the above simple equations. In which the variable amount was in one side.

Example: $2x+1=4$, $x+4=10$, $10 - x=3$

Let us learn how to solve equations by revising the concepts learnt in previous class.

Example : Solve the equation 2x+1=9.

Solution: **Step 1** – Subtracting 1 from both sides of the equation

$$2x + 1 - 1 \quad = \quad 9 - 1$$

$$2x \quad = \quad 8$$

Step 2 – Dividing both sides of the equation by 2

$$\frac{2x}{2} \quad = \quad \frac{8}{2}$$

$$x \quad = \quad 4$$

Hence, the Solution of the given equation is x=4.

We know that by either adding/ subtracting same number on both sides of the equation or multiplying/ dividing by same non- zero number on both the sides of the equation, the equality of the equation does not change. This is proved as shown below by solving the same equation in four different ways –

1) $2x + 1 = 9$

adding 2 to both sides

$$2x + 1 + 2 = 9 + 2$$

$$2x + 3 = 11$$

$$2x = 11 - 3$$

$$2x = 8$$

$$x = \frac{8}{2}$$

$$x = 4$$

2) $2x + 1 = 9$

subtracting 3 from both sides

$$2x + 1 - 3 = 9 - 3$$

$$2x + (-2) = 6$$

$$2x = 6 + 2$$

$$2x = 8$$

$$x = \frac{8}{2}$$

$$x = 4$$

3) $2x + 1 = 9$

Multiplying both sides by 2

$$2(2x + 1) = 2(9)$$

$$4x + 2 = 18$$

$$4x = 18 - 2$$

$$4x = 16$$

$$x = \frac{16}{4}$$

$$x = 4$$

4) $2x + 1 = 9$

Dividing both sides by 3

$$\frac{2x+1}{3} = \frac{9}{3}$$

$$\frac{2x+1}{3} = \frac{3}{1}$$

(cross Multiplying)

$$1(2x + 1) = 3 \times 3$$

$$2x + 1 = 9$$

$$2x = 9 - 1$$

$$2x = 8$$

$$x = \frac{8}{2}$$

$$x = 4$$

Hence, we take the help of these operations to solve the equations as and when required. Now we will learn to solve such simple equations in which there are variables on both the sides, using an example.

Solve the following equation –

$$8x + 1 = 4x + 13$$

This means solving the equation.

The above equation has to be written such that all the terms containing variable are on one side and all the constant terms are on the other side of the equation, to get in the form of $x =$ ……….

We can do this by transposing the terms. Not that while transposing the terms the sign of the terms changes from positive to negative and vice – versa.

Transposing Method —

Example: solve the equation 8x + 1 = 4x + 13 by transposing method

Solution: transposing the term $4x$ and 1

or $8x - 4x = 13 - 1$

or $4x = 12$

dividing both sides by 4

or $\frac{4x}{4} = \frac{12}{4}$

$x = 3$

Balancing Method:

Example: Find the Solution of the equation 7x+2 = 3x+5.

Solution: By balancing method:

Given: $7x + 2 = 3x + 5$

subtracting 2 from both sides

or $7x + 2 - 2 = 3x + 5 - 2$

or $7x = 3x + 3$

Subtracting 3x from both sides

or $7x - 3x = 3x - 3x + 3$

or $4x = 3$

dividing both sides by 4

$\frac{4x}{4} = \frac{3}{4}$

$x = \frac{3}{4}$

Example : Solve the equation $-\frac{x}{2} = \frac{x+1}{5}$

Solution: The least common multiple of the denominators 2 and 5 is 10.

on multiplying both sides by 10 or cross multiplying,

$$10 \times \frac{x}{2} = \frac{x+1}{5} \times 10$$
$$5x = 2\,(x+1)$$
$$5x = 2x + 2$$

Transposing $2x$ from RHS to LHS-

$$5x - 2x = 2$$

$$3x = 2$$

On dividing both the sides by 3 -

$$\frac{3x}{3} = \frac{2}{3}$$
$$x = \frac{2}{3}$$

Example : Solve the equation $\frac{x+1}{2} = \frac{x+2}{3}$

Solution: Given: $\frac{x+1}{2} = \frac{x+2}{3}$

on cross multiplying

$$3(x+1) = 2(x+2)$$
$$3x + 3 = 2x + 4$$

On transposing 2x, 3

$$3x - 2x = 4 - 3$$
$$x = 1$$

Do and learn

Solve the following equations –

1) $2x + 11 = 9$

2) $\frac{x}{2} = \frac{x+4}{3}$

Exercise 2.1

1. Select the correct option for the following multiple-choice questions.

(a) What will be the Solution of the equation 2x+3=5?

(I) 1 (II) 2 (III) 5 (IV) 4

(b) In the equation y+8=6, the value of y will be -

(I) 2 (II) -2 (III) 14 (IV) 48

(c) In 3x = 18, the value of x will be -

(I) 6 (II) 18 (III) 54 (IV) Not possible.

(d) In $\frac{x}{3} = 7$, the value of x will be -

(I) 21 (II) 3 (III) 7 (IV) 7/3

(e) On solving 5(x+4) = 9(4x-5), the value of x will be -

(I) 65 (II) 31 (III) $\frac{31}{65}$ (IV) $\frac{65}{31}$

2. Solve the following equations.

1) $7x + 3 = 3x + 5$ 2) $3(x + 1) = x + 5$

3) $3x + 4 = x + 9$ 4) $\frac{x+1}{2} = \frac{x}{4}$

5) $\frac{x}{5} = \frac{x+1}{9}$ 6) $\frac{x+1}{2} = \frac{x+2}{5}$

7) $\frac{x+4}{2} = \frac{x+3}{3}$ 8) $\frac{x+8}{5} = \frac{x+3}{4}$

Applications of the equation –

In daily lives, we use equation to solve many problems, riddles, etc, To find out the value of an unknown quantity. In the previous class, you have learned to find the value of an unknown quantity in an equation. In general, the unknown quantity is represented by x.

Let us revise the method of forming equations from the statements.

Complete the following table.

Let the unknown quantity (unknown number) be considered as x–

Sl.	problem statement	mathematical statement
1	3 more than number	$x + 3$
2	4 less than a number	$x - 4$
3	3 times the number	$3x$
4	5 more than 3 times the number	$3x + 5$
5	7 less than 4 times a number	…………….
6	7 more than the number	……………
7	5 less than the number	……………
8	7 less than 7 times a number	……………

The following steps are followed before solving mathematical problems based on sentences.

1) Read the given question carefully and find out the known and unknown quantity .
2) Now write the unknown quantity as x (variable quantity).
3) Write the problem statements in the form of mathematical equation (algebraic terms and expressions).
4) Equate the expressions formed, as per the conditions given in the problem / quantities.
5) Solve the equation, to get the value of the variable.
6) Check the answers by substituting the value of the variable in the equation.

Example: The sum of two numbers is 40. Among them the second number is 10 more than the first number. Find the numbers?

It is like a puzzle, in which you do not know both the numbers, and the conditions are given to find them.

1) The first number is 10 more than the second number.

2) The sum of both the numbers is 40.

Solution: The above conditions help us to get the Solution of the question.

Let us consider the first number as x

and second number = 10 more than first number or

or = first number + 10

= (x+10)

According to the question,

The sum of both the numbers is 40.

First number + second number = 40

Then, $x + (x + 10) = 40$

$2x + 10 = 40$ (Transpose 10)

$2x = 40 - 10$

$2x = 30$

$x = \frac{30}{2}$

$x = 15$

Hence, first number = 15,

second number = first number + 10

15 + 10 = 25

Thus, the numbers obtained are 15 and 25. Which satisfiy both the conditions.

15 + 25 = 40, first number + second number = 40

Let us discuss some more examples using this method.

Example: Multiplying a number by 4 gives 20. Find that number.

Solution: Let the unknown number be x. Number multiplied by 4 i.e., 4x according to the question,

$$4x = 20$$

$$\frac{4x}{4} = \frac{20}{4} \quad \text{(Dividing both sides by 4)}$$

$$x = 5$$

Hence, unknown number is 5.

Example: Vishnu's age is 12 years less than Jagdish's age. If the sum of the ages of both is 50, find the ages of Vishnu and Jagdish ?

Solution: Let the age of Jagdish be x years.

Then, Vishnu's age = x-12 years.

according to the question,

The sum of the ages of both is 50.

Then, $x + (x - 12) = 50$

$$x + x - 12 = 50$$

$$2x - 12 = 50 \quad \text{(Transpose –12)}$$

$$2x = 50 + 12$$

$$2x = 62 \quad \text{(Dividing both sides by 2)}$$

$$x = 31$$

That means Jagdish's age = x years i.e. 31 years.

And Vishnu's age = $x - 12 = 31 - 12 = 19$ years.

Example: If the ratio of income of Pawan and Jitendra is 5 : 3. If the difference of their income is 2000, then find the income of both.

Solution: The ratio of income of Pawan and Jitendra is 5 : 3.

Consider Pawan's income as 5x and Jitendra's income as 3x.

according to the question,

$$5x - 3x = 2000$$

$$2x = 2000$$

$$\frac{2x}{2} = \frac{2000}{2} \qquad \text{(dividing both sides by 2)}$$

$$x = 1000$$

Hence, Pawan's income = 5x = 5×1000 = Rs.5000.

And Jitendra's income = 3x = 3×1000 = Rs.3000.

Exercise 2.2

1. The sum of two numbers is 60. If the first number is 20 more than the second number then find both the numbers ?
2. If 10 more than twice the age of Ram is 40 years, find is age.
3. If a number is multiplied by 3 to get 21, find the number?
4. If the ratio of the lengths of the upvastra and dhoti is 5: 4, and sum of their lengths is 36 meters, find the length of upvastra and dhoti.
5. The difference between the heights of Ashish and Rahul is 40 cm. If the ratio of height of both is 5 : 3, what are their heights in cm.
6. On increasing 4 times a number by 10, the number obtained is 50, find that number?
7. The sum of the ages of Utsav and Aaradhya is 60years. If the age of Utsav is 10 more than that of Aaradhya, find the ages of Utsav and Aaradhya ?
8. The sum of three consecutive integers is 51, and then find the integer.
9. On adding 8 to the double of a number, 32 is obtained, find the number.

> **We learned -**

1) If the degree of variable in an equation is one, then it is called a linear equation.

For example : $3x+2 = 8$

variables

the degree of the variable x is 1.

2) If a linear equation has only one variable, then it is called a **linear equation in one variable.**

3) The equation has two sides. In this, numbers and variables can be transposed from one side to the other. On transposing the signs of the terms change.

$8x + 1 = 5x + 2$

On transposing 5x and 1-

$8x - 5x = 2 - 1$

$3x = 1$

4) In applications of linear equations, we find the value of an unknown quantity by making mathematical equations.

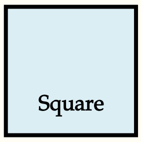
5) Many types of problems based on numbers, ages, dimensions and coins and notes used as currency can be solved using linear equations.

Chapter 3

Square and Square Root

My dear students! You have studied about the shape of a square in the previous class, a square is a quadrilateral in which all the sides are equal. In the previous class, in the chapter on perimeter and area, you have learned to find the area of a square.

Area of square = side × side



Where side means the length of one side.

If one side of the square is 8 cm. what will be the area of the square

Area of square = side × side

$= 8 \times 8 = 64\ cm^2$

Complete the following table.

Side of the Square (in cm)	Area of the Square (in cm^2
1	$1 \times 1 = 1 = 1^2$
2	$2 \times 2 = 4 = 2^2$
3	$3 \times 3 = 9 = 3^2$
....	....
....	....
a	$a \times a = a^2$

Can you complete it?

In the above table the numbers 4, 9, 16, 25 are .... And similarly what find the squares of next five numbers ?

Since 4, 25 can be expressed as $4 = 2 \times 2 = 2^2$, $25 = 5 \times 5 = 5^2$... etc. The product of a number by itself can be expressed as $a \times a = a^2$.

Squares -

The method of finding the square of a number is explained in the first line of following verse, which is taken from Lilavati maths.

समद्विघात: कृति: उच्यते इति प्रथम: प्रकार: ।

(लीलावती, वर्गे करणसूत्रं वृत्तद्वयम् : 9 अ)

Meaning, when a number is multiplied by itself, then the number obtained from it is called the square of that number. In other words, the product of two equal numbers is called a **square**.

Example: Find the square of 3 ?

Solution: Then, $3 \times 3 = 9$

Or it can be expressed in the following form.

$3^2 = 9$

We read this as the square of 3.

In the above, the natural number 9 is expressed as 3^2 (Square of 3). where 3 is also a natural number. Then 9 is a square number.

$$3^2 = 3 \times 3 = 9$$

Here the square of 3 is 9.

In general, if $\mathbf{A = B^2 (B \times B)}$ the square number of B is A. Can you tell the Square of 17?

Considering the numbers and their squares in the following table, Complete the following table.

Number	Square of the Number
1	$1 \times 1 = 1$
2	$2 \times 2 = 4$
3	$3 \times 3 = 9$
4	$4 \times 4 = 16$
5	$5 \times 5 = 25$
6	…………
7	……….
8	…………
9	……….
10	……….

In the above table you will find that only 10 numbers between 1 to 100 are square numbers, the remaining numbers are not square numbers. The numbers 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100 are square numbers and they are also called **perfect square** numbers.

Do and learn

Find the perfect square numbers between the given numbers ?

1) 10 to 20 ………. 2) 30 to 40 ………..

Example: Find the square of 15.

Solution: Square of 15

$15^2 = 15 \times 15 = 225$

Hence, the square of 15 is 225.

Example: What is the unit digit of the square of 12?

Solution: Square of 12

$12^2 = 12 \times 12 = 144$

Hence, the unit digit of the square of 12 is 4.

Example question given in Lilavati ganit:

सखे नवानां च चतुर्दशानां ब्रूहि त्रिहीनस्य शतत्रयस्य।

पञ्चोत्तरस्याप्ययुतस्य वर्गं जानासि चेद्वर्गविधानमार्गम्॥

Hey friend, if you know the method of doing square, find the squares of 9, 14, 297 and 10005 ?

Solution: (Students will solve by themselves.)

Numbers between square numbers –

How many numbers are not square numbers between two consecutive square numbers?

$1^2 = 1$

$2^2 = \underline{2, 3,}\ 4$

$3^2 = \underline{5, 6, 7, 8}, 9$

$4^2 = \underline{10, 11, 12, 13, 14, 15}, 16$

$5^2 = \underline{17, 18, 19, 20, 21, 22, 23, 24}, 25$

In the above,there are four numbers 5, 6, 7, 8 between 2^2 and 3^2 are not square numbers. The six numbers between $3^2$and 4^2, and ten numbers between 5^2 and 6^2 are not square numbers.

We can also find those numbers which are not square numbers between ant two given perfect square by an interesting method.

- Count of numbers between 1^2 and 2^2 is 2 (2, 3)

  Difference $2^2 - 1^2 = 4 - 1 = 3$

  Hence, the count of numbers between 1^2 and 2^2 is one less than the difference between them.

- Count of numbers between 2^2 and 3^2 is 4 (5, 6, 7, and 8)

  Difference $3^2 - 2^2 = 9 – 4 = 5$

  Hence, the count of numbers between 3^2 and 2^2 is one less than the difference between them.

- Count of numbers between 3^2 and 4^2 is 6 (10, 11, 12, 13, 14, 15)

  Difference $4^2 - 3^2 = 16 – 9 = 7$

  Hence, the count of numbers between 3^2 and 4^2 is one less than the difference between them.

- Count of numbers between 4^2 and 5^2 is 8 (17, 18, 19, 20, 21, 22, 23, 24). Difference $5^2 - 4^2 = 25 - 16 = 9$

  Hence, the count of numbers between 5^2 and 4^2 is one less than the difference between them.

If we look carefully, we will find that there is one less than the difference between the two square numbers. is not a square number.

So we can say in general that let's take natural numbers n and n+1.

When, $(n+1)^2 - n^2 = n^2 + 1 + 2n – n^2 = 2n + 1$

We find 2n numbers between n^2 and $(n+1)^2$. Which is one less than the difference between the square numbers.

❖ Do and learn

How many numbers are there between 5^2 and 6^2 which is not a square number?

Properties of square numbers –

Table of Square :-

The squares of the numbers 1 to 20 are tabulated below.

Number	Square	Number	Square
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

We observe that –

1) The unit digit of the square is 0, 1, 4, 5, 6 or 9. Therefore, if 2, 3, 7, 8 appear in unit digit of a number, then that number is not a perfect square. We can find whether a given number possibly a perfect square is not by looking at its unit digit.

2) In the square of all even numbers, even numbers are obtained, and in the square of odd numbers, only odd numbers are obtained.

3) If 1 or 9 is at unit's place of a number, then 1 comes at the end of its square number **Example**: 1, 9, 11, 19

Number	Square
1	1
9	81
11	121
19	361

4) If 4 or 6 is at unit's place of a number, then 6 comes at the end of its square number.

Number	Square
4	16
6	36
14	196
16	256

❖ **Do and learn**

Mark (✓) which of the following numbers are perfect squares.

A) 16 B) 17 C) 35 D) 49

Exercise 3.1

1. What are the unit digits of the squares of the following numbers?

   a) 4 b) 13 c) 18 d) 11

2. Find the square of the number given below?

   A) 16 B) 9 C) 35 D) 19

3. Mark the perfect square number in the following numbers by identifying the unit digit.

   a) 6400 b) 2309 c) 342 d) 434

4. The square of which of the following numbers will be an odd number?

   a) 431 b) 2826 c) 7779 d) 82004

Square Root – The square root of a number is that number which when multiplied by itself gives the given number.

Another method of finding square root in Lilavati ganit is explained in the following verse. To increase your knowledge, you can study the following verses.

त्यक्त्वाऽन्त्याद्विषमात्कृतिं द्विगुणयेन्मूलं समे तद्धृते
त्यक्त्वा लब्धकृतिं तदाद्यविषमाल्लब्धं द्विनिघ्नं न्यसेत्।
पङ्क्त्यां पङ्क्तिहृते समेऽन्यविषमात् त्यक्त्वाऽऽप्तवर्गं फलं
पङ्क्त्यां तद्विगुणं न्यसेदिति मुहुः पंक्तेर्दलं स्यात् पदम् ॥

(लीलावती, वर्गमूले करणसूत्रं वृत्तम् : 10)

In the above verse, the method of finding the square root of a number has been explained.

Observe the squares of the following numbers.

$(4)^2 = 4 \times 4 = 16$

$(5)^2 = 5 \times 5 = 25$

$(3)^2 = 3 \times 3 = 9$

In the above example we see that

$(3)^2 = 3 \times 3 = 9$

Here the square of 3 is 9. Conversely, we can say that the square root of 9 is 3.

$(3)^2 = 3 \times 3 = 9$

Here the square root of 9 is 3.

That is, the **square root is the inverse operation of the square**. the square root sign "$\sqrt{\ }$") by showing Are.

Example : square root of 9 or $\sqrt{9} = 3$

❖ **Do and learn**

Looking at the square table, what will be the square root of the following ?

a) 49 b) 121 c) 256

Squareroot by Prime Factorisation:

Factors of some numbers and their squares are given in the following table.

Number	prime factorisation of number square	square number	prime factorisation of number
4	2×2	16	$2 \times 2 \times 2 \times 2$
6	2×3	36	$2 \times 2 \times 3 \times 3$
10	2×5	100	$2 \times 2 \times 5 \times 5$

You see that the prime factors of a number occur twice in the prime factors of the square of that number.

Example: Prime factors of 10 = 2×5

square of 10

↓ square of 10

$100 = 2 \times 2 \times 5 \times 5$

2	10
5	5
	1

2	100
2	50
5	25
5	5
	1

Let us learn to find the squareroot of a number by the prime factorisation method.

Example: Find the squareroot of 64.

Solution: We know that the prime factorisation of 64

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

2	64
2	32
2	16
2	8
2	4
2	2
	1

on pairing of prime factors

$64 = (2^2 \times 2^2 \times 2^2)$ or $(2 \times 2 \times 2)^2$

or $\sqrt[2]{64} = 2 \times 2 \times 2$

$\sqrt[2]{64} = 8$

So the square root of 64 is 8.

Example: Is 125 a perfect square number ?

Solution: Prime factors of 125

$125 = \underbrace{5 \times 5} \times 5$

All the prime factors are not in pairs.

Hence 125 is not a perfect square number. On pairing the prime factors of 125, we observe that 5 is the only number that does not have a pair, so we need to multiply by 5 to complete the pair.

Hence, $125 \times 5 = 625$

$625 = \underbrace{5 \times 5} \times \underbrace{5 \times 5}$

You can also make 125 a perfect square by dividing it by anpaired number to make it a perfect square.

$125 \div 5 = 25$

Hence, $25 = 5 \times 5$ is a perfect square.

Or $25 = 5^2$

$\sqrt[2]{25} = 5$

Example: Is 40 a perfect square number ?

Solution: $40 = (2 \times 2) \times 2 \times 5$

2	40
2	20
2	10
5	5

Only one pair is formed in the prime factorization. But 2 and 5 do not have pairs in the prime factorization, so 40 is not a square number.

References about square and square roots in Vedic literature:

Dear students ! The order of square and square root is mentioned in the 24th mantra of the 18th chapter of Yajurveda- **'एका च मे तिस्रश्च'**, which also makes the order of square and square root. The sequence of Yajurveda mantra is as follows -

एका च मे तिस्रश्च मे तिस्रश्च मे पञ्च च मे पञ्च च मे सप्त च मे सप्त च मे नव च मे नव च म ऽ एकादश च म ऽ एकादश च मे त्रयोदश च मे त्रयोदश च मे पञ्चदश च मे पञ्चदश च मे सप्तदश च मे सप्तदश च मे नववश च मे नवदश च म एकवि ँ शतिश्च म ऽ एकवि ँ शतिश्च मे त्रयोवि ँ शतिश्च मे त्रयोवि ँ शतिश्च मे पञ्च वि ँ शतिश्च मे पञ्चवि ँ शतिश्च मे सप्त वि ँ शतिश्च मे सप्तवि ँ शतिश्च मे नववि ँ शतिश्च मे नववि ँ शतिश्च म ऽ एक त्रि ँ शच्च म ऽ एकत्रि ँ शच्च मे त्रयस्त्रि ँ शच्च मे यज्ञेन कल्पन्ताम् । **(18/24 यजुर्वेद)**

Meaning, in एका च मे + तिस्रश्च: $1 + 3 = 4$. Square of 2 is 4 and square root of 4 is 2. The above mantra suggests the method of forming squares and the corresponding square roots.

Example: Square of 2 is the sum of first tow odd numbers namely $(1+3 = 4)$. The smallest square number is four and its square root is two. To find the next square number, add the next odd number (5) to the previous result (4), i.e., $5+4 = 9$ is the square of 3 is obtained by adding the first three odd numbers $(1 + 3 + 5)$. This sequence is followed to find the successive squares.

number of square roots	Numbers According to the mantra	Explanation	Squares
2	एका च मे+ तिस्रश्च मे	4 = 3 + 1	4
3	previous sum + पञ्च च मे	9 = 5 + 4	9
4	previous sum + सप्त च मे	16 = 7 + 9	16
5	previous sum+ नव च मे	25 = 9 + 16	25
6	previous sum + एकादश च मे	36 = 11 + 25	36
7	previous sum + त्रयोदश च मे	49 = 13 + 36	49
8	previous sum + पञ्चदश च मे	64 =15 + 49	64
9	previous sum + सप्तदश च मे	81 = 17+ 64	81
10	previous sum + नवदश च मे	100 = 19 +81	100
11	previous sum + एक विंशतिश्च च मे	121= 21 +100	121
12	previous sum + त्रयोविंशतिश्च च मे	144 = 23+121	144
13	previous sum + पञ्चविंशतिश्च च मे	169= 25 +144	169
14	previous sum + सप्तविंशतिश्च च मे	196 = 27+169	196
15	previous sum + नवविंशतिश्च च मे	225 = 29+169	225
16	previous sum+ एकत्रिंशच्च मे	256 =31+ 225	256
17	previous sum + त्रयस्त्रिंशच्च च मे यज्ञेन कल्पन्ताम्	289 = 33+256	289
18	....	324 = 35+289	324
19	....	361 = 37+324	361

20	....	400 = 39+361	400

By adding the next odd number to the previous square, the table of square and square root is formed. Through this sequence, the square of all the numbers can be found by the mantra.

For odd numbers, according to the mantra the squares are

तिस्रश्च मे	×	तिस्रश्च में	=	3	×	3	=	9
पञ्च च मे	×	पञ्च च मे	=	5	×	5	=	25
सप्त च मे	×	सप्त च मे	=	7	×	7	=	49
नव च मे	×	नव च मे	=	9	×	9	=	81
एकादश च मे	×	एकादश च मे	=	11	×	11	=	121
त्रयोदश च मे	×	त्रयोदश च मे	=	13	×	13	=	169
पञ्चदश च मे	×	पञ्चदश च मे	=	15	×	15	=	225
सप्तदश च मे	×	सप्तदश च मे	=	17	×	17	=	289

....

According to the above mentioned Yajurveda mantra, multiplying the odd numbers by itself from 1 to 33 will give their squares.

Exercise 3.2

1. Select the correct option for the following multiple-choice questions.

(a) The square root of 8100 will be -

(I) 900 (II) 90 (III) 9 (IV) Not possible

(b) What is the value of $\sqrt{90000}$ –

(I) 30 (II) 300 (III) 90 (IV) 81

(c) The square root of 900 will be –

(I) 3 (II) 30 (III) 300 (IV) None of these

(d) The square root of $\frac{256}{441}$ will be-

(I) $\frac{21}{16}$ (II) $\frac{16}{21}$ (III) $\frac{13}{9}$ (IV) None of these

(e) The square root of 0.25 will be -

(I) 5 (II) 5.0 (III) 0.5 (IV) 0.25

(f) The square root of 196 will be-

(I) 8 (II) 12 (III) 14 (IV) 18

2. How many natural numbers are there between 53^2 and 54^2 ?

(i) 106 (II) 108 (III) 107 (IV) 109

3. What can be the unit's digit in the square root of the following numbers ?

a) 256 b) 10000 c) 625 d) 289

4. Which of the following numbers cannot be a perfect square number? (**Hint:** Identify the ones digit)

a) 441 b) 257 c) 408 d) 102

5. By what number should the following number be multiplied to

make it a perfect square?

a) 90 b) 60 c) 95

6. Find the square root of the following numbers.

a) 729 b) 576 c) 900

We learned -

1) If a number is multiplied by itself, then the number obtained is called its square.

Like : square of 5

or $5^2 = 5 \times 5 = 25$

Here the square of 5 is 25.

So if a natural number can be expressed in the form $a = b^2$. where B is also a natural number then A is a square number.

2) All those square numbers which end with 0, 1, 4, 5, 6, 9 at the unit digit.

3) The square root of a number is that number which when multiplied by itself gives the given number. The square root is called the inverse operation of the square.

$$5^2 = 5 \times 5 = 25$$

Here the square root of 25 is 5.

4) There are 2n numbers between any two consecutive square numbers i.e. between n^2 and $(n+1)^2$ which is 1 less than the difference between two square numbers $(n)^2$ and $(n+1)^2$ which are not square numbers.

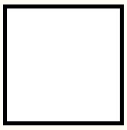
$$(n+1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1$$

1 less than the difference = 2n + 1 - 1 = 2n

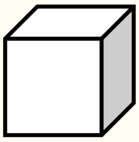
Chapter 4

Cube and Cube Root

Dear Students! We used the term cube in the chapter on three dimensional solid figures. We know that a square is a two-dimensional shape whose solid or three-dimensional shape is called a cube.



square



cube

Do you know anything more about the solid figures of the cube ?

Ashish – Solid shapes have length, width and height.

Babita – All the sides of a cube are equal or the length, breadth and height of a cube are equal.

Guruji - Very good! Correct.

You are right that cube is a solid figure. Whose all sides are equal. In the previous chapter, we have discussed that - When a number is multiplied by itself, the number obtained is called the square of that number.

Cube -

The method of finding the cube of a number has been explained in the first line of the following verse take and from Lilavati maths.

समत्रिघातश्च घन: प्रदिष्ट: इति प्रथम: प्रकार: ।

(लीलावती, घने करणसूत्रं वृत्तत्रयम् 11)

Meaning, when a number is multiplied by itself for three times, the number obtained is called the **cube** of that number. In other words, the repeated product of a number for three times is called its **cube**.

Let us learn to find the cube using examples.

Example: Find the cube of the number 2.

Solution: Cube of 2 or 2^3

$$2^3 = 2 \times 2 \times 2 = 8$$

Here the cube of 2 is 8.

Table 1 – Complete the following table.

Number	Cube of the number
1	$1^3 = 1 \times 1 \times 1 = 1$
2	$2^3 = 2 \times 2 \times 2 = 8$
3	$3^3 = 3 \times 3 \times 3 =$........
4	$4^3 = 4 \times 4 \times 4 =$........
5	$5^3 = 5 \times 5 \times 5 = 125$
6	$6^3 = 6 \times 6 \times 6 =$........
7	$7^3 = 7 \times 7 \times 7 = 343$
8	$8^3 = 8 \times 8 \times 8 = 512$
9	$9^3 = 9 \times 9 \times 9 =$........
10	$10^3 = 10 \times 10 \times 10 = 1000$

we know $3^2 = 9$ where $9 = 3 \times 3$

Thus, $3^3 = 27$ where $27 = 3 \times 3 \times 3$

If we observe in the table, we will find that the cube of an even number is always an even number and the cube of an odd number is always an odd number.

Table 2 – In this table now the cubes of numbers from 11 to 20 are given below.

Number	Cube number
11	$11^3 = 1331$
12	$12^3 = 1728$
13	$13^3 = 2197$
14	$14^3 = 2744$
15	$15^3 = 3375$
16	$16^3 = 4096$
17	$17^3 = 4913$
18	$18^3 = 5832$
19	$19^3 = 6859$
20	$20^3 = 8000$

Observe in the above table carefully, that if the unit digit of the number is either 0, 1, 4, 5, 6 or 9, the unit digit of their cubes is the same respectively.

❖ Do and learn

What is the unit digit of the cube of the numbers given below ?

a) 35 b) 44

Example: Is 216 a perfect cube ? Think about it.

Solution: Prime factorisation of 216

$$216 = \underbrace{2 \times 2 \times 2} \times \underbrace{3 \times 3 \times 3}$$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Observe that in the prime factorisation of a perfect cube, every prime factor occurs three times in their product form.

Hence 216 is a perfect cube number.

Example: Is 432 a perfect cube ?

Solution: On prime factorisation

$$432 = 2 \times \underbrace{2 \times 2 \times 2} \times \underbrace{3 \times 3 \times 3}$$

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

In this, after grouping three identical numbers, 2 remains alone.

Hence 432 is not a perfect cube.

Exercise 4.1

1. Find the cube of the following numbers?

   A) 12 B) 17 C) 9 D) 13

2. Which of the following number is not a perfect cube?

   a) 100 b) 243 c) 423 d) 512

Cube Root

The cube root of a number is that number which when multiplied three times by itself gives the given number.

Another method of finding cube root in Lilavati maths is explained in the following verse. To increase your knowledge, you can study the following verses.

आद्यं घनस्थानमथाघने द्वे पुनस्तथाऽन्त्याद् घनतो विशोध्य।
घनै पृथक्स्थं पदमस्य कृत्या त्रिघ्न्या तदाद्यं विभजयेत् फलं तु॥
पङ्क्त्यां न्यसेत तत्कृतिमन्त्यनिघ्नीं त्रिघ्नीं त्यजेत् तत्प्रथमात् फलस्य।
घनं तदाद्याद् घनमूलेवं पङ्क्तिर्भवेदेवमतः पुनश्च॥

(लीलावती, घनमूल करणसूत्रं वृत्तम् : 15)

In the above verse, the method of finding the cube root of a number has been explained.

As we know that square and square root are inverse of each other, cube and cube root are also inverse operation of each other.

we know -

$$2^3 = 2 \times 2 \times 2 = 8$$

Hence, we say that the cube root of 8 is 2.

It is written as $\sqrt[3]{8}=2$. The symbol for the cube root is " $\sqrt[3]{}$ ".

Observe the following table.

Cube of a number	Cube root	cube of a number	Cube root
$1^3 = 1$	$\sqrt[3]{1} = 1$	$6^3 = 216$	$\sqrt[3]{216} = 6$
$2^3 = 8$	$\sqrt[3]{8} = 2$	$7^3 = 343$	$\sqrt[3]{343} = 7$
$3^3 = 27$	$\sqrt[3]{27} = 3$	$8^3 = 512$	$\sqrt[3]{512} = 8$
$4^3 = 64$	$\sqrt[3]{64} = 4$	$9^3 = 729$	$\sqrt[3]{729} = 9$
$5^3 = 125$	$\sqrt[3]{125} = 5$	$10^3 = 1000$	$\sqrt[3]{1000} = 10$

To find the cube root of a number by the prime factorisation method –

Consider the number 3375, find its cube root by prime factorisation.

$$3375 = \underbrace{3 \times 3 \times 3}\times\underbrace{5 \times 5 \times 5}$$

$$3375 = 3^3 \times 5^3$$

$$3375 = (3 \times 5)^3$$

$$3375 = 15^3$$

Hence, $\sqrt[3]{3375} = 15$.

5	3375
5	675
5	135
3	27
3	9
3	3
	1

Finding the cube root of a number using the Vilokanam Sutra:

We know the factorisation method to find the cube root. Now, we will learn to find the cube root of a perfect cube number having 6 digits by Vilokanam Upsutra.

Memorize the table.

Number	1	2	3	4	5	6	7	8	9	10
cube of number	1	8	27	64	125	216	343	512	729	1000

unit digit of a number	unit digit of cube root
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9
0	0

It is clear that the unit digit of the cube root of the number 0, 1, 4, 5, 6, 9 are also 0, 1, 4, 5, 6, 9. When the unit digit of a number is 2, the unit digit of its cube is 8 and vice-versa. Similarly, when the unit digit of the number is 3, the unit digit of cube is 7 and vice-versa.

Example: If the given number 29791 is a perfect cube, then find its cube root.

Solution: $\sqrt[3]{29791} = 31$

procedure

1. If the number of digits in the given number is less than 6, write as many zeros as required before the first digit on the left to make it a six- digit number.

$$29791 = 029791$$

2. Groups of three digits are formed from the left side of the number.

029	791
Left side	Right side

The number of digits in the cube root is as many groups as formed.

3. Determining the unit digit of cube root:

Find the unit digit of the right group. Using the relationship between the unit digits of cube and cube roots, find the unit digit of the cube root of the given number. The unit digit of the right group is 1,hence, the unit digit of the cube root of the given number is 1.

4. Determining the tens digit of the cube root:

Find the number which is equal to or a perfect cube which is less than and nearest to the left group of numbers. Find the cube root of the cube found.

The perfect cubes 0, 1, 8 and 27 are less than 29. Out of which 27 is the nearest to 29. Hence, the tens digit is 3.

Thus, $\sqrt[3]{29791} = 31$.

Example find the cube root of the given number 941192 by Vilokanam.

Solution:

1. Groups of three digits are formed from the left side of the number.

941 192

Left side right side

The number of digits in the cube root is as many groups as formed.

2. Determining the unit digit of cube root:

Find the unit digit of the right group. Using the relationship between the unit digits of cube and cube roots, find the unit digit of the cube root of the given number.

The unit digit of the right group is 2,hence, the unit digit of the cube root of the given number is 8.

4. Determining the tens digit of the cube root:

Find the number which is equal to or a perfect cube which is less than and nearest to the right group of numbers. Find the cube root of the cube found.

The left group will determine the tens digit of the cube root from 941.

$$9^3 = 729$$
$$10^3 = 1000$$

10^3 is going to be greater than 941, so less than 10, that is, 9 will be ten times the cube root, that is, $\sqrt[3]{941192} = 98$

Do and learn

1) Find the cube root of 729.

Exercise 4.2

1. Find the cube root of the following numbers by factorisation method?

   a) $\sqrt[3]{74088}$ (b) $\sqrt[3]{1728}$

   c) Find the cube root of 8000 ?

   d) Find the cube root of 512 ?

2. Find the cube root of 64 and its unit digit.
3. Find the cube root of 125 and its unit digit.
4. Find the cube root from the Vilokanam formula -

(1)	3375	(2)	1728	(3)	1331	(4)	64000
(5)	970299	(6)	68921	(7)	287496	(8)	2197

- **We learned -**

1) A number multiplied by itself thrice is called a **cube** number.

   Example: 1, 8, 27

   $3^3 = 3\times3\times3 = 27$

   Here 3 cubed is 27.

2) If each of the prime factors occurs thrice in the prime factorisation of a number, the number is called a **perfect cube.**

3) Cube root of a number is that number which when multiplied three times with each other gives the given number. The cube root is the reverse process of finding the cube.

   Example: $3^3 = 3\times3\times3 = 27$

   Here the cube root of 27 is 3.

4) The symbol " $\sqrt[3]{}$ " To is used to express cube root. **Example**: $\sqrt[3]{27}=3$.

Chapter 5

Powers and Exponents

Abhishek and Ankit, two students of Ved Bhushan third year, are questioning each other. Abhishek !

What was the population of India in the 2011 census?

Ankit! 121 crores (approximately)

Abhishek asks again ?

Abhishek : What is the distance between Sun and Earth ?

Ankit: About 150 million kms.

Abhishek: Now write the answers in the form of numbers ?

Ankit : It is difficult to write this number.

Can these numbers be easily written or read and understood? We can easily write or read such large numbers with the help of powers and exponents.

In this chapter, we will study about the base (power) and exponent of integers and rational numbers.

Do you know how to write numbers in square and cube forms? You must remember that-

Example: Write the cube of 12.

Solution: 12^3

$12^3 = 12 \times 12 \times 12 = 1728$

cube of 12

Consider the following numbers.

1) $2 \times 2 \times 2 = 8$

2) $2 \times 2 \times 2 \times 2 \times 2 = 32$

3) $4 \times 4 \times 4 = 64$

4) $5 \times 5 \times 5 = 125$

5) $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

You can also write them like this.

1) $2 \times 2 \times 2 = 2^3$

2) $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

3) $4 \times 4 \times 4 = 4^3$

4) $5 \times 5 \times 5 = 5^3$

5) $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

Similarly,

Hundred (100) = $10 \times 10 = 10^2$

One thousand (1000) = $10 \times 10 \times 10 = 10^3$

One lakh (100000) = $10\times10\times10\times10\times10 = 10^5$

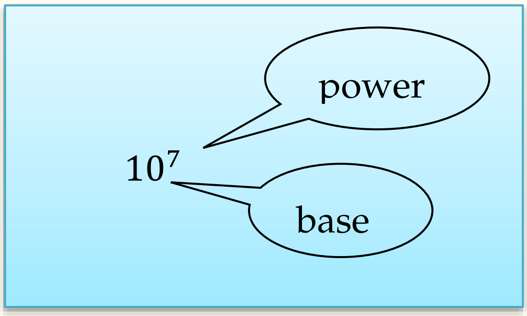
Vishal-

Wow !. This means 1 crore can be written as 10^7

It is very simple.

In 2^3 the base is 2 and the exponent is 3. In 5^3 the base is 5 and the exponent is 3. We read 5^3 as 5 raised to the power of 3.

ie.



We read 10^7 as 10 raised to the power of 7.

Example: Write 81 in exponent form.

Solution: $81 = 3 \times 3 \times 3 \times 3$

Hence, $81 = 3^4$

In 3^4 3 is the base and 4 is the exponent. We read 3^4 as 3 raised to the power of 4.

Example: Express the following as an exponent.

a) $2 \times 2 \times 2 \times 5 \times 5$

b) $3 \times 3 \times 7 \times 7 \times 7 \times 7$

Solution: a) $2 \times 2 \times 2 \times 5 \times 5$

$= 2^3 \times 5^2$

Solution: b) $3 \times 3 \times 7 \times 7 \times 7 \times 7$

$= 3^2 \times 7^4$

Example: Which number is bigger in 2^3 and 3^2 and why ?

Solution: $2^3 = 2 \times 2 \times 2 = 8$

$3^2 = 3 \times 3 = 9$

We know $8 < 9$

Hence, $2^3 < 3^2$

Means 3^2 is greater than 2^3.

Example: Simplify.

a) 2×5^2 b) $2^2 \times 5^2$

Solution: a) 2×5^2

$= 2 \times 5 \times 5$

$= 50$

Solution: b) $2^2 \times 5^2$

$= 2 \times 2 \times 5 \times 5$

$= 100$

Exponent of negative integer -

a) $(-1)^3 = (-1) \times (-1) \times (-1) = -1$

b) $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$

c) $(-3)^3 = (-3) \times (-3) \times (-3) = -27$

d) $\left(\frac{-4}{5}\right)^2 = \left(\frac{-4}{5}\right) \times \left(\frac{-4}{5}\right) = \frac{16}{25}$

e) $\left(\frac{-2}{3}\right)^3 = \frac{-2}{3} \times \frac{-2}{3} \times \frac{-2}{3} = \frac{-(2\times2\times2)}{3\times3\times3} = \frac{-8}{27}$

Observe that for the negative base, if the exponent is an even number the value is positive and if the exponent is an odd number the value is negative.

Exercise 5.1

1. Write the following in exponential form.

   1) $3 \times 3 \times 3 \times 3$

   2) $4 \times 4 \times 4 \times 2 \times 2$

   3) $a \times a \times a \times a$

   4) $8 \times 8 \times t \times t \times t$

2. Find the bigger number in the following pairs.

   1) 2^5 or 5^2 2) 3^5 or 5^3

3. Simplify the following.

   1) 2×3^2 2) $3^2 \times 2^2$

   3) $5^2 \times 2$ 4) $7^2 \times 2$

   5) 0×2^3 6) $5^2 \times 3^2$

4. Find the value.

   1) $(-1)^5$ 2) $(-2)^3$

   3) $(-4)^2$ 4) $(-3)^6$

Laws of Exponents –

Rule 1: Multiplication of exponential numbers with the same base

first principles,

$$अ^{म} \times अ^{न} = अ^{म+न}$$ **म, न इत्यनयोः सर्वस्मिन् माने।**

(बीजगणितम्, प्रघातमापकगणितम् (क), पृ. 198)

Example: Find the value of $2^5 \times 2^3$.

Solution: $2^5 \times 2^3 = (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^8 = 256$

$2^5 \times 2^3 = 2^{5+3} = 2^8$

Note that in 2^5 and 2^3 the base digit (2) is the same and the sum of the exponents 5 and 3 is 8.

Example: Find the value of $a^2 \times a^4$.

Solution: $a^2 \times a^4 = (a \times a) \times (a \times a \times a \times a)$

$= a \times a \times a \times a \times a \times a$

$= a^6$

$a^2 \times a^4 = a^{2+4} = a^6$

In this, the base is the same. We know that the exponents add up when the bases are the same and are in the product form. We can say generally. For a non-zero number a where m and n are any two positive integers, then $\mathbf{a^m \times a^n = a^{m+n}}$

❖ **Do and learn**

Simplify

1) $3^4 \times 3^2$ 2) $7^7 \times 7^2$

Rule 2: Division of exponential numbers with the same base

Example: Solve $3^5 \div 3^3$.

Solution: $3^5 \div 3^3 = \frac{3^5}{3^3} = \frac{3\times3\times3\times3}{3\times3\times3}$

$= 3 \times 3$

$= 3^2$

Hence,

$$3^5 \div 3^3 = 3^{5-3} = 3^2$$

Example: Find $5^7 \div 5^4$.

Solution: $5^7 \div 5^4 = \frac{5^7}{5^4} = \frac{5\times5\times5\times5\times5\times5\times5}{5\times5\times5\times5}$

$= 5 \times 5 \times 5$

$= 5^3$

So $5^7 \div 5^4 = 5^{7-4} = 5^3$

Observe that -

5^7 and 5^4 have the same base and the difference between the exponents 7 and 4 is 3.

So we can say that the exponent decreases in the division of exponential numbers with the same base. We can say generally. For a non-zero number a where m and n are any two positive integers, then

$$\mathbf{a^{m} \div a^{n} = a^{m-n}}$$

Rule 3: Number with zero exponent

$$अ^{0} = \frac{अ^{न}}{अ^{न}} = 1$$ **अतः कस्याप्यङ्कस्य शून्यघातः रूपसमोभवति।**

(बीजगणितम्, प्रघातमापकगणितम् (ख), पृ. 198)

If the exponent of any non-zero number is zero (0), then its value is 1. i.e.

$$\mathbf{a^0 = 1}$$

Example: Solve $3^5 \div 3^5$.

Solution: $3^5 \div 3^5 = \frac{3^5}{3^5} = \frac{3\times3\times3\times3\times3}{3\times3\times3\times3\times3} = 1$

$= 3^{5-5} = 3^0$

Therefore $3^0 = 1$

$a^0 = 1$ for any non-zero integer a

Rule 4: Exponent of an exponential number

$\mathbf{(a^m)^n = a^{m\times n} = a^{m.n}}$ for some non-zero integer a.where m and n are whole numbers.

Example: Solve $(7^2)^3$.

Solution: $(7^2)^3 = 7^2 \times 7^2 \times 7^2$

$= 7^{2+2+2} = 7^6$

or $(7^2)^3 = 7^{2\times3} = 7^6$

Rule 5: Multiplication of numbers with different base but same exponent

Example: Solve $3^4 \times 2^4$.

Note: Here the base numbers are not the same but the exponents are the same.

Solution: $3^4 \times 2^4$

$$= (3 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2)$$
$$= (3 \times 2) \times (3 \times 2) \times (3 \times 2) \times (3 \times 2)$$
$$= (3 \times 2)^4 \quad = (6)^4$$

From the above it follows that if a and b are any two non-zero numbers and n is a positive integer, then $\mathbf{a^n \times b^n = (a \times b)^n}$

Rule 6 - Division of numbers of different base but same exponent

Example: Solve $5^4 \div 2^4$.

Solution: $5^4 \div 2^4 = \frac{5^4}{2^4} = \frac{5 \times 5 \times 5 \times 5}{2 \times 2 \times 2 \times 2}$

$$= \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$$
$$= \left(\frac{5}{2}\right)^4$$

Then $5^4 \div 2^4 = \frac{5^4}{2^4} = \left(\frac{5}{2}\right)^4$

Then it follows that if a and b are any two non-zero numbers and n is a positive integer, then $\boldsymbol{a^n \div b^n = \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n}$

Rule 7 - Power of negative exponent $\mathbf{a^{-n}} = \frac{1}{a^n}$ for any non-zero rational number a where n is a positive rational number a^{-n}, is the multiplicative inverse of a^n.

Example: Solve 3^{-1}.

Solution: $3^{-1} = \frac{1}{3}$ (since $a^{-1} = \frac{1}{a}$)

Thus, $10^{-2} = \frac{1}{10^2}$

❖ **do and learn**

Simplify the following using the law of exponentials -

1) 10^{-3} 2) $5^4 \times 4^4$

3) $5^{10} \times 5^{15}$ 4) $6^{15} \div 6^{10}$

Exercise 5.2

1. Simplify using the laws of exponents and write the answer in exponential form. (When the base is the same.)

1) $3^4 \times 3^5$ 2) $4^7 \times 4^2$ 3) $2^5 \times 2^{10}$

4) $6^{12} \div 6^9$ 5) $a^b \div a^2$ 6) $2^{20} \div 2^{10}$

7) $(7^4)^3$ 8) $(3^4)^2$ 9) $(4^5)^2$

10) $(13)^0$ 11) $(13)^{0\times2}$ 12) $(-2)^0$

2. Simplify using the rules of exponents and write the answer in exponential form. (When the base is different but the degree is the same.)

1)1) $3^5 \times 4^5$ 2) $6^{10} \times 2^{10}$ 3) $4^2 \times 8^2$

4) $2^5 \div 3^5$ 5) $3^4 \div 5^4$ 6) $2^7 \div 9^7$

3. Simplify using the rule of negative exponents.

1) 3^{-4} 2) 2^{-2} 3) 5^{-1} 4) 8^{-3}

Exponentiation of large numbers

In the following mantras taken from Vedas, details of livestock have been given in large (greatest) numbers received from the famous king Prithrisrava (initial Richa).

षष्टिं सहस्राश्व्यस्यायुतासन मुष्ट्रानां विंशतिं शता।

दश श्यावीनां शता दश त्र्यरूपीणां दश गवां सहस्ता॥

(ऋग्वेद : 8/46/22)

Meaning, I got sixty thousand and ayut (ten thousand) horses and twenty hundred (two thousand) camels. Received ten hundred (one hundred) Ashwinis of Shyam Varna and ten thousand cows marked at three places. In addition to the above-mentioned mantras, there is mention of the largest number **in Rigveda in 8/2/41 and 20/29/9**.

Let us read and understand the following -

$$75 = \frac{75}{10} \times 10 = 7.5 \times 10^1$$

$$750 = \frac{750}{100} \times 100 = 7.5 \times 10^2$$

$$7500 = \frac{7500}{1000} \times 1000 = 7.5 \times 10^3$$

Here we have expressed 75, 750, 7500 in standard form.

When a number is expressed as the product of 1.0 or a decimal number greater than 1.0 or less than 10 and a power of 10, then this form of the number is called the standard form.

- **Writing the largest number in standard form -**

Let us learn to express the largest numbers in the standard form with the help of examples-

$$6578 = 6.578 \times 1000 = 6.578 \times 10^3$$

$$31000 = 3.1 \times 10000 = 3.1 \times 10^4$$

$$300000 = 3 \times 100000 = 3 \times 10^5$$

$$2340000 = 2.34 \times 1000000 = 2.34 \times 10^6$$

Thus in standard form,

- The diameter of the earth is 127576000 m. Is. Then the standard form of the diameter of the earth is 1.27576×10^8 m.
- The distance between the Earth and the Moon is 384000000 m. Is. Then the standard form is 3.84×10^8 m.

Expressing the smallest number in standard form –

Let us learn to express the smallest numbers in its standard form by taking examples. Express in standard form.

Example: Express in standard form - 0.0004

Solution: $\frac{0.0004}{10000} = \frac{4}{10000} = \frac{4}{10^4}$

$= 4 \times 10^{-4}$

Example: average diameter of red blood cells is 0.000007 mm

Solution: Then in standard form,

$$0.000007 = \frac{7}{1000000}$$

$$= \frac{7}{10^6}$$

$$= 7 \times 10^{-6}$$

you know that- $a^{-1} = \frac{1}{a}$

or $a^{-2} = \frac{1}{a^2}$

Example: The thickness of a paper is 0.00016.

Solution: Then in standard form,

$$0.00016 = \frac{16}{100000} = \frac{16}{10^5}$$

$$= 16 \times 10^{-5}$$

or $\frac{1.6}{10000} = \frac{1.6}{10^4} = 1.6 \times 10^{-4}$

Example: Convert the following standard form to normal number.

1) 5.9×10^{-4} 2) 5.9×10^{4} 3) 2×10^{4}

4) 2.1×10^{-5} 5) 7×10^{-3}

Solution: 1) $5.9 \times 10^{-4} = \frac{5.9}{10^4} = 0.00059$

2) $5.9 \times 10^{4} = \frac{5.9}{10} \times 10^{4} = \frac{59 \times 10000}{100} = 59000$

3) $2 \times 10^{4} = 2 \times 10000 = 20000$

4) $2.1 \times 10^{-5} = \frac{2.1}{10^5} = 0.000021$

5) $7 \times 10^{-3} = \frac{7}{10^3} = 0.007$

Do and learn

Convert the following to a normal number.

1) 3×10^{-5} 2) 4.12×10^{5}

EXERCISE 5.3

1. Select the correct option for the following multiple-choice questions.

(a) The value of 5^{-3} will be -

(I) 5 (II) 125 (III) $\frac{1}{125}$ (IV) None of these

(b) The value of $(-7)^3$ will be-

(I) 243 (II) - 343 (III) - 49 (IV) 343

(c) If a=1 and b=2 then $a^b + b^a$ will be -

(I) -1 (II) 2 (III) 3 (IV) 4

(d) If a=-1 and b=2 then $a^b + b^a$ will be -

(I) $\frac{3}{2}$ (II) $\frac{2}{3}$ (III) $\frac{1}{2}$ (IV) $\frac{-1}{2}$

(e) The standard form of 0.00000015 shall be—

(I) 0.5×10^{-6} (II)) 1.5×10^{-6}

(III) 0.5×10^{-7} (IV)) 1.5×10^{-7}

(f) The standard form of 0.00000065 shall be—

(I) 65×10^{-6} (II)) 6.5×10^{-6}

(III) 6.5×10^{-5} (IV)) 65×10^{-5}

(g) Which of the following will be the general form of 7×10^{-3}?

(I)) 0.007 (II) 0.07 (III) 0.7 (IV) 7

(h) The standard form of the number 1000 shall be-

(I)) 1×10^{-1} (II) 1×10^{-3}

(III) 1×10^{3} (IV) 1×10^{-4}

2. Express the following large numbers in standard form?

1) 700000 2) 2180000

3) 48000 4) 418000000

3. Express the following small numbers in standard form ?

1) 0.00027 2) 0.00004

3) 0.0001 4) 0.0000418

4. Convert the following standard form to normal number?

1) 2.4×10^{-4} 2) 3×10^{-3} 3) 7.1×10^{5}

4) 4.8×10^{3} 5) 2.46×10^{-5} 6) 1.3×10^{6}

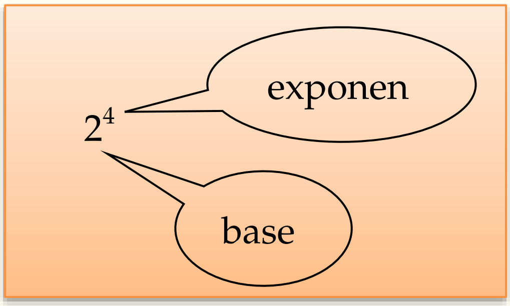
5. Express in standard form?

a) The diameter of the Sun is 14,00,00,000 m.

b) The thickness of a human hair is about 0.0002 cm.

c) The size of the bacteria is 0.000005 m.

- **We learned –**



We read 2^4 as 2 raised to the power of 4.

1) Using exponents to simplify large numbers makes the number easier to read and write.

2) Following are some exponential forms.

$$1000 = 10 \times 10 \times 10 = 10^3$$

Here 10^3 has 10 base and 3 exponent. We read this as 10 raised to the power of 3.

3) **Law of Exponents** - For any non-zero integer a and b and for whole numbers m and n -

- Same base and different exponents

a) $a^m \times a^n = a^{m+n}$ b) $a^m \div a^n = a^{m-n}$

c) $a^0 = 1$ d) $(a^m)^n = a^{mn}$

e) $a^{-1} = \frac{1}{a}$ (a^{-1} is the multiplicative inverse of a)

- When the base is different and the exponent is the same -

a) $a^n \times b^n = (a \times b)^n$ b) $a^n \div b^n = \frac{a^n}{b^n}$

$(-1)^{\text{even number}} = 1$

$(-1)^{\text{odd number}} = -1$

4) To express a number in scientific notation or standard form, it is expressed as the product of a decimal number between 1.0 and 10.0 (1.0 inclusive) and a power of 10.

Example :

- **standard form of the largest number**

a) $48000000 = 4.8 \times 10^7$

b) $3.1 \times 10^3 = \frac{31}{10} \times 1000 = 3100$

- **Standard form of smallest number:**

a) $0.000005 = \frac{5}{1000000} = \frac{5}{10^6} = 5 \times 10^{-6}$

b) $2.3 \times 10^{-3} = 0.0023$

Chapter 6

Vedic Mathematics

In this chapter of Vedic Mathematics, we will study in detail the formulas mentioned in the previous class on multiplication of two and three-digits numbers, square and square root as well as division by the formula "Urdhwatiryagbhayam".

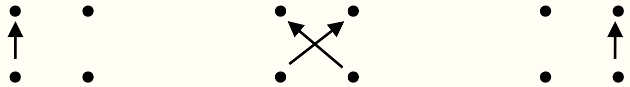
Multiplication Operation – Urdhvatiryagbhayam Sutra

Now we are going to introduce you to one such general formula of multiplication. With the help of which you can solve any multiplication problems in an easy way.

The first word 'Urdhva' of the formula means just above (multiplication of the numbers written above and below) similarly the second word 'Tiryak' means oblique (multiplication of numbers written diagonally).

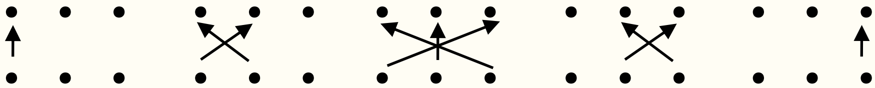
Dotstick Method –

- Multiplication of two digit number –



Step (III) Step (II) Step (I)

- Multiplication of three digit numbers –



Step (V) Step IV) Step (III) Step (II) Step (I)

Example: Find the product of 32 and 15.

Solution:

Step 1 – Write the product 32 and the multiplier 15 in the following way.

$$\begin{array}{r} 3\ 2 \\ \times\ 1\ 5 \\ \hline \\ \hline \end{array}$$

Step 2 – Grouping the numbers and multiplying: three groups will be formed. Which is shown in III, II, I.

III	II	I
3	32	2
1	15	5

Step 3 – Multiplication $3 \times 1 \ / \ 3 \times 5 + 2 \times 1 \ / \ 2 \times 5$

Step 4 - Multiplication $3/15 + 2/10$

$3/17/10$

By writing the numbers sequentially and adding them one digit at a time from right to left, we get the result.

3 / 17 / 10

+1 +1

= 480

Hence, required product of $32 \times 15 = 480$.

Example:Multiply 135 with 123 using the formula Urdhvatiryagbhayam.

Solution: Step 1 – Write the product of 123 and 135 as follows.

$$\begin{array}{r} 1\ 2\ 3 \\ \times\ 1\ 3\ 5 \\ \hline \\ \hline \end{array}$$

Step 2 – Five groups will be formed in the multiplication of three digits by three digits. Which are shown as follows V, IV, III, II and I.

V	IV	III	II	I
1	12	123	23	3
1	13	135	35	5

Step 3 - Procedure to multiply

Multiplication : 1+1/1×3+1×2/1×5+2×3+3×1/2×5+3×3/3×5

Product : 1 / 3+2 / 5 + 6 + 3 / 10 + 9 / 15

Procedure to add: 1 / 5 / 14 / 19 / 15

By writing the number sequentially and then adding one digit at a time from right to left, we get the final result.

1 / 5 / 14 / 19 / 15

+1 +2 +1

= 1 6 6 0 5

Hence, the required product of 123 × 135 is 16605.

❖ **Do and learn**

Find the product using the Urdhvatiryagbhayam formula.

1) 2 3
× 3 1

2) 1 2 5
× 2 0 1

Multiply by Nikhilam (substrate) –

Multiply it by the Anurupena formula –

Large deviations of a question are obtained by " Nikhilam". That their multiplication becomes difficult, like in 32 × 34, if base 10 is taken, then the deviation of 22 and 24 becomes bigger. Therefore, on the basis of the nearest ten, the sub-basis is selected. Due to which the deviation from the sub-basis is small.

As : In 88×81, the sub-base is 8×10=80 and in 32×27, the sub-base is 3×10=30. The rest of the method is the same as before.

Example: Find the value of 32 × 33.

Step 1: Number

$$\begin{array}{rr} & 32+2 \\ \times & 33+3 \end{array}$$

Step 2: The Solution will be in two sections-

$$\begin{array}{rrrr} & 32 & + & 2 \\ \times & 33 & + & 3 \\ \hline & & / & 6 \end{array}$$

Hint –

1. Base = 10, sub-base = 3×10 = 30
2. Subadhar points = Subadhar/Base = 30/10 = 3

3.Deviation from base = number – base = 32–30 = +2/ = 33–30 = +3

Step 3: By adding any one number and the deviation of the remaining number on the left hand side, multiply it by the base digit 3.

$$\begin{array}{rrrr} & 32 & + & 2 \\ \times & 33 & + & 3 \\ \hline & 105 & / & \end{array}$$

(where 32 + 3 = 35 or 33 + 2 = 35 and 3 × 35 = 105)

Step 4: Step (1) and step (3) are integrated and operationally arranged.

$$\begin{array}{r} 32 + 2 \\ \times\ 33 + 3 \\ \hline 105\ /\ 6 \end{array}$$ (removing slant line)

= 1056

Hence, the required product of 32×33 is 1056.

Example: Solve 55×57

Using step 4 of the previous example directly.

Solution:

Number	Deviation
55	+ 5
× 57	+ 7

5 × (55 + 7 or 57 + 5) / 5 × 7

5 × 62 / 35

310 / 35

= 3135

> **Hint -**
>
> 1) Base = 10, sub-base = 5×10 = 50
>
> 2) Base number = 5
>
> 3) Deviation from base = +5 and +7
>
> 4) In base 10, there will be only one digit to the right of zero.
>
> 5) Adding the tens digit on the left side = 310 + 3 = 313

Example: Solve 66 × 67.

Solution:

Number	Deviation
66	– 4
× 67	– 3

7× (66 – 3) / (–3) × (–4)

7 × 63 / 12

441 / 12

= 4422

> **Hint -**
>
> 1) Base = 10, sub-base = 7×10 = 70
>
> 2) Base number = 7
>
> 3) Deviation from base = –4 and –3

Example: Find the product - 64 × 58

Solution:

Number	Deviation
64	+ 4
× 58	– 2

6 × (64 – 2)/(– 2) × (+ 4)

6 × 62 / – 8

372 / – 8

371 + 1 / – 8

371 / 10 – 8

= 3712

Hint -

1) Base = 10, sub-base = 6×10 = 60

2) Base number = 6

3) Deviation from base 64 = 64–60 = 4

58 = 58 – 60 = –2

4) Multiplication of deviations in base right side

= 4 × 2 = –8

Which is a negative, so for positive, 1 is shown from the left side and taking 10 in the ones side will be 10 – 6 = 4.

5) Hence product = 3712

❖ Do and learn

Find the product of the following by the Anurupena formula.

1) 6 4 × 6 2 = ______

2) 4 3 × 4 6 = ______

3) 5 1 × 4 9 = ______

Exercise 6.1

1. Multiply using the Urdhvatiryagbhayam formula.

Multiplication of two-digits -

1) 32 × 43 = ____

2) 31 × 42 = ____

3) 45 × 21 = ____

5) 89 × 43 = ____

6) 56 × 45 = ____

7) 45 × 27 = ____

Multiplication of three-digits -

4) 123 × 301 = ____

5) 401 × 104 = ____

6) 312 × 121 = ____

5) 109 × 243 = ____

6) 150 × 240 = ____

7) 045 × 127 = ____

2. Multiply using the Anurupena formula.

1) 66 × 63 = ____

2) 35 × 32 = ____

3) 72 × 73 = ____

4) 71 × 82 = ____

Urdhvatiryagbhayam formula-

This application is based on Urdhvatiryagbhayam and **Dhwajank** Sutra. By this method every question of division can be done very easily. Before starting the operation, the following points are to be noted while writing the question.

1) First divide the divisor into two convenient parts, the unit digit of the number is called dwajank of the divisor and the remaining part is called the main number or modified divisor. The dwajank may have only one or several digits containing units.

2) Similar to the previous methods of division operation, in this method the fixed space is divided into three parts.

   a) In the first section, both the parts of the divisor will be taken. The main number will be written below i.e. at the base place and the flag number will be written above it i.e. at the exponent place.

   b) As many digits as there are in the dwajank, the same number of last digits of the dividend will be written in the third block and its remaining digits will be written in the middle block.

   c) In dwajank method, 524 ÷ 23 can be written in the following form.

	first coloum	middle coloum		third coloum
dwajank	3	5	2	4
main number	2			
		quotient		remainder

Method -

1) In middle block, the first digit of the quotient obtained by dividing the dividend by the left most digit of the divisor, is written at the fixed place of the quotient below the horizontal line.

2) The remainder obtained is written before and below the second digit from the left. Which becomes the new dividend.

3) The following formula gives the modified quotient from the new quotient.

   Revised Quotient = New Quotient – Quotient Number × dwajank

4) By dividing the prime number again in the modified divisor, the previous operations are repeated.

Let the method be explained with the following example.

Example: Find the quotient using the dwhijank formula –

$552 \div 23$

Solution:

dwhijank	3	5	5	2
main number	2	1		1
		2	4	0

Hint –

1) Divide 5 by 2.

2) The first digit of the quotient, 2, will be written below the horizontal line.

3) Remainder = 1 written before 5 of the middle section and new dividend = 15 is obtained.

4) Revised Quotient = New Quotient – First Quotient × dwhijank

$$=15 - 2 \times 3 = 9$$

5) Divide 9 by 2. Write the quotient (4) below the line.

6) Remainder = 1 before 2 of third section and new dividend = 12.

7) Revised Quotient = New Quotient – Second Quotient × dwhijank

$$= 12 - 4 \times 3 = 0$$

8) Quotient = 24, Remainder = 0

Example: Find the quotient and remainder by dividing 4096 by 64 using the dwhijank method.

Solution:

Dwhijank	4		
main number	6	4 0 9	6
		4	1
		6 4	0

Hint –

1) On dividing 6 by quotient = 40, we get quotient = 6 and remainder = 4.

2) Revised dividend = 49 – 6 × 4 = 25.

3) Divide the prime number 6 in 25. The second digit of the quotient will be written below in 4 horizontal lines.

4) Remainder = 1 obtained before third division 6 and new dividend = 16.

5) Revised dividend = 16 – 4 × 4 = 0

6) Hence quotient = is 64, remainder = 0

Exercise 6.2

1. Divide using the dwhijank methode.

1)	1764 ÷ 42	2)	7396 ÷ 82	3)	992 ÷ 62
4)	945 ÷ 21	5)	6417 ÷ 24	6)	5004 ÷ 42
7)	459 ÷ 23	8)	4167 ÷ 42	9)	7008 ÷ 24

Squares and Square roots:

In the previous lessons, you have studied squares and square roots.

Let us learn to find the square root of a number by the Vedic method.

You will remember about the previous exercise that -

- If you take the square of 2, we get 4, and if we take the square root of 4, we get 2.
- If we take the square of 3, we get 9, and if we take the square root of 9, we get 3.

That is, $2^2 = 2\times 2 = 4$ and $\sqrt[2]{4} = 2$

$3^2 = 3\times 3 = 9$ and $\sqrt[2]{9} = 3$

In general, finding the square root of a number $\left(\sqrt[2]{9} = 3\right)$ is considered a more difficult task. While observation is helped by the Vedic Sutras (vilokanam).

Method to find the square root –

1) First find whether the number is a perfect square or not.
2) The unit digit of a perfect square is from 0, 1, 4, 5, 6 and 9.
3) If the number ends with 2, 3, 7 or 8 then it is not a perfect square.
4) The number of zeros at the end of a perfect square number is

even and the number preceding the zeros should be a square number.

5) A number which ends with an odd number of zeros is not a perfect square number.

6) If the number is a perfect square, then find the number of digits in its square root.

7) Find out the unit's digit of the square root. Using the following able.

Table – 1

units digit	Unit digit of the square root
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7

In addition to the above table, we look at another table carefully, with the help of which you can find the square root of a number in a few moments.

Table - 2

Number	to the nearest square root
1 – 3	1
4 – 8	2
9 – 15	3
16 – 24	4
25 – 35	5
36 – 48	6
49 – 63	7
64 – 80	8
81 – 99	9

Rules for finding the correct square root of a 3-4 digit number from Sutra Vilaokanam –

1) Starting from the right side make a group of two numbers each.
2) Look at the unit's digit of the number and check your answer with Table1 which will help you to decide the units digit of your answer.
3) Now refer to the second group and find the tens digit of your square root by giving table 2.

Example: Find the square root of 2116.

Solution: Make a group of two numbers each from the right.

21	16
group 2	**group 1**

- Here the unit's digit is 6, then the square root will end with 4 or 6. (see Table 1)
- Now look at the second group, since 16 < 21 < 25, the ten's digit is 4 (see Table 2).
- Now you have two options $\sqrt{2116} = 44$ or 46

We know 452 = 2025 since 2116 > 2025 so expected square root will be greater than 45 so

$$\sqrt{2116} = 46$$

Example: Find the square root of 5184.

Solution: Make a group of two numbers each from the right.

51	84
group 2	group 1

- If the unit digit of the first pair is 4, then the end (unit digit) of the square root will be 2 or 8.
- Now look at the second group, since 49 < 51 < 64, the tens digit is 7 (see Table 2).
- Now we have two options. $\sqrt{5184} = 72$ or 78

We know 752 = 5625 since 5184 < 5625 so expected square root will be less than 75 so

$$\sqrt{5184} = 72$$

Example: Find the square root of 9216.

Solution : 92 16

group 2 **group 1**

- If the unit digit of the first group is 6, then the end (unit digit) of the square root will be 4 or 6.
- Now consider the second group, since 81 < 92 < 100, the tens digit is 9 (see Table 2).
- Now we have two options $\sqrt{9216} = 94$ or 96

We know $95^2 = 9025$ since 9216 > 9025 so expected square root will be greater than 95 so -

$$\sqrt{9216} = 96$$

EXERCISE 6.3

1. Find the square root from the formula Vilokanam.

1)	$\sqrt{169}$	2)	$\sqrt{324}$	3)	$\sqrt{2025}$
4)	$\sqrt{1024}$	5)	$\sqrt{9025}$	6)	$\sqrt{400}$

- **We learned -**

1) The word UrdhvatiryaBhyam is made up of two words 'Urdhva' and 'Tiryak'. The meaning writing the numbers one below the other and the meaning of tirayak is the product of the number written diagonally.

2) Numbers are grouped while multiplying by Urdhvatiryagbhayam formula, while multiplying two digits with two digits three groups and while multiplying three digits with three digits, groups five are groups formed.

3) Learned to multiply by corresponding formula.

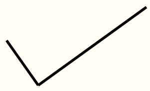
4) Studied the general method of finding the quotient from the dwhajank formula.

5) Learned to find the square root of a 3 - 4-digit number by Vilokanam Sutra.

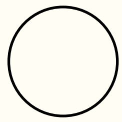
Chapter 7

Understanding Quadrilaterals

Dear students! You have studied different types of figures in the previous class. You will also remember about curves. Is a circle a two-dimensional shape? Do circles have sides? No, circle is a Curve-shaped two-dimensional figure. Circle can be understood as a simple closed figure.

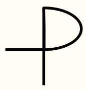


This is an open shape.



It is a simple closed curved figure. (Circle)

It is a closed figure.

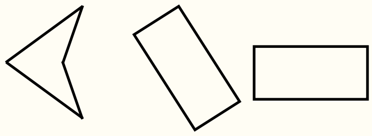


It is a closed curved figure but not simple.

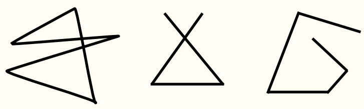
Discuss with your friends by drawing other figures like this.

Polygon

Polygon is made up of two words. Namely 'poly' meaning many and 'gon' meaning sides. A simple closed curve made of only line segments is called a polygon.

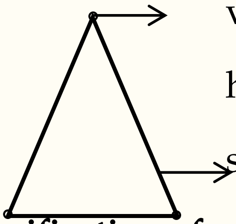


Curve that is a polygon.



Curve that is not a polygon.

Identification of sides and vertices of a polygon –

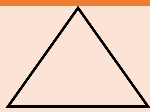
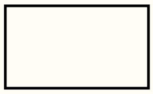
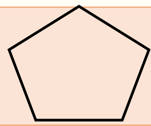
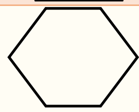


vertices

he given polygon is a 'triangle' which has 3 sides and 3

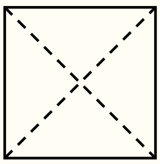
sides

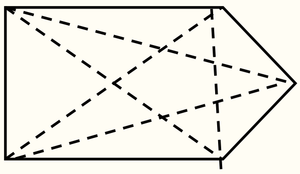
Classification of polygons based on the number of sides

Number of sides or vertices	Classification	figure
3	Triangle	
4	Quadrilaterals	
5	Pentagons	
6	Hexagon	
⋮	⋮	⋮
n	Polygon	……..

Diagonal –

The diagonal of a polygon is the line segment obtained by joining any two of its non-adjacent vertices (excluding consecutive vertices).



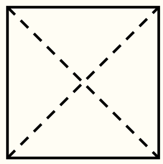


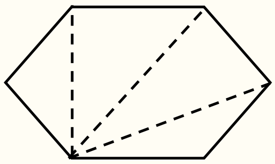
There are 2 diagonals in a quadrilateral. In a pentagon there are.......... diagonals.

Convex and concave polygons –

Convex polygon

A polygon in which all interior angles are less than 180° or none of the diagonals are in the exterior region of the polygon. is called a convex polygon.

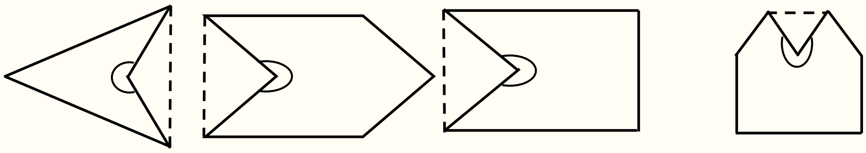




There are 2 diagonals in a quadrilateral. There are.......... diagonals in a hexagon.

Concave polygon

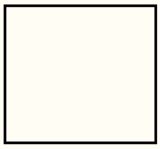
A polygon whose one or more interior angles is greater (more) than 180^0 or one are more diagonals is in the exterior region of the polygon, is called a concave polygon.



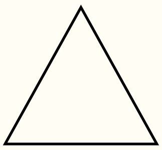
The given interior angle is greater than 180^0 and one of the diagonals is in the exterior region.

Regular polygon

A polygon that has equal sides and equal angles. is called a regular polygon. Like a square equilateral triangle



4 sides and 4 angles are equal in a square.

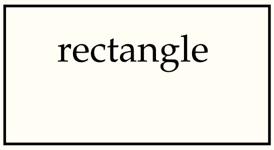


All the three sides and all the three angles are equal in an equilateral triangle.

Irregular polygon

A polygon in which the sides and angles are Not equal. is called an irregular polygon.

such as:



Four angles (90^0) in a rectangle are equal, but the sides are not. This is a irregular polygon.

Exercise 7.1

1. Fill in the blanks.

   1) A polygon is a simple closed curve made up of ………… (line / line segment)

   2) A polygon made of five sides is called …………………… (quadrilateral / pentagon)

   3) A polygon whose all angles and sides are equal is called .... (regular polygon / irregular polygon)

   4) The line segment joining the vertices (excluding consecutive vertices) of a polygon is called............ (Diagonal / Triangle)

   5) A polygon in which the diagonal is exterior is called.................. (convex / concave)

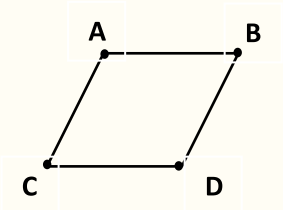
   6) A polygon whose each interior angle is less than 180° and all diagonals are interior is called a …………… polygon. (convex / concave)

2. Name the polygon in which the following sides are given.

   a) 4 sides b) 6 sides c) 8 sides

Quadrilateral –

A quadrilateral is a closed figure that has four sides. A polygon with four edges (sides) and four vertices (corners) is called a quadrilateral.



In the figure AB, BC, CD and DA are the four sides.

From which quadrilateral ABCD is formed.

The characteristic of quadrilateral is :

1) Every quadrilateral has four sides.

2) Every quadrilateral has four angles.

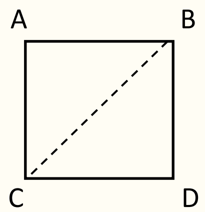
3) Every quadrilateral has four vertices.

4) The sum of all four interior angles in a quadrilateral is 360°.

Angle sum property of a quadrilateral:

Let us understand the angle sum property of a quadrilateral.

In a quadrilateral, a diagonal divide it in too unique triangles. Since, the sum of all the interior angles of a triangle is 180°, the sum of all the interior angles of a quadrilateral is 2×180° = 360°. This can be done by drawing a diagonal of the quadrilateral and dividing it into two triangles.



BD is the diagonal in the above quadrilateral ABCD. Then you see that on drawing the diagonal BD, two triangles (Δ ABD, Δ BCD) are formed. We know that the sum of all three angles of a triangle is 180^0. The sum of the angles of both the triangles is-

$\Delta ABD + \Delta BCD = 180^0 + 180^0 = 360°$

$\angle ABD + \angle ADB + \angle A + \angle BDC + \angle DBC + \angle C = 360^0$

$\angle A + \angle ABD + \angle DBC + \angle C + \angle ADB + \angle BDC = 360^0$

where $\angle ABD + \angle DBC = \angle B$

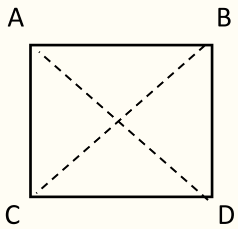
$\angle ADB + \angle BDC = \angle D$

$\angle A + \angle B + \angle C + \angle D = 360^0$

Hence, the sum of the four sides of a quadrilateral is 360^0.

Types of quadrilateral –

1) **Square** - When all four sides and angles of a quadrilateral are equal (right angles), the quadrilateral is called a square.



All four sides AB = BC = CD = DA

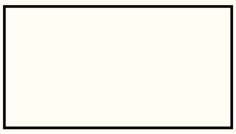
Four angles ∠ A = ∠ B = ∠ C = ∠ D = 90^0

The diagonals of a square are perpendicular bisectors.

In this the diagonals are AC and BD.

2) **Rectangle** -

A quadrilateral with equal sides and four angles is called a quadrilateral rectangle.

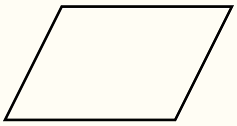


Here opposite sides AB = DC and CB = DA

Four angles ∠ A = ∠ B = ∠ C = ∠ D = 90^0

3) **Parallelogram** - Such a quadrilateral in which the opposite sides are equal and parallel and the opposite angles are equal, then this quadrilateral is called a parallelogram. Parallel sides are denoted by the symbol ‘ || '.

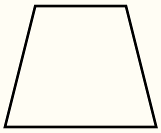
arms opposite here



AD || BC and AB || DC

Opposite angles ∠ A = ∠ C and ∠ B = ∠ D

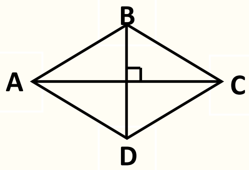
4) **Trapezium** - Such a quadrilateral in which only one pair of opposite sides are parallel, then that quadrilateral is called a trapezium.



Opposite side in this

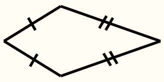
AB | | DC

5) **Rhombus -** A quadrilateral with all four sides equal and both diagonals bisecting each other at 90° degrees, as well as unequal diagonals. The shape is then known as a rhombus.



In this diagonals AC and BD bisect each other at 90^0.

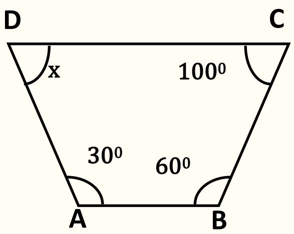
6) **Kite –** A kite is a quadrilateral with two pairs of different adjacent sides that are equal in length. And the diagonals ntersect each other at 90^0.



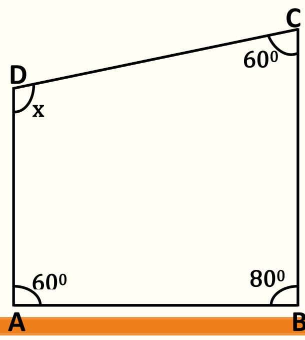
Let us learn to solve problems based on angles in a quadrilateral.

1) Find the measure of x (the measure of the angle) in the following figures.

(a)



(b)



Solution: (a) In figure,

We know that the sum of the four angles of a quadrilateral = 360^0.

then, $\angle A + \angle B + \angle C + \angle D = 360^0$

$30^0 + 60^0 + 100^0 + x = 360^0$

or $190^0 + x = 360^0$

or $x = 360^0 - 190^0 = 170^0$

$x = 170^0$

Solution: (b) $\angle A = 60^0$, $\angle B = 80°$ and $\angle C = 60°$ are given in the figure.

We know that the sum of the four angles of a quadrilateral is 360^0.

Then, $\angle A + \angle B + \angle C + \angle D = 360^0$

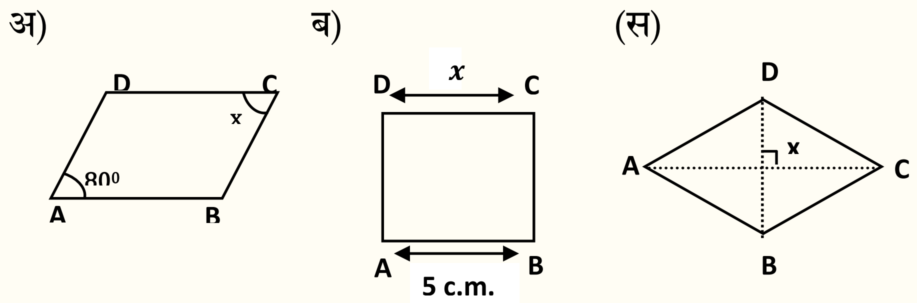
$60^0 + 80^0 + 60^0 + x = 360^0$

or $200^0 + x = 360^0$

or $x = 360^0 - 200^0 = 160^0$

$x = 160^0$

Example: Identifying the shapes of the given quadrilateral, find the unknown values using the definition or quadrilaterals.



Solution: a) The given figure (a) is a parallelogram.

To find the value of x

Then we know, opposite angles of a parallelogram are equal, Hence,

$\angle A = \angle C$

$80^0 = x$

Hence, in figure (a), the value of x is 80°.

Solution: b) The given figure (b) is a square.

Given – where AB = 5 cm.

To find - DC = ? (measurement of arm DC)

Then we know that all sides of a square are equal. Hence,

DC = 5 cm

Hence, in figure (b), side DC = 5 cm.

Solution: c) The given figure (c) is a rhombus.

To find the value of *x*

We know that, in a rhombus the diagonals bisect each other at 90^0.

Hence, $\angle x = 90^0$

Hence, in figure (c) angle $x = 90^0$.

Exercise 7.2

1. Select the correct option for the following multiple-choice questions.

(a) A parallelogram in which each angle is a right angle is called …….

(I) Rectangle (II) Rhombus

(III) Trapezium (IV) Kite

(b) The parallelogram whose four sides are equal and the diagonals are perpendicular bisectors of each other-

(I) Square (II) Trapezium

(III) Rhombus (IV) Rectangle

(c) A quadrilateral in which only one pair of opposite sides are parallel is called.

(I) rhombus (II) parallelogram

(III) Trapezium (IV) rectangle

(d) What is the sum of the measures of the four interior angles of a quadrilateral?

(I) 360^0 (II) 540^0 (III) 180^0 (IV) 90^0

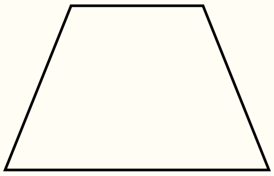
(e) How many triangles can be formed by the diagonals of a pentagon?

(I) 3 (II) 4 (III) 2 (IV) 1

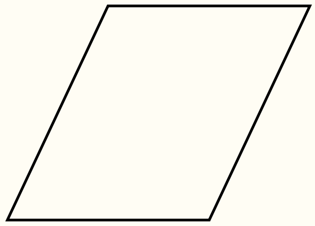
2. Identify the shape of the following quadrilateral -

(Square, rectangle, parallelogram, kite, rhombus, trapezium)

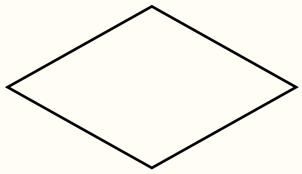
a)



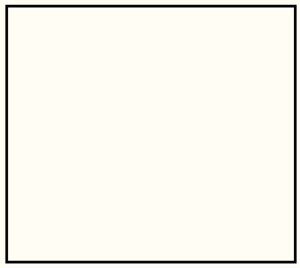
b)



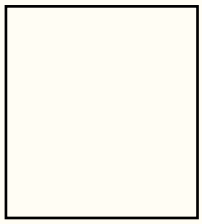
c)



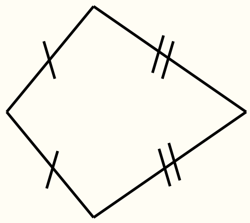
d)



e)

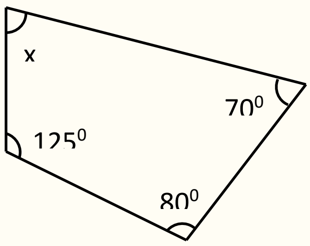


f)

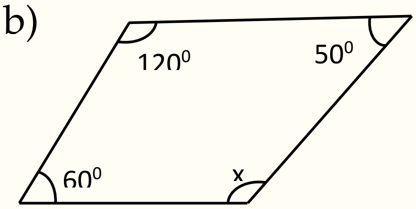


3. Identifying the shape of the following quadrilateral, find the values of the unknown angles (x).

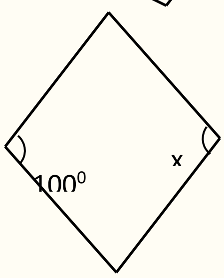
a)



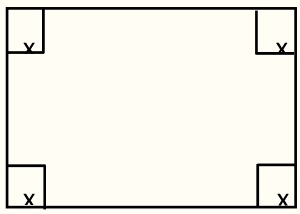
b)

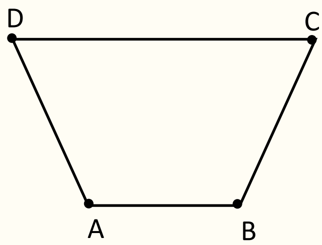


c)



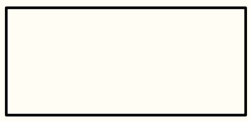
d)



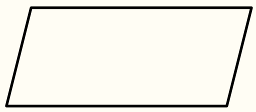


4. Is the following figure a parallelogram? If not, then give the reason.

1.



2.



5. State whether it is true or false.

   a) All squares are rectangles.

   b) All sides of a rectangle are equal.

   c) The shape of a rhombus is like that of a kite.

   d) All parallelograms are squares.

6. State the difference between parallelogram and trapezium.

- **We learned -**

  1) **Polygon –** A simple closed figure made of line segments is called a polygon.

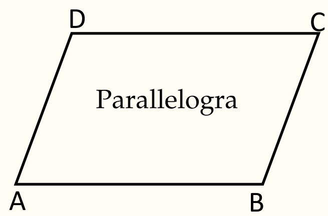
     Such as – triangle, quadrilateral, pentagon etc.

  2) **Types of polygons –** Convex polygon, concave polygon, Regular polygon, Irregular polygon

  3) **Quadrilateral-** A closed figure bounded by four sides is called a quadrilateral. Special names have been given to a quadrilateral on the basis of the sides and angles.

Quadrilaterals and Properties –

1) **Parallelogram –** A quadrilateral in which each pair of opposite sides are equal and parallel.

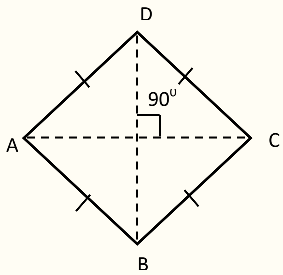


Properties - a) Opposite sides are equal.

b) Opposite angles are equal.

c) Diagonals bisect each other.

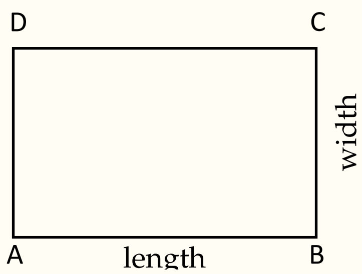
2) **Rhombus –** A quadrilateral whose all sides are of equal measure.



Properties - a) Has all the properties of the Parallelogram.

b) Diagonals bisect each other at 90^0.

3) **Rectangle –** A parallelogram in which each angle is a right angle.

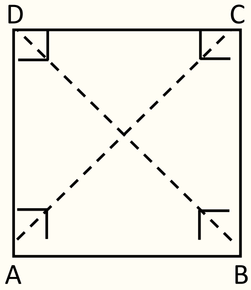


Properties – a) All properties of parallelogram

b) Each angle is a right angle of 90°.

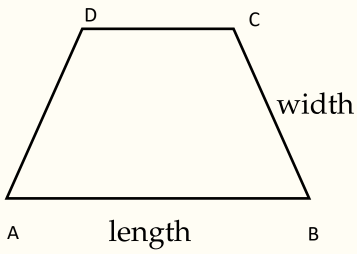
c) The diagonals are of equal measure.

4) **Square –** A rectangle whose all sides are equal.



Properties – All properties of parallelogram, rhombus and rectangle.

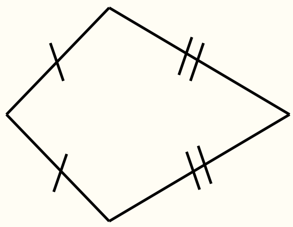
5) **Trapezium -** In this quadrilateral at least one pair of opposite sides are parallel.



Here the opposite sides AB || DC

6) **Kite –** A quadrilateral in which two pairs of adjacent sides are equal.

a) Diagonals intersect each other at 90^o.



Chapter 8

Algebraic expressions and identities

Dear students! We discussed variable and constant quantities in the chapters on simple equations, where we learned that a variable quantity is one whose value changes. The variable quantity is represented by the letters a, b, x, y, and so on. A constant quantity, on the other hand, is one whose value does not change. Examples include 4, -10, 100, and so on. You are now comfortable with basic algebraic expressions. To solve puzzles and problems, you write mathematical statements in the form of algebraic expressions. $x+2$, $3x+4$, $x+5$, and so on.

In this chapter, we will study algebraic expressions and identities in detail.

algebraic expression

Algebraic expressions are created by combining variables and constants. We use addition, subtraction, multiplication, and division to accomplish this. We've seen expressions like $x+5$, $2x+1$, and $3x-2$.

Adding 5 to x yields the 'variable' in the expression $x+5$. $2x+1$ is calculated by multiplying the variable x by 2 and then adding the constant 1 to the product. Thus, the variable in the expression $3x-2$ is obtained by multiplying the constant (- 2) by three times the variable x.

$3x + (-2) = 3x - 2$

Note: An algebraic expression must contain at least one variable.

Terms of algebraic expression –

An algebraic term has more than one number of variables (l, m, n, x, y, z ……..) and constants. Any algebraic expression has sub parts.

For example: Consider $5x + 4$.

In this case, we multiply 5 and x to get 5x and then add 4. Similarly, in 3y2 + 2z, we multiply y and y by three to get 3y2. Then we multiplied 2 and z separately to get 2z, and then we added 3y2 and 2z to get 3y2+ 2z.

Terms of an algebraic expression are the sub-parts of an expression that are formed separately and then added together.

Example: $4xy,\ 3\mathrm{y}^2 + 2\mathrm{z},\ 3\mathrm{yz} + 2x$

An expression containing only one term. It is called monomial. An expression with two terms is called a **binomial** and an expression with three terms is called a **trinomial**. Thus the number of terms in a polynomial can be one or more than one. We will discuss about polynomials in the next class.

Examples of monomials: $3y^2, 3xy,\ 4x,\ 5xyz, 5y^2z$

Examples of binomials: $3x + y, 3xy + 2z, 3y^2 + 4z, 3y^2 - z^2$

Examples of trinomials : $a + b + c,\ 3a + 3x + z,\ a + y + b,\ x^2 + y + z$

Factors of an algebraic term –

A term in an algebraic expression can be the product of several variables and constants. An expression can be formed as a product of factors and terms.

Ex: $3y^2 + 2xz$

position	variable	constant
$\mathbf{3y^2, 2xz}$	x, y, z	$3, 2$

$3y^2 + 2xz$

$= 3 \times y \times y + 2 \times x \times z$

Similarly, factors of $10x^2y = 2 \times 5 \times x \times x \times y$

➢ Do and learn

expression	number of terms	terms	factors	variables	Constants
$\mathbf{2x + 3z}$	2	$2x, 3z$	$2x = 2 \times x$ $3z = 3 \times z$	x, z	2,3
$\boldsymbol{a + 3b + c}$					
$\boldsymbol{2x^2 + t^2}$					

Coefficient –

The coefficient of any of the factors of a term is equal to the product of the remaining factors of that term. Coefficients can be of both algebraic and numerical types.

Exmple: coefficient of $x\,y$ in 3xy = 3

Coefficient of $3x$ in $3xy$ $= y$

Coefficient of 3 in $3xy$ $= xy$

coefficient of x^2 *in* $10x^2y$ $= 10y$

Example: What is the coefficient of y in the following expressions ?

Solution: $3x - 4y, \quad 3x + y + z, \quad 2x^2y + z$

Expression	Term	Coefficient
$\mathbf{3x - 4y}$	$-4y$	3
$\mathbf{3x + y + z}$	y	1
$\mathbf{2x^2y + z}$	$2x^2y$	$2x^2$

Do and learn

Match the correct coefficients in the following expression

$3x^2y^3 + 4xy + 15.$

1) Coefficient of x^2y^3 : 4
2) Coefficient of $3x^2$: x
3) Coefficient of xy : 3
4) Coefficient of 4y : y^3

Like and unlike terms

When the algebraic factors of two terms are the same, they are referred to as like terms. When the factors are different, the terms are referred to as unlike terms.

Let us understand with an example.

$2xy\,,\ 7x\,,\ 12x\,,\ 5x^2,\ -3x,\ 9y^2,\ -4xy,\ -4x^2$

The similar terms are as follows.

1) $7x, 12x$ and $-3x$
2) $5x^2$ and $-4x^2$
3) $2xy$ and $4xy$

In contrast, $4xy$ and $9y^2$ have different algebraic factors, they are unlike terms. Thus $12x$ and $-4x^2$ are also unlike terms.

- **Do and learn**

Match the like terms correctly.

1. a^2b : $15a^2bc$
2. a^2bc : $-3a/b$
3. $\frac{2a}{b}$: $4a^2b$

Addition and subtraction of algebraic expressions –

The following verse is found in the Brahmasphutasiddhanta regarding the addition and subtraction of algebraic terms.

अव्यक्त वर्ग घनवर्ग वर्ग पञ्चगत षड्गतादीनाम ।

तुल्यानां सङ्कलितव्यवकलिते पृथगतुल्यानाम्॥

(ब्रह्मस्फुटसिद्धान्त, कुट्टकाध्याय :41)

It has been said in the above verse that the addition and subtraction of terms having same powers, cube, 3, 4, 5, 6 etc. is done.

"योगोऽन्तरं तेषु समान जात्यो र्विभिन्नजात्योस्तु पृथक् स्थितिश्च "

(बीजगणितम्,अव्यक्तसङ्कलनव्यवकलने सूत्रम्,7)

In the above verse of beeja ganitam, Bharakayacharya uses the same terms for sum and difference. The sum of like terms is done in algebraic expression addition to obtain a new term with the same algebraic factors (like term), whose coefficient is equal to the sum (or subtraction) of the coefficients of like terms. and the terms that do not match are written as they are.

Addition:

Example: Find the sum of $8xy$ and $3xy$.

Solution: 8xy + 3xy = (8+3) xy = 11xy

Example: Subtract 6ab from 11ab.

Solution: 11ab – 6ab = (11–6) ab = 5ab

Example: Find the sum of x+2y and $3x+4y$.

Solution: $(3x+4y) + (x+2y)$

On arranging the same post-

$= (3x + x) + (4y+2y)$

$= 4x + 6y$

Example: Add $3y+2z$ and $8x+4y+z$.

Solution: $(8x+4y+z) + (3y+2z)$

On arranging like terms together -

$= 8x+4y+3y+z+2z$

$= 8x+7y+3z$

Alternatively :

$$\begin{array}{r} 8x + 4y + z \\ + \quad 3y + 2z \\ \hline 8x + 7y + 3z \\ \hline \end{array}$$

subtraction

Example: Subtract $3x + 2a$ from $8x + 5a$.

Solution: $8x + 5a – (3x + 2a)$

$= 8x + 5a – 3x – 2a$

on arranging like terms together

$= 8x– 3x + 5a – 2a$

$= 5x + 3a$

Example: Subtract $2a – c$ from $3a + b – 3c$.

Solution: $3a + b – 3c – (2a – c)$

$= 3a + b – 3c – 2a + c$

on arranging like terms together

$= 3a - 2a + b - 3c + c$

$= a + b - 2c$

Remember:

The signs of algebraic terms are treated in the same way as the signs of numbers.

Let us see and understand the following examples carefully.

Example: $-(-7) = +7$

$-(-2a) = 2a$

$-(5 - x) = -5 + x$

$-(a - b) = -a + b$

Multiplication of algebraic terms –

The following verse is found in the Brahmasphutasiddhanta regarding the product of algebraic terms.

सदृशद्विवधो वर्गस्त्र्यादिवधस्तद् गतोऽन्यजातिवधः।

अन्योऽन्यवर्णघातो भावितकः पूर्ववच्छेषम्॥

(ब्राह्मस्फुटसिद्धान्त,कुट्टकाध्याय, 42)

Meaning, the multiplication of two identical quantities is the square of that quantity. Multiplication of three identical is a cube of the quantity. The product of same four like quantities is a fourth power of the quantity. If there are different variables,the terms are return as they are in product form.

$$\boldsymbol{x \times x = x^2} \qquad \boldsymbol{x \times x \times y = x^2y}$$
$$\boldsymbol{x \times x \times x = x^3} \qquad \boldsymbol{x \times y \times y \times z = xy^2z}$$

In addition to Brahmasphutasiddhanta, Bijaganitam (Avyaktadi Gunne

Karansutra, 8) has told about the multiplication of the unlike terms.

Example: Multiply 3xy and 2y.

Solution: $(3xy) \times (2y)$

$= 3 \times 2 \times x \times y \times y$

$= 6xy^2$

Example : Multiply (x + y) by (2x + 3y).

Solution: $(x + y)(2x + 3y)$

$= x(2x + 3y) + y(2x + 3y)$

$= (x \times 2x) + (x \times 3y) + (y \times 2x) + (y \times 3y)$

$= 2x^2 + 3xy + 2xy + 3y^2$

(Two like terms are added)

$= 2x^2 + (3xy + 2xy) + 3y^2$

$= 2x^2 + 5xy + 3y^2$

- **Do and learn**

Multiply.

1) $(xy) \times 4xy$

2) $(2x + y) \times (a + b)$

3) $(x^2y)xz$

4) $(3x + y)(5x)$

Exercise 8.1

1. Select the correct option for the following multiple-choice questions.

(a) 8xy is an algebraic expression ……….

(I) Monomial (II) Binomial (III) Trinomial (IV) None

(b) $2x^2 - x + 1$ is an algebraic expression ………..-

(I) Monomial (II) Binomial (III) Trinomial (IV) None

(c) Substituting x=2, the value of the expression x^2-2 will be-

(I) 2 (II) 4 (III) 8 (IV) 10

(d) The numerical coefficient in the expression $\frac{x}{2}$ will be

I) x (II)$\frac{1}{2}$ (III) 7 (IV) 2

(e) Subtracting $3m^2 - n^2$ −4 from $8m^2$+$3n^2$+1 from will give-

(I) $5m^2$+$2n^2$+5 (II) $5m^2$+$4n^2$+5

(III) $5m^2$+$2n^2$+3 (IV) $5m^2 - 2n^2$ −3

2. Classify the following algebraic expressions as monomial, binomial and trinomial.

$4x^2,\ 3a + b,\ 3t^2,\ 4x + z,\ a + b + c,\ 2c + a + x,$

$P + 3x,\ 9x - 2z\ ,\ 2z^2y^2,\ 9x - y + 2,\ 10x^2y^2z,$

$4x - 12y^2z,\ \ 4x\ +\ 3a^2b\ +\ c$

3. What is the coefficient in the given terms?

1) Z in $10x^2y^2z$ 2) xy^2 In $9xy^2z$

3) y^2In$-\frac{8}{7}y^2z$ 4) $5xy$ In y

5) $3x$ In $3xy + 4z$ 6) $3y$ In $4x + 3yz$

7) x^2 In $-4x^2y + z$

4. Match the like terms in the following.

1)	$7a^2b$	:	$15x^2y^2z^2$
2)	$5x^2y^2z^2$	:	$5nm$
3)	$\frac{3p}{q}$	:	$\frac{-7p}{q}$
4)	$2nm$	:	$-11a^2b$
5)	$8xyz$	:	$\frac{4a^2}{b^2}$
6)	$\frac{2a^2}{3b^2}$	:	$2xyz$

5. Find the sum of the following algebraic expressions.

1) $(x + 2y)$ and $(3x + y)$
2) $8a^2t$ and $12a^2t$
3) $(7y + 8z)$ and $(y - 2z)$
4) $8pq$ and $7qp$
5) $(x + 3a^2b)$ and $(x + y + 2a^2b)$

6. Subtract the following algebraic expressions. (subtraction)

1) $(11a^2bc) - (3a^2bc)$
2) $(7pq^2) - (-3pq^2)$
3) $(3x + 3y) - (x + y)$
4) $(3x + 4y) - (x + y)$

7. Multiply the expression in the following pairs.

1) $8xy, 2x$
2) $(4x + y), 5x$
3) $10x, (3x + 2)$
4) $(3x + y), (x + y)$
5) $(a + b), (a + 2b)$
6) $-4x, 2x$

Identities -

An identity is an expression which is true for all values of the variables.

Standard Identities –

In Lilavati Maths, the first step of the following verse is the general method of squaring a number to form the first and second identities.

खण्डद्वयस्याभिहतिर्द्विनिघ्नी तत्खण्डवर्गैक्ययुता कृतिर्वा।

(लीलावती गणित,वर्गे करणसूत्रं वृत्तद्वयम्, 9 (स))

Meaning, divide the number whose square is to be found into two parts, by adding twice the product of those parts to the sum of the squares of the parts, the square of the whole number is obtained. The identity obtained from this can be written in the language of mathematics as follows.

$$\mathbf{(a \pm b)^2 = a^2 + b^2 \pm 2ab}$$

Let us practice the formula of Identities 1 and 2.

$$x \times x = x^2$$

Thus,

$$(a + b) \times (a + b) = (a + b)^2$$

let's discuss the product of (a+ b) (a+ b) or $(a + b)^2$

$$(a + b)^2 = (a + b)(a + b)$$
$$= a(a + b) + b(a + b)$$
$$= a^2 + ab + ba + b^2$$
$$= a^2 + 2ab + b^2 \text{ (because ab = ba)}$$

Therefore $\boldsymbol{(a + b)^2 = a^2 + b^2 + 2ab}$ **(First identity)**

let's discuss about the product in $(a - b)(a - b)$ or $(a - b)^2$.

$$(a - b)^2 = (a - b)(a - b)$$

$= \quad a(a-b) - b(a-b)$

$= \quad a^2 - ab - ba + b^2$

$= \quad a^2 - 2ab + b^2$

Therefore $\boldsymbol{(a-b)^2 = a^2 + b^2 - 2ab}$ **(Second identity)**

From the following verse of Lilavati Mathematics, we get the reference of formation of 3rd identity.

तयोर्योगान्तराहतिः॥3॥

वर्गान्तरं भवेदेवं ज्ञेयं सर्वत्र धीमता।

(लीलावती गणित, तज्ज्ञानाय करणसूत्रं, पृ.174)

Meaning, the difference between the square of the two variables is the product of sum of the variables and difference between the variables. Let us consider the product of $(a+b)\,(a-b)$.

$(a+b)\,(a-b) \quad = \quad a(a-b) + b(a-b)$

$= \quad a^2 - ab + ba - b^2$

$= \quad a^2 - b^2$

Therefore $\boldsymbol{(a+b)\,(a-b) = a^2 - b^2}$ **(Third identity)**

In all the above three identities, the left and right sides are obtained by multiplication. You can verify that for any value the value on both sides of the identity is the same.

Use of identity –

Multiplication is simplified by using identities in the multiplication of binomial expressions and the multiplication of numbers.

Example : Solve with the help of identity 1 [$(\boldsymbol{a}+\boldsymbol{b})^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2 + 2\boldsymbol{ab}$].

1) $(2x+y)^2$ 2) $(102)^2$

Solution 1: $(2x+y)^2$

we know -

$$(\boldsymbol{a}+\boldsymbol{b})^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2 + 2\boldsymbol{ab}$$
$$a = 2x\,,\ b = y$$
$$(2x + y)^2 = (2x)^2 + y^2 + 2 \times 2x \times y$$
$$= 4x^2 + y^2 + 4xy$$

Solution 2: $(102)^2 = (100 + 2)^2$

we know

$$(\boldsymbol{a}+\boldsymbol{b})^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2 + 2\boldsymbol{ab}$$
$$(100+2)^2 = 100^2 + 2^2 + 2 \times 100 \times 2$$
$$= 10000 + 4 + 400$$
$$= 10404$$

Example : Solve using identity 2 [$(\boldsymbol{a}- \boldsymbol{b})^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2 - 2\boldsymbol{ab}$]

1) $(4x - 3y)^2$ 2) $(98)^2$

Solution 1: (4x – 3y)²

we know -

$$(\boldsymbol{a}-\boldsymbol{b})^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2 - 2\boldsymbol{ab}$$
$$a = 4x,\ b = 3y$$
$$(4x - 3y)^2 = (4x)^2 + (3y)^2 - 2 \times 4x \times 3y$$
$$= 16x^2 + 9y^2 - 24xy$$

Solution 2: $(98)^2 = (100-2)^2$

$$(\boldsymbol{a}-\boldsymbol{b})^2 = \boldsymbol{a}^2 + \boldsymbol{b}^2 - 2\boldsymbol{ab}$$
$$= 10000 + 4 - 400$$
$$= 10004 - 400$$

$= \quad 9604$

Example : Solve using identity 3 $[(\boldsymbol{a}+\boldsymbol{b})\ (\boldsymbol{a}-\boldsymbol{b}) = \boldsymbol{a}^2 - \boldsymbol{b}^2]$.

1) $(3x - 4y)\ (3x + 4y)$ 2) $16x^2 - 25y^2$

Solution 1: (3x – 4y) (3x + 4y)

we know -

$$(\boldsymbol{a}+\boldsymbol{b})\ (\boldsymbol{a}-\boldsymbol{b}) \quad = \quad \boldsymbol{a}^2 - \boldsymbol{b}^2$$

$$a = 3x, \quad b = 4y$$

$$(3x - 4y)\ (3x + 4y) = (3x)^2 - (4y)^2$$

$$= 9x^2 - 16y^2$$

Solution 2: $16x^2 - 25y^2$

we know

$$(\boldsymbol{a}+\boldsymbol{b})\ (\boldsymbol{a}-\boldsymbol{b}) = \boldsymbol{a}^2 - \boldsymbol{b}^2$$

$$16x^2 - 25y^2 = (4x)^2 - (5y)^2$$

$$a = 4x, \quad b = 5y$$

$$= (4x + 5y)\ (4x - 5y)$$

- **Do and learn.**

Simplify using appropriate identities.

1) $(3x + y)\ (3x + y)$ or $(3x + y)^2$

2) $(4x - 2)^2$

3) $(3a - b)(3a - b) = (3a - b)^2$

4) $x^2 - y^2$

5) $(5x + 2)\ (5x - 2)$

Exercise 8.2

1. Find the product using the identity.

Using Identity 1 : $(a+b)^2 = a^2+b^2+2ab$

1) $(x+4)(x+4)$ 2) $(3x+2)(3x+2)$

Using Identity 2 : $(a-b)^2 = a^2+b^2-2ab$

3) $(3x-4)(3x-4)$ 4) $(4x-y)(4x-y)$

Using Identity 3 $(a+b)(a-b) = a^2-b^2$

5) $(2x+4)(2x-4)$ 6) $(3y+2)(3y-2)$

2. Evaluate using the identities.

1) $(2+y)^2$ 2) $(P+4)^2$ 3) $(2x+3y)^2$

4) $(2x-y)^2$ 5) $(2p-3)^2$ 6) $(y-3x)^2$

7) 102×98 8) $(104)^2$ 9) $(99)^2$

3. Find the value of 107×93.

- **We learned -**

1) Algebraic expressions consist of variables and constants. To make expressions, the operations of +, -, ×, ÷ are performed on variables and constants.

   For example: (1) 2x+1 (2) 4xy

2) Expressions are made up of terms. Expressions are made by adding or subtracting terms.

3) Factors of variables is called algebraic factors.

4) The coefficient of a term is its numerical factor or sometimes any one factor of the term is called the coefficient of the remainder of the term.

   Example: Coefficient of x y in 10xy is = 10

   Coefficient of x in 10xy is = 10y

5) An algebraic expression with one term is called a monomial, an expression with two terms is called a binomial, and an expression with three terms is called a trinomial.

   Example: Monomial : $4xy,\ a^2b$

   Binomial : (x + y), 4x+2y

   Trinomial : 3x + y + z, a + b + c

6) Those terms in which the algebraic factors are the same are called **like terms** and when they are different, they are called **unlike terms.**

7) An algebraic expression with two like terms is added or subtracted. The unlike terms are written as they are in the final expression.

8) Multiplying a monomial by a monomial gives a monomial

Example: $2x \times 3y = 6xy$

9) The product of two binomials is obtained using the distributive law.

10) An identity is an expression which is true for all values of the variables.

11) Standard Identities -

First $(a+b)^2 = a^2 + 2ab + b^2$

Second $(a-b)^2 = a^2 - 2ab + b^2$

Third $(a+b)(a+b) = a^2 - b^2$

Chapter 9

Comparison of quantities

Percent -

You will recall that proportion means a comparison of two or more similar quantities. If Raman has 4 pens and Asha has 8 pencils, then the ratio of their pens and pencils will be 4 : 8 i.e. 1 : 2 in simple form. We can also represent this as a percentage. Percent means how much out of 100. We can represent 1 out of 2 as 50 out of 100 also.

That is, in $\frac{1}{2}$ the denominator is made 100 by multiplying 50 with both numerator and denominator as shown below.

$$= \frac{(1\times50)}{(2\times50)}$$

$$= \frac{50}{100}$$

Hence, percentage form of $\frac{1}{2}$ is 50%. it is read as 50%.

let us understand the percentage in detail by taking some more such examples.

Example: Out of 20 fruits kept in a basket 14 are mangoes and 6 are apples.

Solution: Compare (ratio) of the number of mangos to the number of apples.

= 14 : 6

or = 7 : 3

Similarly, comparing the number of apples and mangoes, we will get 3 : 7. Let us see this comparison in percentage as well.

Solution: Total number of fruits = 20

Number of apples in 20 fruits = 6.

When we consider all the 20 fruits as 1 fruit than the part of apple in it is

$$= \frac{6}{20}$$

Hence for out of 100 fruits,

$$= \frac{6}{20} \times 100\ \%$$

$$= 6 \times 5\ \%$$

$$= 30\%$$

There are mangoes and apples in the basket. Therefore -

Percentage of Mango + Percentage of Apple = 100%

or percentage of mangoes + 30% = 100%

or Percentage of mango = 100 – 30 = 70%

Hence, the basket contains 70 percent of mangoes and 30 percent of apples.

❖ **Do and learn**

A compass box contains 12 ball pens and 8 pencils. Find, the percentage of the ball pen in the compass box.

Example: In a school of Ved Pratishthan Ujjain, 25% Jamun, 15% Neem and the rest are Peepal plants, have been planted under plantation programme? If the number of plants is 80,

1) What is the number of Jamun plants?
2) What is the number of Neem plants?
3) What is the ratio of Jamun and Neem plants?
4) Find the number of Peepal plants?

Solution: Given - Total number of plants is 80?

1) Number of Jamun plants = 25% of 80

$$= 80 \times \frac{25}{100}$$

$$= 80 \times \frac{1}{4}$$

$$= 20$$

Hence the number of Jamun plants is 20.

2) Number of Neem plants = 15% of 80

$$= 80 \times \frac{15}{100}$$

$$= 80 \times \frac{3}{20}$$

$$= 4 \times 3 = 12$$

Hence the number of neem plants is 12.

3) Ratio of Jamun and Neem plants = 20 : 12

or = 5 : 3

4) Number of Peepal plants = Total no. of plants – (Neem plants + Jamun plants) no. of

$$= 80 - (20 + 12)$$

$$= 80 - 32$$

= 48 plants

Hence the number of Peepal plants is 48.

Exercise 9.1

1. Select the correct option for the following multiple-choice questions.

(a) 50% of 1200 = …………….

(I) 60 (II) 600 (III) 6000 (IV) 120

(b) What percent of 40 is 16?

(I) 20% (II) 40% (III) 60% (IV) 80%

(c) 40% of x = 200 then the value of x is -

(I) 240 (II) 800 (III) 500 (IV) 5

2. Convert the following ratio to percentage.

a) 1 : 3 b) 2 : 5

3. Out of total 70 students in Mouni Baba Ved Pathshala, 20% are Ved Bhushan third year students. how many students study in Ved Bhushan third year.

4. There is total 100 trees in Ganesha's field; out of which 30% trees are shady and the rest are fruit trees, answer the following question: what is the number of fruit trees in Ganesha's field.

5. A group of 50 students of Shukla Yajurveda were asked their favorite sweet. Where 20% students like Gulab Jamun, 10% students like Rasgulla.

   a) What is the number of students who like Gulab Jamun ?

   b) What is the number of students who like Rasgulla ?

Increase percentage or decrease percentage –

You would often get the following information in daily life -

Example: a) 15% decrease on marked price

b) 10% increase in the price of petrol

Example: The cost of a ball is 20. After one year the cost of the ball has increased by 30%, then what is the new cost of the ball ?

Aaradhya says- First of all we will find out the increase in the price which is Rs.20, and add 30% of 20 to it.

20 Rs. 30% of = $20 \times \frac{30}{100}$

= 6

new price = old price + increase amount

= 20 + 6

= 26 Rs.

On the contrary, find the price if it is reduced by 10%.

Reduction in price = 10% of 20

$= 20 \times \frac{10}{100} = 2$

New Price = Old Price – Decrease amount

= 20 – 2

= 18

Finding discount

Generally, when a trader sells an item to his customer, he gives a discount on the marked price, this discount is called discount. Students! You must have heard the word discount from the shopkeeper while shopping goods.

Example: 1) 20% discount on this shirt.

2) A discount of 10% will be given on the marked price of this dhoti.

therefore,

Discount = Marked Price – Selling Price

Note: Discount is always given on marked price only.

Example: Marked priceo of a packet of biscuits is Rs. 50 and it is sold for Rs.40, what will be the discount amount and the discount percent.

Solution: Discount = Marked Price – Selling Price

= 50 – 40 Rs.

= 10 Rs.

Hence, the discount amount Rs. 10.

> **keep in mind** that the discount here is on the marked price, so the amount we get will have to be used as the basis.

For Rs.50, a discount of Rs.10 is offered. To calculate the percentage of discount, we must first determine the cost price, which is Rs. 100.

Thus,

Discount percent = $\frac{10}{50} \times 100$

Discount percent = 20 %

❖ **Do and learn**

If the marked price of an article is Rs.60. and selling price is Rs.40. find the discount and discount percent.

Example: A kurta costs Rs.400 at retail. Find the amount of the discount and the selling price of the kurta if the trader offers a 20% discount.

Solution: Given: Marked price = Rs.400.

Amount of discount = 20% of marked price

= 400× 20%

$= 400 \times \frac{20}{100}$

= Rs 80

Selling price = Marked price – Discount

= 400 – 80

= Rs 320

Hence the amount of discount on kurta is Rs.80. and selling price is Rs.320.

Example: A dupatta costs Rs.400 after a 20% discount is applied to the marked price. Find the dupatta's marked price.

Solution: Selling price = Rs.400.

Since 20% discount means Rs.20 discount on marked price of Rs.100, selling price = Rs.100 – 20 = Rs.80

If the selling price is Rs.80, the marked price of the dupatta is Rs.100.

Therefore,

the selling price is Rs.1, then the marked price of the dupatta = Rs. $\frac{100}{80}$.

Therefore,

the selling price is Rs.400, then the marked price of the dupatta is

$$= \frac{100}{80} \times 400$$

= Rs 500

Hence, the marked price of the dupatta = Rs.500.

Exercise 9.2

1. Select the correct option for the following multiple-choice questions.

(a) If a chair with a marked price of Rs.350 is available for Rs.300, what will be the amount of the discount?

(I) 20 (II) 30 (III) 40 (IV) 50

(b) 10% of the tomatoes in 250 kg were rotten and had to be removed. How much tomato remains?

(I) 240 kg (II) 225 kg (III) 200 kg (IV) 275 kg

(c) Due to the rainy season's depression, the price of a garment available for Rs.500 is reduced by 30%, and the new price is -

(I) 300 (II) 350 (III) 410 (IV) 260

2. If the following marked price and selling price are given on an article, find the discount amount of the following?

a) Marked price = Rs.500., selling price = Rs.450.

b) Marked price = Rs.600., selling price = Rs.480.

c) Marked price = Rs.750., selling price = Rs.710.

D) Marked price = Rs.890., selling price = Rs.780.

3. The marked price of an article is Rs.1500. And the customer got for Rs.1000. What will be the discount and the discount percentage?

4. The cost of a toothpaste is Rs.60. What will be the new price of toothpaste if its price is increased by 20%?

5. A set of soap Rs.80. If the price of the soap set is reduced by 20%. Then what will be the new price of the soap set?

6. At a discount of 30% on the marked price, a set of greeting cards is sold at Rs.140. What will be the marked price of the set of five such cards.

Remember points:

Principal amount:

When a person borrows some money from a moneylender or moneylender or a bank, the money borrowed is called the principal. It is denoted by 'p'.

Interest Amount :

When the borrowed money is returned after a certain time, some more money has to be paid in addition to the principal, this extra money is called interest amount. which is denoted by 'I'.

Compound Interest:

When the interest is calculated at regular intervals and added to the principle (P) for further calculation, the difference between final amount and the principal amount is called compound interest. The final amount is called compound amount. And it is denoted by 'A'.

Time :

The period for which money is borrowed is called time. It is denoted by ‘t’. Time is calculated only in years.

Rate of Interest :

It is the percentage at which the interest is calculated per annum. It is denoted by 'r'.

Simple Interest :

When interest is charged only on the principal at the same rate for a fixed time period, then it is called simple interest. It is denoted by 'S.I'.

Formula: Simple Interest = $\frac{(\text{Principal} \times \text{Rate} \times \text{Time})}{100}$

or $S.I. = \frac{p \times r \times t}{100}$ **Or** $S.I. = p \times r\,\% \times t$

where **Amount = Principal + Interest**

or A = P + SI

For example: Rajesh borrows Rs.1200 at the rate of 3% if he wants to repay the amount in 2 years and 6 months. How much interest will he has to pay ?

Rajesh goes to Guruji–

Guruji, how will the interest be calculated for 2 years 6 months ?

Guruji - Convert the time into years, there are 12 months in a year.

To convert months to years, divide by 12.

Borrowed money (Principal) = Rs.1200.

Rate of interest = 3% per annum

time = 2 years 6 months

= 2 years + $\frac{6}{12}$ years

= $\left(2 + \frac{1}{2}\right)$ years or $\frac{5}{2}$ years

Simple Interest = $\frac{(\text{Principal} \times \text{Rate} \times \text{Time})}{100}$

$= 1200 \times \frac{3}{100} \times \frac{5}{2}$

= Rs.90

Amount = principal + interest

= 1200 + 90

= 1290 Rs.

Example: For the principle of Rs. 2500, what will be the simple interest for 4 years at the rate of 5% per annum?

Solution: Given: Principal = Rs.2500., rate = 5%, time = 4 years

Then we know that –

$$\textbf{Simple Interest} = \frac{(\textbf{Principal} \times \textbf{Rate} \times \textbf{Time})}{\textbf{100}}$$

$$= \frac{2500 \times 5 \times 4}{100}$$

$$= \text{Rs. } 500$$

Do and learn

Find the simple interest.

If principal = Rs.500, Rate = 5% per annum, Time = 1 year 6 months.

Compound interest -

The difference between the final amount and the principle, in which interest is calculated and added to the principle at regular intervals for further calculation is called compound interest. Compound interest, is denoted by 'C.I.'.

Formula to calculate compound interest:

- **Compound Amount** $= \text{Principal}\left(1 + \frac{\text{Rate}}{100}\right)^n$

  or $A = P\left(1 + \frac{R}{100}\right)^n$

- **Compound Interest** $= \text{Principal}\left(1 + \frac{\text{Rate}}{100}\right)^n - \text{Principal}$

  or $CI = P\left(1 + \frac{R}{100}\right)^n - P$

Where, A = Amount, P = Principal, R = Rate, n = Time

Example: Find the total amount to be paid on Rs. 10000 at the rate of 2% per annum for 1 year, if the interest compounded annually.

Solution: Given - Principal = Rs.10000., rate = 2%, time = 1 year

we know,

$$\textbf{Amount} = \mathbf{P\left(1+\frac{R}{100}\right)^{n}}$$

$$= 10000\left(1+\frac{2}{100}\right)^{1}$$

$$= 10000\left(\frac{100+2}{100}\right)^{1}$$

$$= 10000\left(\frac{102}{100}\right)$$

$$= 10200 \text{ Rs.}$$

Hence, after 1-year Rs 10200 has to be returned.

Example: What is the compound interest at 3% per annum if 20000 Rs. is borrowed at compound interest for two years?

Solution: Given - Principal = 20000, Rate = 3% per annum, Time = 2 years

$$\textbf{Amount} = \mathbf{P\left(1+\frac{R}{100}\right)^{n}}$$

$$= 20000\left(1+\frac{3}{100}\right)^{2}$$

$$= 20000\left(\frac{100+3}{100}\right)^{2}$$

$$= 20000\left(\frac{103}{100}\right)^{2}$$

$$= 20000\times\frac{103}{100}\times\frac{103}{100}$$

$$= \text{Rs.}21218$$

compound interest = total amount – principal

CI = A – P

= 21218 – 20000 = Rs.1218

EXERCISE 9.3

1. Find the simple interest.

   1) Principal = 1500, Rate = 1%, Time = 2 years 6 months

   2) Principal = 2000, Rate = 2%, Time = 1 year 6 months

   3) Principal = 3000, Rate = 3%, Time = 3 years 6 months

   4) Principal = 10000, Rate = 4%, Time = 2 years

   5) Principal = 800, Rate = 3%, Time = 5 years

2. Find the amount of money when taken at compound interest.

   1) Principal = 50000, Rate = 2%, Time = 1 year

   2) Principal = 40000, Rate = 3%, Time = 2 years

   3) Principal = 60000, Rate = 3%, Time = 1 year

   4) Principal = 80000, Rate = 4%, Time = 2 years

3. Find the compound interest.

   1) Principal = 20000, Rate = 3%, Time = 1 year

   2) Principal = 40000, Rate = 4%, Time = 2 years

4. Find the total amount and interest for Rs.30000 compounded at 5% per year for 3 years.

5. Mahesh borrowed $70,000 at 4% annual compound interest for three years. what will be the amount of compound interest ?

- **We learned -**

1) Discount given on marked price is called discount.

   Discount amount = Marked Price – Selling Price

2) Percent discount = $\frac{\text{Discount amount}}{\text{Marked Price}} \times 100$

3) Simple Interest = $\frac{(\text{Principal} \times \text{Rate} \times \text{Time})}{100}$

   Total Amount = principal + interest

4) When interest is calculated on this amount by adding the interest accumulated from time to time to the principal, then it is called compound interest.

5) The total amount when the interest is compounded annually

$$A = P\left(1 + \frac{R}{100}\right)^n$$

where P = principal, R = rate, n = time in years

$$\text{Compound interest} = P\left(1 + \frac{R}{100}\right)^n - P$$

compound interest (CI) = A – P

Chapter 10

Direct and inverse proportion

We can observe many situations in daily life, where we need to know the relation between the varying quantities, such as due to change in one quantity there is also change in the other.

Example :

1) The amount of money deposited in your account and amount of money accrued in it.
2) It takes 15 days to complete a piece of work. If the number of people working is increased, then the work will be completed in less time.
3) Greater the number of items purchased, greater is the total amount to be paid.
4) The more items of the same value, the higher their value will be.
5) The more money you deposit in the bank account for a longer period, the more will be the interest.

Remember: -

Situation in which, on changing one quantity, the other quantity also changes is called variations. This can be either direct or inverse variations. To answer these types of questions, we will study some variation including the concept of direct and inverse proportion.

Direct proportion -

Variations are situations in which when one quantity changes, the other quantity also changes. Variations can be either direct or inverse. To answer these kinds of questions, we'll look at some variations, such as the concepts of direct and inverse proportion.

If there is a direct relationship between two quantities, we can use the following Aryabhatiyam verses to calculate the value of the unknown quantities.

त्रैराशिकफलराशिं तमथेच्छाराशिना हतं कृत्वा ।

लब्धं प्रमाणभाजितं तस्मादिच्छाफलमिदं स्यात् ॥

(आर्यभट्टीयम्, गणितपाद : 26)

Meaning, by multiplying the 2nd known quantity by 1st desired quantity and dividing by the 1st known quantity, we get the 2nd desired quantity.

$$requisition\ (ichhaphala) = \frac{\text{fruit (phala)} \times requisition(\ ichharashi)}{argument(pramana)}$$

In addition to Aryabhatiyam, the Lilavati math composed by Bhaskaracharya has also been described as the Trirashike Karanasutram.

Example: The work of painting is going on in the school for Diwali. The labourers painted 15 rooms in 3 days, find the number of days required by them to paint 20 rooms of the school.

Abhishek – Calculate the time required to paint 20 rooms if 15 rooms are painted in three days.

Ankit – **Number of days :: Number of rooms Painting rooms**

Number of days	Number of rooms
3	15
x	20

In the above, the number of days and the number of rooms painted are proportional to each other. They are in direct proportion as the number of rooms to be painted increases, the number of days required also increases.

3 : x :: 15 : 20

Product of outer terms = Product of inner terms

$$3 \times 20 = 15 \times x$$

$$\frac{3 \times 20}{15} = x$$

Or $x = 4$

Abhishek - Yes it is correct. 20 rooms will be painted in 4 days.

Ayush - Is there any relation between the number of workers and the time taken to complete the work ?

Abhishek – I think that if there are more labourers, then the work will be completed soon. Let's ask Guruji.

Ayush – Guruji, is there a direct relation between the number of days and the number of laborers engaged in the completion of the work?

Guruji - Yes, there is a relation.

The relationship between two variables is such that increasing one variable increases the other variable, on increasing one variable the other also increases, or on decreasing the other also decreases, is called direct relation.

But sometimes when one variable increases, the second variable decreases and when the first variable decreases, the second variable increases. This relation is called inverse relation.

❖ Do and learn

Read the following statements to find out whether it is a direct/inverse relationship.

S.no.	Statement	Relationship (direct / inverse)
1)	The cost of 3 meters of cloth for a kurta is Rs.300 Find the price of 5 meter of such cloth and the relation between the cloth for the kurta and its price?	....
2)	In a piece of work, 2 workers complete the piece of work in 20 days.Is there a relation between the number of workers and the number of days in which the work is completed?	....
3)	What is the relationship between the amount of water in the tank and the time taken to empty it?	....

Example: If the cost of four pens is Rs.20, the total cost is Rs.40. How many pens can be purchased?

Solution: Let the number of pens bought for Rs. 40 be x.

The number of pens and the total cost of the pens have a direct relationship.

pen number	**::**	**pen Did worth**
↑ 4		20 ↑
x		40

4 : x :: 20 : 40

product of outer terms = product of middle terms

$$4 \times 40 = 20 \times x$$

$$\frac{4 \times 40}{20} = x$$

or $x = 8$

So Rs.40, 8 pen will be purchased.

Example: If a passenger train travels 50 kilometers in 20 minutes at a constant speed, how long will it take to travel 100 kilometers?

Solution: Let the train cover a distance of 100 km in x minutes.

The distance traveled and the time taken have a direct relationship, which means that as the distance increases, so does the time taken by the train.

Ratio of distance covered by train :: Ratio of times taken

50 : 100 :: 20 : x

product of outer terms = product of middle terms

$$50 \times x = 20 \times 100$$

$$x = \frac{20 \times 100}{50}$$

Or $x = 40$ minutes

Hence the train will cover a distance of 100 km in 40 minutes.

EXERCISE 10.1

1. If 7 kg of tomatoes cost Rs 35, how much should we pay for 10 kg of tomatoes?
2. If a quintal of soyabean costs Rs 6,000, how many quintals can be purchased for Rs 4,000?
3. If the cost of two mobile phones is Rs 10,000, how much money will be required to purchase five such phones?
4. If 2 kg of sugar costs Rs.80, how many kg of sugar can be purchased for Rs.160?
5. A car covers a distance of 100 km in 2 liters of petrol, how many kilometers will the car cover in 5 liters of petrol?
6. If the cost of 16 pencils is Rs 64, how many pencils can be bought for Rs 144?

Inverse relationship

In Lilavati Mathematics, the questions based on inverse relation have been mentioned in the following verses to know the answers.

इच्छावृद्धौ फले ह्रासो ह्रासे वृद्धिः फलस्य तु।

व्यस्तं त्रैराशिकं तत्र ज्ञेयं गणितकोविदैः ।।

(लीलावती गणित, उदाहरण 102)

Meaning: out of two quantities, if one increase as and the other decreases or one decreases and the other increase, it is required of us to know the relationship between them.

When one of the quantities increases/decreases and the other quantity decreases/increases respectively, they are in inverse proportion.

In this, one variable is said to be inversely proportional to the other variable.

For example: It takes 15 days for 5 men to complete a piece of work, in how many days will 10 men complete the same work?

After reading the preceding question, you may have concluded that by increasing the number of people, the same work could be completed in fewer days. That is, when one variable is increased, the other variable decreases. As a result, the two variables have an inverse relationship.

Example: In 8 days, 4 laborers complete the task of weeding a field. How many days will it take to complete the same work if 16 laborers do it?

Solution: The number of workers and the time required to complete the weeding job have an inverse relationship.

Number of workers ↑	Time taken to complete the work (in days) ↓
4	8
16	x

Let the number of days required to do the work be x.

$$16 : 4 :: 8 : x$$

$$\frac{16}{4} = \frac{8}{x}$$

Product of outer terms = product of middle terms

$$16 \times x = 4 \times 8$$

$$x = \frac{4 \times 8}{16}$$

Or $x = \frac{32}{16}$

Hence, it will take 2 days to complete the work by 16 workers.

Example: A tank can be filled in 6 hours by two identical pipes. How long will it take to fill the same tank with 3 such pipes ?

Solution: Let the time taken (hours) to fill the tank be x.

By increasing the number of pipes, it will take less time to fill the water tank. So it is a situation of an inverse relationship.

Number of Pipes ↑	::	Time ↓
2		6
3		x

Let it take x time (hours) to fill the tank here.

$3 : 2 \quad :: \quad 6 : x$

$\frac{3}{2} = \frac{6}{x}$

Product of outer terms = product of middle terms

$3 \times x = 2 \times 6$

$x = \frac{2 \times 6}{3}$

or $x = 4$

Hence, it will take 4 hours to fill the tank by3 pipes.

EXERCISE 10.2

1. A car covers a distance of 20 km/hr to reach a place, say 60 km. How much time will it take?
2. If there are 100 students in a hostel and their food supplies are sufficient for 20 days, how many days will this food item last if 25 more students join the group (total 125)?
3. A cowshed contains enough food for 20 cows for 6 days. How many days will the food last if 10 more cows arrive at this cowshed?
4. It takes five men 15 days to complete a piece of work. How long will it take 15 men to complete the same task?
5. In India, 15 volunteers can clean their village in four days; how many volunteers are needed if the same village is to be cleaned in three days?

We learned -

1) When two quantities are related in such a way that when the value of one increases or decreases in the same proportion, the value of the other also increases or decreases in the same proportion, they are said to be **directly proportional**.
2) An inverse relation or inversely proportional relationship exists when two quantities are related in such a way that increasing or decreasing the value of one causes the value of the other to decrease or increase in the **inverse proportion.**

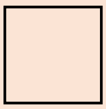
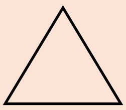
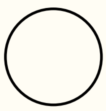
Chapter 11

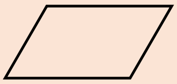
Mensuration

Dear Students! We've learned that the distance around the boundary of a closed plane figure (two-dimensional figure) is referred to as its perimeter, and the region encircled by that figure is referred to as its area. We learned how to calculate the perimeter and area of various plane figures in two dimensions, such as triangles, squares, and rectangles. In this chapter, we will learn how to calculate the surface area of a cube, cuboid, and cylinder, three-dimensional solid figures.

In Manaw Shulb sutra10.2.5.5. And in 10.3.2.11 the method of finding the area has been explained.

You may recall from a previous class the formula for the area of a two-dimensional figure.

Sl.	shape	size	Area
1)		square	side × side $a \times a$
2)		rectangle	length × width $\ell \times b$
3)		triangle	$\frac{1}{2} \times$ base $\times$ height $\frac{1}{2} \times b \times h$
4)		circle	π (radius)^2 πr^2

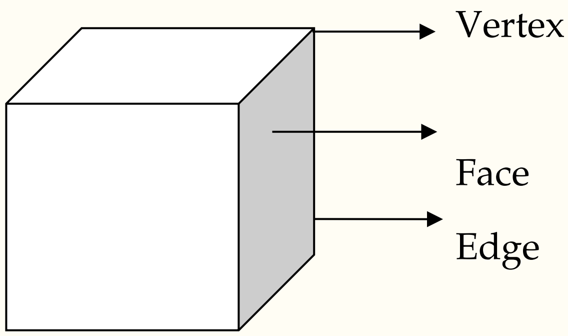
5)		parallelogram	base × height $b \times h$

Recall the formulas for the perimeter of the figures mentioned in the table.

Face, Edge and Vertices:

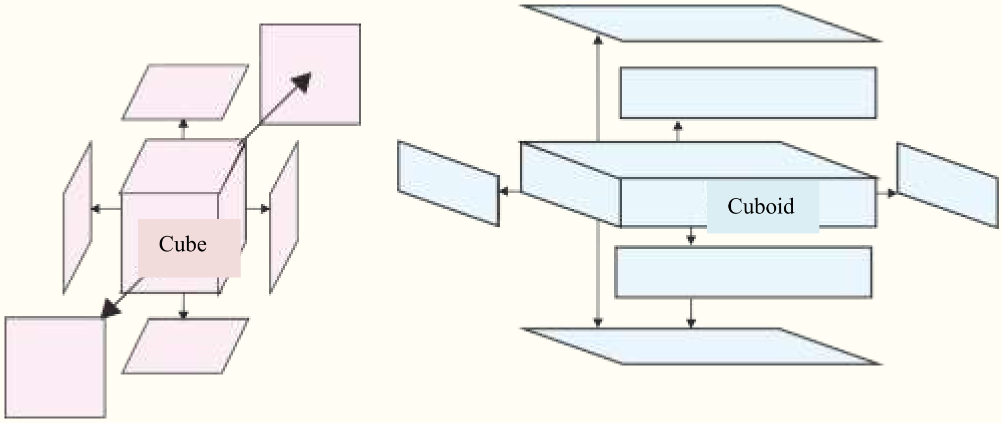
Every three-dimensional object has length, breadth and height. The solid shapes can we classified based on their faces, edges and vertices. Vertices are the corners; edges are line segments joining any two adjacent vertices and faces are the surfaces joining edges.

Number of Faces, edges and vertices are unique for any given solid shape. WE can easily identify the faces, edges, vertices of solid shapes.



You must have seen that in some shapes two or more faces or surfaces are identical (congruent). All the three-dimensional shapes can be ripped open such that their faces can be laid on a two-dimensional plane called nets of solid shape.

Let us understand nets of solid shape as given below:



Identify the congruent faces in t he following figures.

Surface area of cube and cuboid -

Bhola and Atharva made the shape of a cube and cuboid respectively out of card board to keep their Veda book and other books on which they want to paste colored paper. Bhola prepared a cube of 40 cm edge, and Atharva prepared a cuboid of length 30 cm, breadth of 40 cm and height of 5o cm. They are in a problem that how much paper should be brought from the market to cover the cube and cuboid.

Discussion between Bhola and Atharv -

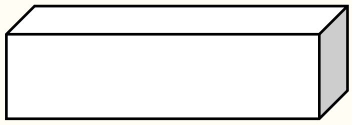
Bhola - Why don't we calculate the area of each face separately and add them?

Atharva – Bhaiyya (Bhola) This is called surface area.

Guru ji - Very well discussed!

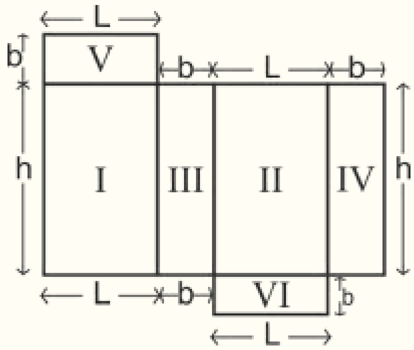
The area enclosed by the entire surface of a solid is called its surface area.

Surface Area of cuboid –



cuboid

On cutting open the cuboidal box made by Atharva, the picture shown below is obtained. You see that in the cuboid we get three pairs of two congruent faces each. Where length is denoted by *l*, width *b* and height *h*.



Area of cuboid=Area 1+ Area 2+ Area 3+ Area 4+ Area 5+ Area 6

$$= h \times l + h \times l + b \times h + b \times h + l \times b + l \times b$$
$$= 2hl + 2bh + 2lb$$

Total surface area of cuboid = 2 (lb + bh + hl)

Where *l*, *b* and *h* are the length, width and height respectively.

the total surface area of the cuboid formed by Atharva

$= 2(30 \times 40 + 40 \times 50 + 50 \times 30)$ Sq. cm

$= 2(1200 + 2000 + 1500)$

$= 2\ (4700)$

$= 9400$ sq cm

For painting, all around the room, excluding the floor (floor) and the ceiling, we paint only the area of the four walls of the room.it is called the lateral surface area of the cuboid.

lateral surface area of cuboid = total surface area – roof and floor area

$$= 2 (lb + bh + hl) – 2lb$$

$$= 2bh + 2lh$$

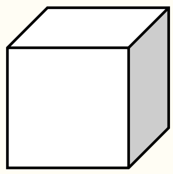
Lateral surface area of cuboid = $2 \times (b + l) \times h$

or

lateral surface area of cuboid = perimeter of rectangle (floor) × height

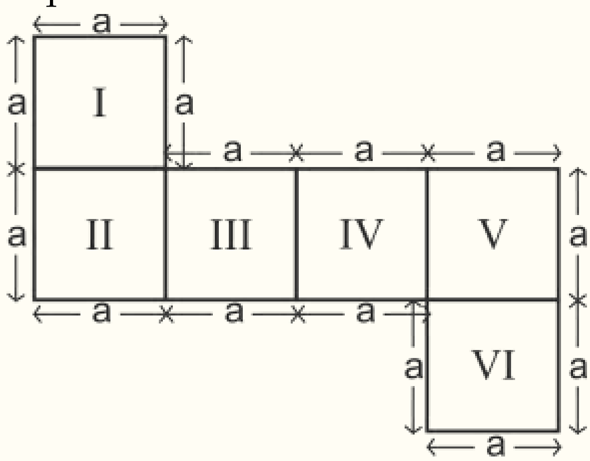
Here the area of roof and floor is independent of height and depends only on length and width.

Total surface area of cube :



cube

You know that a cube is a three-dimensional shape of a square, that is, all sides are equal. There are six faces in a cube.



Area of one face of a cube = side × side = $(side)^2$ and all the six faces are square, each side is denoted by a.

Thus, **total surface area of the cube** = 6 × area of a square face

Thus,

Total surface area of the cube made by Bhola = $6\times (\text{side})^2$

$= 6 \times (40)^2$ (side = 40 cm)

$= 6 \times 1600$

$= 9600$ sq cm

Example: Shubham packed a gift for his brother on his birthday in the shape of a cubical box. Find the area of the paper required to cover it, it its side is 15 cm.

Solution: Given: One side of the cube (a) = 15cm.

we know,

Total surface area of cube = $6a^2$

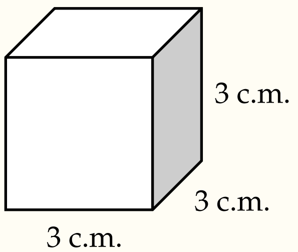
$= 6\times (15)^2$ (a = 15 cm)

$= 6\times 225 = 1350$ sq. cm.

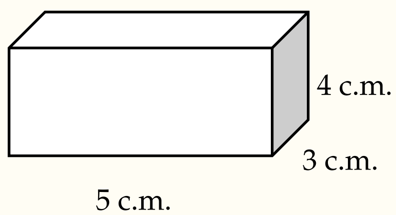
Therefore, the area of colored paper required is 1350 square cm.

Example: Find the surface area of the following figures.

1)



2)



Solution: 1) The shape of the given figure is a cube.

Given – Side of the cube = 3 cm.

we know

Surface area of cube $= \boldsymbol{6a^2}$

$= 6 \times (3)^2$

$= 6 \times 9 = 54$ sq. cm.

2) The shape of the given figure is a cuboid.

Given - Length of the cuboid (l) = 3 cm., breadth (b) = 5 cm.

height (c) = 4 cm.

We know,

Total surface area of cuboid = 2 ($lb + bh + hl$)

= 2 (3×5+5×4+4×3)

= 2 (15+20+12)

= 2 (47)

= 94 sq cm

Example: There is a cuboid shaped suitcase whose length is 30 cm., width 40 cm and height 20 cm. If the suitcase is to be covered with tarpaulin or cloth from above, how much clothes will be needed.

Solution: Given

The length of the suitcase l = 30 cm., breadth b = 40 cm., height h = 20 cm.

Thus, **surface area of suitcase** = 2 ($lb + bh + hl$)

= 2 (30×40+40×20+20×30)

= 2(1200 + 800 + 600)

= 2 (2600)

= 5200 sq. cm.

5200 sq. cm of cloth is required to cover the suitcase.

Example: A classroom is of cuboid shape where the length of the room is 5 m., width 6 m. And height 4 m. If the cost of painting the four walls of that room is Rs 300. per square metre, then what would be the cost of painting the four walls of that room.

Solution: Given- Length of the room = 5 m., breadth = 6 m., height = 4 m.

we know

Surface area of four walls of the room

= Perimeter of the rectangle (base) × Height of the room

$= 2(l + b) \times h$

$= 2(5 + 6) \times 4$

$= 2(11) \times 4$

$= 22 \times 4 = 88$ square meters

If the cost of painting per square meter = Rs.300

therefore,Cost of painting all the four walls of the room = Rs.88 ×300.

= 26400 Rs.

Exercise 11.1

1. Fill in the blanks by writing the following formulas.

(a) Total surface area of the cuboid = ...

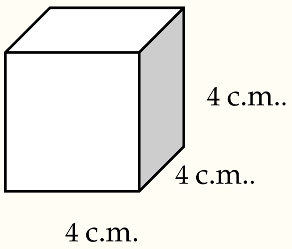
(b) Lateral surface area of cuboid = ...

(a) Area of one face of the cube =...............................

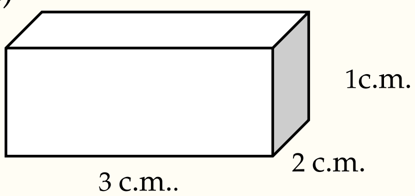
(b) Total surface area of the cube = ...

2. Find the surface area of the following figures.

1)



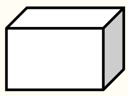
2)



3. Find the surface area of a cube.

Of side 2 cm.

2 cm



4. Rahul painted outside of the box of dimensions 2 m. × 3 m × 4 m. find the surface area painted.

5. A soap box contains 10 soaps of the box is 10 cm. × 20 cm × 10 cm then find the surface area of the soap box.

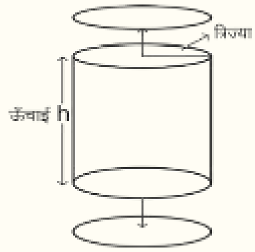
6. The internal measurement of Ganesh's room is 12 m., 10 m. and 7 m., find the total surface area of the room. If he wants to paint the four walls of his room Rupees 20 per square metre, find the cost involved in it.

7. Find the surface area of a cuboid whose length is 3 m., width 10 m. And height 4 m.

8. Find the side of a cube whose surface area is 600 cm^2.

Surface area of cylinder

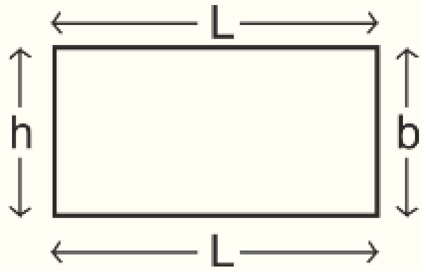
A pipe, circular pole, tubelight etc. are examples of right circular cylinder. In a right circular cylinder, the line segment joining the centers of the circular faces must be perpendicular to the base.

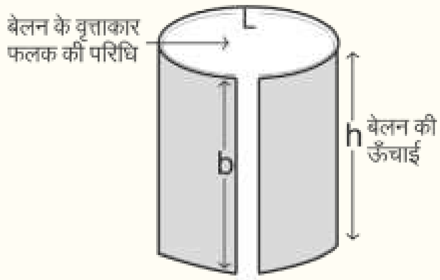


The remaining surface except the two circular faces of the cylinder is called curved surface. Let us find the formula for the curved surface area of a cylinder by an activity.

Procedure: -

Take a rectangular paper, mark its length and width, fold the ends according to the pictures and paste them with the help of overlapping tape. Thus the curved surface of a cylinder will be formed.





One of the sides of the rectangular paper is defined as the circular base circumference ($2\pi r$) of the cylinder and the width (b) is defined as the height (h) of the cylinder.

Thus, curved surface area of the cylinder = area of the rectangular paper

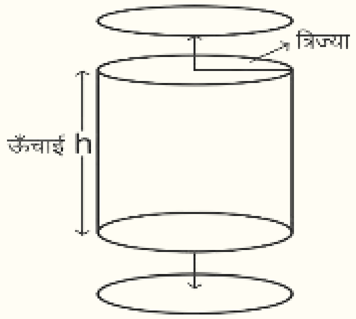
$$= l \times b \qquad (l = 2\pi r \quad b = h)$$

$$= 2\,\pi\, r \times h$$

$$= 2\pi r h$$

Curved surface area of cylinder = 2 π rh square units

If both the circular faces of the cylinder are included in the curved surface of the cylinder, then it is called the complete surface of the cylinder.



total surface area of cylinder

= curved surface area of the cylinder + area of both the circular faces

= $2\pi rh + 2\pi r^2$

= $2 \pi r (h + r)$

Total surface area of cylinder = 2 π r (h + r) square units

where $\pi = \frac{22}{7}$

Example: Find the total surface area of a cylinder whose radius is 14 cm. and height 7 cm. be (where $\pi = \frac{22}{7}$)

Solution: Given, radius of the cylinder = 14 cm, height of cylinder = 7 cm

we know,

Total surface area of the cylinder = 2 π r (r + h)

$= 2 \times \pi \times 14 (7 + 14)$

$= 2 \times \frac{22}{7} \times 14 (21)$

$= 2 \times \frac{22}{7} \times 14 \times 21$

= 1848 sq cm

Therefore, the total surface area of the cylinder is 1848 sq. cm.

Example: Find the curved surface of a hollow pipe on a bridge. Whose radius is 2 m. And height 21 m.

Solution: Given- Radius of hollow pipe = 2 m., height = 21 m.

Then we know, **Curved surface area of hollow pipe (cylinder) = 2 π rh**

$$= 2 \times \frac{22}{7} \times 2 \times 21$$

= 264 square meters

Hence the curved surface of the hollow pipe is 264 square metres.

Exercise 11.2

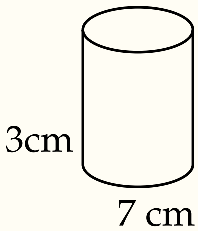
1. Fill in the blanks by writing the following formulas.

(a) Curved surface area of cylinder =..............................

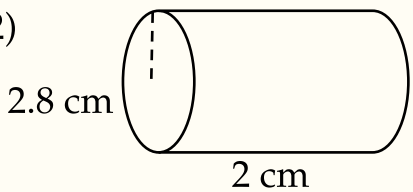
(b) Total surface area of the cylinder =..............................

2. Find the curved surface area of the following cylindrical figures.

(1)



(2)

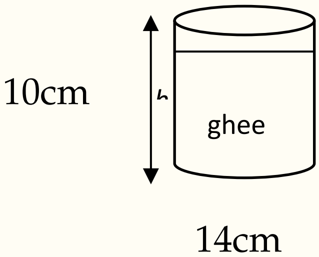


3. What will be the curved surface of that cylinder? Whose height is 14 cm and radius is 4 cm.

4. Find the total surface area of the cylinder. Whose radius is 21 cm. and height is 2 cm.

5. A company packs its pure ghee in such a cylindrical shape. Whose diameter is 14 cm. and height 10 cm. The company puts a label around the pot. (as shown in the figure) If the label is 1 cm from both the oil and top of the vessel. Find the area of the label if it is pasted at a distance of

(**Hint** - The height of the label left on both sides of the vessel has to be subtracted from the height of the label given for the height)



6. A cylinder has a diameter of 14 cm and a height of 5 cm. Determine the curved surface area and total surface area of the cylinder.

7. Find the height of a cylinder whose radius is 7 cm. and total surface area is 968 cm^2.

- **We learned -**

1) The surface area of a solid figure remains equal to the sum of the areas of the faces of that figure.

2) Surface area of cuboid = $2(lb + bh + hl)$

   (where l = length, b = width and h = height)

3) Surface area of cube = $6a^2$

   (where a = of the cube)

4) Curved surface area of cylinder = $2\pi rh$

   where r = radius of the base of the cylinder

   h = height of cylinder

   $\pi = \frac{22}{7}$

5) Total surface area of the cylinder = curved surface area of the cylinder + area of both the circular parts.

$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r(h + r)$$

Introduction and contribution of Indian mathematicians

❖ Brahmagupta -

Brahmagupta was a world famous mathematician. He was born in the year 598 AD in a village named Bhinmal situated between Ayu mountain and river Luni in the state of Rajasthan. His father's name was Jishnu, he wrote books named 'Brahmasphutasiddhanta' and 'Khandakhadyak' on the basis of ancient 'Brahmapitamahsiddhanta'. Is another book is also known as 'Dhyanagrahopadesh'. Brahmagupta composed a book named 'Brahmasphutasiddhanta' at the young age of 30 years. There is total 221 chapters in this book. Two chapters of the entire book is deouted to mathematical subjects and the

19 chapters, or on astrology related subjects. The 12th chapter of this book is famously known as 'Ganitadhyay'. Translation of this book was done by T. Olebrooke in English, which was published from London in 1927. In his book, Brahmagupta has given the main rules related to mathematics such as addition, subtraction, multiplication, division, square and square root, cube and cube root, addition of fractions, subtraction, etc., Trirashik rules, inverse Trirashik, Bhaand, Pratibandh, Mishrak rule explain the concept triangle, quadrilateral etc. Chiti Vyavahar discusses (method of finding the volume of a sloped trench), Khat Vyavahar explains (calculation of the area of the trench), Krakchik Vyavahar is useful mathematics for saw-millers, Rashi Vyavahar is, method of knowing the volume of a heap of grain, Shadow behavior (Question related to lamp pillar and its shadow) etc. Acharya Brahmagupta has also described 'Ganak', 8 vyavhar and 20 activities of

mathematics.

20 Recurring - Compiled, subtracted, generated, division, square root, cube, cube root, Bhagjaati, Prabhagjaati, Bhaganubandha caste, Bhagapawah caste, Bhagamata caste Trirashik, Busy Trirashik, Pancharashik, Saptarashik, Navarashik, Ekadasharashik and Bhandapratibhaand.

vyavhar -Mixtures, Shredhi, Kshetra, Khat, Chiti, Krakchik rashik and Chaya.

The 18th chapter of 'Brahmasphutasiddhanta' is known as 'Kutakadhyaya'. It has 103 verses. Indeterminate equation is a part of algebra. Brahmagupta has described many methods of solving equations. Brahmagupta gave the theory of linear equation-

अव्यक्तान्तरभक्तं व्यस्तं रूपान्तरं समेऽव्यक्तः ।
वर्गाव्यक्ता शोध्याः यस्मात् रूपाणि तदधस्तात् ।।

(ब्राह्मस्फुटसिद्धान्त, 18.43)

वर्गचतुर्गुणितानां रूपाणां मध्यवर्गसहितानाम् ।
मूलं मध्येनोनं वर्गद्विगुणोद्धृतं मध्यः ।।

(ब्राह्मस्फुटसिद्धान्त, 18.44)

❖ **Awadhesh Narayan Singh (1901-1954):**

Indian mathematician Awadhesh Narayan Singh studied mathematics at the University of Lucknow and then joined the Mathematics Department of the University of Calcutta. He died in 1954. At the time of his death, he was working on the completion of the third volume of Dutt and Singh, which included the history of geometry, trigonometry and calculus, as well as Maya classes, series and sequences, set and combination theory, and other topics. which has been mentioned in the Preamble.

when the book was reprinted in 1961, the third volume was not included. It seems that before his death in 1958, Swami Vidyaranya gave the manuscript of the third volume to Kripa Shankar. A copy was published in 1979 by S.C. Gupta. N. Singh who is the son of Awadhesh Narayan Singh. Shukla later published a revised version of the third volume as a series of seven articles in the Indian Journal of History of Science during 1980–1994.

- **Ram Varma Maruthampuran**

1948 with a detailed mathematical explanation, which laid the groundwork for all subsequent works on Yuktibhashya (Malyalam). The Indian National Commission for the History of Science was constituted by the Indian National Science Academy in 1965. The commission actively promotes research on the history of science in India. In 1971, the Academy published A Concise History of Science. It is a landmark publication in this field by D. M. Bose, S. N. Sen and B. B. Edited by Subbarappa. In 2009, Su Babarappa published a revised edition with substantial additions. In 1966, the Academy started publishing the Indian Journal of History of the Science which has been very successful and is counted among the leading journals of India.

आदर्श प्रश्नपत्र/ Model Que. Paper / गणित /
वेदभूषण तृतीय-वर्ष / Vedabhushan Third Year/
कक्षा 8वीं / प्रथमा - III / Class 8th / Prathama - III
विषय - गणित

पूर्णांक – 100 **समय – 3 घण्टे**

प्रश्न - 01. सही विकल्प के सामने (✓) चिह्न लगाइए – **10 × 2 = 20**

Questions - 01. Put a (✓) mark against the correct option -

1. निम्न में अभाज्य संख्या के समूह को पहचान कर सही विकल्प का चयन करें-

   Identify the set of prime numbers in the following and select the correct option -

   (i) 3 एवं 5 / 3 and 5 (ii) 9 एवं 15 / 9 and 15 (iii) 11 एवं 15 / 11 and 15 (iv) 17 एवं 19 / 17 and 19

   (अ) केवल (i) / Only (i) (आ) केवल (iv) / Only (iv)

   (इ) दोनों (i) एवं (iv) / Both (i) and (iv) (ई) केवल (ii) / Only (ii)

2. निम्न में से सत्य नहीं है।

   Which of the following is not true?

   (अ) 2×3^2 का मान 18 है।
   The value of 2 is 18.

   (आ) $16 = 4 \times 2^2$

   (इ) आधार समान होने पर गुणन में घात परस्पर घटती है ।
   When the bases are the same, the powers in multiplication decrease mutually.

   (ई) $3^{12} \times 3^2 = 3^{14}$

3. निम्न में से कौन-सा कथन सत्य नहीं है-

   Which of the following statement is not true -

(अ) ऐसी राशि जिसका मान परिवर्तित नहीं होता है, उसे अचर राशि कहलाती है।

A quantity whose value does not change is called a constant quantity.

(आ) ऐसी राशि जिसका मान परिवर्तित होता है, उसे चर राशि कहलाती है।

A quantity whose value changes is called a variable quantity.

(इ) समीकरण में सदैव बराबर का चिह्न होता है ।

There is always an equal sign in an equation.

(ई) $\frac{x}{3} = 7$ में x का मान 3 होगा ।

In $\frac{x}{3} = 7$, value of x will be 3.

4. वर्ग एवं वर्गमूल के सम्बन्ध में निम्न में से कौन-सा कथन सही है।

Which of the following statement is correct regarding square and square root?

(अ) 8100 का वर्गमूल 81 है।

The square root of 8100 is 81.

(आ) 9 का वर्ग 81 है परन्तु 81 का वर्गमूल 9 नहीं है ।

The square of 9 is 81 but the square root of 81 is not 9.

(इ) 5 एवं 6 का वर्ग क्रमशः 25 एवं 36 है।

The square of 5 and 6 are 25 and 36 respectively.

(ई) 25 का वर्ग 5 है तथा 5 का वर्गमूल 25 है।

The square of 25 is 5 and the square root of 5 is 25.

5. निम्न में से द्विपदी बीजीय व्यंजक का चयन करें ।

Select the binomial algebraic expression from the following.

(अ) $8xy + 3xy$

(आ) $8x^2y^2 + x^2y^2$

(इ) $8xy + x^2y^2$

(ई) इनमें से कोई नहीं

None of these

6. घन एवं घनमूल के सम्बन्ध में निम्न में से कौन-सा कथन सही नहीं है।

Which of the following statement is not correct regarding cube and cube root?

(अ) 216 का घनमूल 8 है ।

The cube root of 216 is 8.

(आ) 4 का घन 64 एवं 64 का घनमूल 4 है।

Cube of 4 is 64 and cube root of 64 is 4.

(इ) 6 का घन 216 है तथा 216 का घनमूल 6 है ।

The cube of 6 is 216 and the cube root of 216 is 6.

(ई) 27 का घनमूल 3 है ।

The cube root of 27 is 3.

7. चतुर्भुज के सम्बन्ध में निम्न से कौन-सा विकल्प सही नहीं है।

Which of the following option is not correct regarding the quadrilateral?

(अ) चतुर्भुज के चारों अन्तः कोणों की मापों का योगफल 180 अंश होता है ।

The sum of the measures of the four interior angles of a quadrilateral is 180 degrees.

(आ) चतुर्भुज के चारों अन्तः कोणों की मापों का योगफल 360 अंश होता है ।

The sum of the measures of the four interior angles of a quadrilateral is 360 degrees.

(इ) वह चतुर्भुज जिसकी चारों भुजाएँ समान हो, वर्गाकार चतुर्भुज कहलाता है ।

A quadrilateral whose all four sides are equal is called a square quadrilateral.

(ई) वह चतुर्भुज जिसमें आमने-सामने की भुजा समान हो, आयताकार चतुर्भुज कहलाता है ।

A quadrilateral in which opposite sides are equal is called a rectangular quadrilateral.

8. निम्न में से कौन-सा विकल्प सही नहीं है ।

Which of the following option is not correct?

(अ) 80 का 25% = 25 (आ) 80 का 20% = 16

(इ) 800 का 30% = 240 (ई) 400 का 100% = 400

9. निम्न में से कौन-सा सूत्र सही नहीं है –

Which of the following formula is not correct?

(अ) बेलन का वक्रपृष्ठ = $2\pi rh$

Curved surface of a cylinder = $2\pi rh$

(आ) वृत्त की परिधि = $2\pi r$

Circumference of circle = $2\pi r$

(इ) वृत्त का क्षेत्रफल = πr^2

Area of circle = πr^2

(ई) घनाभ का आयतन = लम्बाई × ऊँचाई

Volume of cuboid = length × height

10. निम्न में से कौन-सा सूत्र सही है –

Which of the following formula is correct

(अ) वर्ग का क्षेत्रफल = भुजा × भुजा

Area of square = side × side

(आ) त्रिभुज का क्षेत्रफल = $\frac{1}{2}$ (आधार × ऊँचाई)

Area of triangle = $\frac{1}{2}$ (Base × Height)

(इ) आयत का क्षेत्रफल = लम्बाई × चौडाई

Area of rectangle = length × breadth

(ई) उपर्युक्त तीनों / The above three

प्रश्न - 02. रिक्त स्थानों की पूर्ति कीजिए – 5 × 2 = 10

Questions - 02. Fill in the blanks -

1. परिमेय संख्या .... के रूप में होती है ।

   A rational number is of the form ....

2. दो परिमेय के मध्य .... संख्या परिमेय संख्या होती है ।

   A number between two rational is a rational number.

3. त्रिभुज में .... भुजाएँ होती है ।

   A triangle has .... sides.

4. ऐसा बहुभुज जिसमें विकर्ण बाह्य हो .... बहुभुज कहलाता है ।

   A polygon in which the diagonal is exterior is called a .... polygon.

5. $-(a-b) =$ ....

प्रश्न - 03. निम्नलिखित युग्मों के मिलान पर विचार कीजिए – 5 × 2 = 10

Questions - 03. Consider matching the following pairs -

1.	x^2y^3 में y^3 का गुणांक Coefficient of y^3 in x^2y^3	क.	21
2.	$x + 4 = 25$ तब, x का मान $x + 4 = 25$ Then, the value of, x	ख.	1
3.	वर्ग के प्रतिलोम संक्रिया Inverse operation of squares	ग.	4
4.	$a^0 =$	घ.	x^2
5.	चतुर्भुज के शीर्ष Vertex of the quadrilateral	ङ.	वर्गमूल Square root

उपर्युक्त युग्मों के आधार पर सही विकल्प का चयन कीजिए –

Select the correct option based on the above pairs -

(अ) (1) (ङ), (2) (ग), (3) (क), (4) (ख), (5) (घ)

(आ) (1) (घ), (2) (क), (3) (ङ), (4) (ग), (5) (ख)

(इ) (1) (ङ), (2) (ग), (3) (घ), (4) (क), (5) (ख)

(ई) (1) (घ), (2) (क), (3) (ङ), (4) (ख), (5) (ग)

प्रश्न - 04. सत्य / असत्य कथन पर विचार कीजिए – $5 \times 1 = 5$

Questions - 04. Consider the statement true / false -

1. सभी वर्ग, आयत होते हैं। / All squares are rectangles.
2. आयत की सभी भुजाएँ बराबर होती है।
   All sides of a rectangle are equal.
3. $10x^2y$ में x^2 का गुणांक $10y$ है। / The coefficient of x^2 in $10x^2y$ is $10y$
4. मूलधन और ब्याज के योग को मिश्रधन कहते हैं।
   The sum of principal and interest is called compound money.
5. घन का पृष्ठीय क्षेत्रफल = $6a^2$ होता है।
   The surface area of a cube = $6a^2$.

उपर्युक्त कथनों को पढ़कर सही विकल्प का चयन कीजिए –

Read the above statements and choose the correct option -

(अ) (1) सत्य, (2) सत्य, (3) सत्य, (4) सत्य, (5) सत्य
(1) True, (2) True, (3) True, (4) True, (5) True

(आ) (1) सत्य, (2) सत्य, (3) सत्य, (4) असत्य, (5) सत्य
(1) True, (2) True, (3) True, (4) False, (5) True

(इ) (1) सत्य, (2) असत्य, (3) सत्य, (4) सत्य, (5) सत्य
(1) True, (2) False, (3) True, (4) True, (5) True

(ई) (1) असत्य, (2) सत्य, (3) असत्य, (4) असत्य, (5) सत्य
(1) False, (2) True, (3) False, (4) False, (5) True

प्रश्न - 05. अति लघूत्तरीय प्रश्न – **$10 \times 2 = 20$**

Questions - 05. Very short answer type questions -

1. 16 का वर्गमूल ज्ञात कीजिए। / Find the square root of 16.

2. 10^7 में आधार एवं घात बताइये ।/State the base and power in 10.

3. मान ज्ञात करें/ Find the value - $5^0 \times 3$

4. आयत के अन्तः कोणों का योगफल कितना होता है ।
   What is the sum of the interior angles of a rectangle?

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5. संख्या 876854 का बीजांक ज्ञात करें ।/Find the bijaank of the number 876854.

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6. समान्तर चतुर्भुज के कोई दो गुणधर्म लिखिए।/Write any two properties of parallelogram.

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7. ऊर्ध्वतिर्यग्भ्याम सूत्र द्वारा हल कीजिये/Solve by Urdhvatiryagbhyam formula -

$\underline{120 \times 192}$

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8. बहुभुज से आप क्या समझते है। /What do you understand by polygon?

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9. 40 का कितना प्रतिशत 16 है। /What percent of 40 is 16.

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10. ध्वजांक विधि से 4096 मे 64 का भाग देकर भागफल एवं शेषफल ज्ञात कीजिये ।
Find the quotient and remainder by dividing 4096 by 64 using the dhvajaank number method.

प्रश्न - 07. लघूत्तरीय प्रश्न – $5 \times 3 = 15$

Questions - 07. Short answer questions -

1. विलोकनम् सूत्र का अर्थ समझाते हुए, उक्त सूत्र के द्वारा 5184 का वर्गमूल ज्ञात कीजिये ।
   Explaining the meaning of Vilokanam formula, find the square root of 5184 by the above formula.

   ----

2. आनुरूप्येण सूत्र का अर्थ समझाते हुए,निम्न गुणनफल ज्ञात करें-
   Explaining the meaning of the Anurupyena formula, find the following product -

   315×320

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3. निम्न श्लोक के अर्थ को समझाते हुए, दिये गये प्रश्न को हल करें-

Explaining the meaning of the following verse, solve the given question.

$5^3 + 4$

समत्रिघातश्च घनः प्रदिष्टः इति प्रथम प्रकारः ।

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4. यदि उपवस्त्र एवं धोती की लम्बाई का अनुपात 5 : 4 है । यदि दोनों लम्बाईयो को जोड़ने पर 36 मीटर प्राप्त होता हो, तो उपवस्त्र व धोती की लम्बाई ज्ञात करें ।

If the ration of the length of the upavastr and the dhoti is 5 : 4. If 36 meters is obtained by adding both the lengths, then find the length of upavastr and dhoti.

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5. तीन क्रमागत पूर्णांकों का योग 51 है तो पूर्णांक ज्ञात कीजिये ।

The sum of three consecutive integers is 51, then find the integer.

प्रश्न - 08. दीर्घ उत्तरीय प्रश्न – **$4 \times 5 = 20$**

Questions - 08. Long answer type questions -

1. यदि 2000 रु. 2 वर्ष के लिये चक्रवृद्धि ब्याज पर उधार लिया जाये तो 3% वार्षिक दर से चक्रवृद्धि ब्याज कितना देय होगा ।

If Rs. 2000, If the loan is taken on compound interest for 2 years, then how much compound interest will be payable at the rate of 3% per annum.

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2. चतुर्भुज से आप क्या समझते हैं। चतुर्भुज के वर्गीकरण को समझाइए।

What do you understand by quadrilateral? Explain the classification of quadrilaterals.

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3. वेद् का कोई एक मन्त्र लिखिए जिसमें संख्या का बोध हो।

A mantra of the Vedas write, in which there is cognition of number?

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4. निम्न वेद् मन्त्र के द्वारा वर्ग एवं वर्गमूल की अवधारणा को समझाइये।

Explain the concept of square and square root with the help of following Veda Mantra.

एका च मे तिस्त्रश्च मे तिस्त्रश्च मे पञ्च च मे पञ्च च मे सप्त च मे सप्त च मे नव च मे नव च म ऽ एकादश च म ऽ एकादश च मे त्रयोदश च मे त्रयोदश च मे पञ्चदश च मे पञ्चदश च मे सप्तदश च मे सप्तदश च मे नववश च मे नवदश च म एकवि ँ शतिश्च म ऽ एकवि ँ शतिश्च मे त्रयोवि ँ शतिश्च मे त्रयोवि ँ शतिश्च मे पञ्च वि ँ शतिश्च मे पञ्चवि ँ शतिश्च मे सप्त वि ँ शतिश्च मे सप्तवि ँ शतिश्च मे नववि ँ शतिश्च मे नववि ँ शतिश्च म ऽ एक त्रि ँ शच्च म ऽ एकत्रि ँ शच्च मे त्रयस्त्रि ँ शच्च मे यज्ञेन कल्पन्ताम् ॥

Rashtriya Adarsh Veda Vidyalaya Run and Proposed by

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