





# MATHEMATICS TEXTBOOK

(On the basis of vedic Mathematics and Sanskrit Literature of Mathematics)

# Veda Bhushan I Year / Prathama - I Year / Class VI

# MAHARSHI SANDIPANI RASHTRIYA VEDA SANSKRIT SHIKSHA BOARD (Established and Recognized by the Ministry of Education, Government of India)

रातं सहस्रमयुतं न्यर्बुदमसंख्येयं स्वमस्मिन् निविष्टम् । तदस्य घन्त्यभिपश्यत एव तस्माद् देवो रोचत एष एतत् ॥ यवोदरैरक्वुल्रमष्टसंख्यैर्हस्तोऽङ्गुलैः षड्गुणितैश्चतुर्भिः । हस्तैश्चतुर्भिर्भवतीह दण्डः कोशः सहस्रद्वितयेन तेषाम् ॥ राताय स्वाहा सहस्राय स्वाहायुताय स्वाहा नियुताय स्वाहा प्रयुताय स्वाहार्बुदाय स्वाहा न्यर्बुदाय स्वाहा समुद्राय स्वाहा मध्याय स्वाहान्ताय स्वाहा नियुताय स्वाहोषिसे स्वाहा व्युष्ठे स्वाहोदेष्यते स्वाहोचते स्वाहोदिताय स्वाहा सुवर्गाय स्वाहा लोकाय स्वाहा सर्वस्मै स्वाहा। धनयोर्धनमृणमृणयोर्धनर्णयोरन्तरं समैक्चं खम्। ऋणमैक्चं च धनमृणधनशून्ययोः शून्ययोः शून्यम्॥ सप्तास्यासन् परिधयस्तिः सप्त समिधःकृताः। देवा यद्यज्ञं तन्वाना अवध्रन् पुरुषं पशुम्॥ समश्रतौ तुल्यचतर्भुजे च तथायते तद्भजकोटिघातः।



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#### Note : -

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#### PREFACE

#### (In the light of NEP 2020)

The Ministry of Education (Department of Higher Education), Government of India established Rashtriya Veda Vidya Pratishthan in Delhi under the Chairmanship of Hon'ble Education Minister (then Minister of Human Resource Development) under the Societies Registration Act, 1860 (XXI of 1860) on 20th January, 1987. The Government of India notified the resolution in the Gazette of India vide no 6-3/85- SKT-IV dated 30-3-1987 for establishment of the Pratishthan for preservation, conservation, propagation and development of oral Samhita, tradition of Vedic studies (Veda Padapatha to Ghanapatha, Vedanga, Veda Bhashya etc), recitation and intonation of Vedas etc and interpretation of Vedas in scientific lines. In the year 1993 the name of the organization was changed to Maharshi Sandipani Rashtriya Veda Vidya Pratishthan (MSRVVP) and it was shifted to Ujjain, Madhya Pradesh.

The National Education Policy of 1986 and Revised Policy Formulations of 1992 and also Programme of Action (PoA) 1992 have mandated Rashtriya Veda Vidya Pratishthan for promoting Vedic education throughout the country. The importance of India's ancient fund of knowledge, oral tradition and employing traditional Guru's for oral education was also emphasized in the PoA.

In accordance with the aspirations of the nation, national consensus and policy in favour of establishing a Board for Veda and Sanskrit Education at national level, the General Body and the Governing Council of MSRVVP under the Chairmanship of Hon'ble Education Minister, Government of India, have set up "Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board" (MSRVSSB) in tune with the mandate of the Pratishthan and its implementation strategies. The Board is necessary for the fulfillment of the objectives of MSRVVP as envisioned in the MoA and Rules. The Board has been approved by the Ministry of Education, Government of India and recognized by the Association of Indian Universities, New Delhi. The bye-laws of the Board have been vetted by Central Board of Secondary Education and curriculum structure have been concurred by the National Council of Educational Research and Training, New Delhi.

It may also be mentioned here that the committee "Vision and Roadmap for the Development of Sanskrit - Ten year perspective Plan", under the Chairmanship of Shri N. Gopalaswamy, former CEC, constituted by the Ministry of Education Govt. of India in 2015 recommended for establishment of a Board of Examination for standardization, affiliation, examination, recognition, authentication of Veda Sanskrit education up to the secondary school level. The committee was of the opinion that the primary level of Vedic and Sanskrit studies should be inspiring, motivating and joyful. It is also desirable to include subjects of modern education into Vedic and Sanskrit Pathashalas in a balanced manner. The course content of these Pathashalas should be designed to suit to the needs of the contemporary society and also for finding solutions to modern problems by reinventing ancient knowledge.

With regard to Veda Pathashala-s it is felt that they need further standardization of recitation skills along with introduction of graded materials of Sanskrit and modern subjects so that the students can ultimately acquire the capabilities of studying Veda bhashya-s and mainstreaming of students is achieved for their further studies. Due emphasis may also be given for the study of Vikriti Patha of Vedas at an appropriate level. The members of the committee have also expressed their concern that the Vedic recitation studies are not uniformly spread all over India; therefore, due steps may be taken to improve the situation without in anyway interfering with regional variations of recitation styles and teaching method of Vedic recitation.

It was also felt that since Veda and Sanskrit are inseparable and complementary to each other and since the recognition and affiliation problems are same for all the Veda Pathashalas and Sanskrit Pathashalas throughout the country, a Board may be constituted for both together. The committee observed that the examinations conducted by the Board should have legally valid recognition enjoying parity with modern Board system of education. The committee observed that the Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain may be given the status of Board of Examinations with the name "Maharshi Sandipani Rashtriya Veda Sanskrita Vidya Parishat with headquarters in Ujjain which will continue all programs and activities which were being conducted hitherto in addition to being a Board of Examinations.

The promotion of Vedic education is for a comprehensive study of India's glorious knowledge tradition and encompasses multi-layered oral tradition of Vedic Studies (Veda Samhita, Padapatha to Ghanapatha, Vedanga, Veda Bhashy aetc), recitation and intonation, and Sanskrit knowledge system content. In view of the policy of mainstreaming of traditional students and on the basis of national consensus among the policy making bodies focusing on Vedic education, the scheme of study of Veda stretching up to seven years in Pratishthan also entails study of various other modern subjects such as Sanskrit, English, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, etc. as per the syllabus and availability of time. In view of NEP 2020, this scheme of study is with appropriate inputs of Vedic knowledge and drawing the parallels of modern knowledge in curriculum content focusing on Indian Knowledge System.

In Veda Pathashala-s, GSP Units and Gurukula-s of MSRVVP, affiliated to the Board transact the curriculum primarily based on oral tradition of a particular complete Veda Shakha with perfect intonation and memorization, with additional subsidiary modern subjects such as English, Sanskrit, Mathematics, Science, Social Science and SUPW. Gradually, the Veda Pathashala-s will also introduce other skill and vocational subjects as per their resources.

It is a well-known fact that there were 1131 shakha-s or recensions of Vedas; namely 21 in Rigveda, 101in Yajurveda, 1000 in Samaveda and 9

in Atharva Veda. In course of time, a large number of these shakhas became extinct and presently only 10 Shakhas, namely, one in Rigveda, 4 in Yajurveda, 3 in Samaveda and 2 in Atharvaveda are existing in recitation form on which Indian Knowledge System is founded now. Even in regard to these 10 Shakhas, there are very few representative Vedapathis who are continuing the oral Vedic tradition/ Veda recitation/Veda knowledge tradition in its pristine and complete form. Unless there is a full focus for Vedic learning as per oral tradition, the system will vanish in near future. These aspects of Oral Vedic studies are neither taught nor included in the syllabus of any modern system of school education, nor do the schools/Boards have the systemic expertise to incorporate and conduct them in the conventional modern schools.

The Vedic students who learn oral tradition/ recitation of Veda are there in their homes in remote villages, in serene and idyllic locations, in Veda Gurukulas, (GSP Units), in Veda Pathashala-s, in Vedic Ashrams etc. and their effort for Veda study stretches to around 1900 – 2100 hours per year; which is double the time of other conventional school Board's learning system. Vedic students have to have complete Veda by-heart and recite verbatim with intonation *(udatta, anudatta, swaritaetc)*, on the strength of memory and guru parampara, without looking at any book/pothi. Because of unique ways of chanting the Veda mantras, unbroken oral transmission of Vedas and its practices, this has received the recognition in the UNESCO-World Oral Heritage in the list of Intangible Cultural Heritage of Humanity. Therefore, due emphasis is required to be given to maintain the pristine and complete integrity of the centuries old Vedic Education (oral tradition/ recitation/ Veda knowledge Tradition). Keeping this aspect in view the MSRVVP and the Board have adopted unique type of Veda curriculum with modern subjects like Sanskrit, English, Vernacular language, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture etc. as well as skill and vocational subjects as prescribed by NEP 2020.

As per Vedic philosophy, any person can become happy if he or she learns both *Para-Vidya and Apara-Vidya*. The materialistic knowledge from the Vedas, their auxiliary branches and subjects of material interest were called *Apara-Vidya*. The knowledge of supreme reality, the ultimate quest from Vedas, Upanishads is called *Para-Vidya*. In all the total number of subjects to be studied as part of Veda and its auxiliaries are fourteen. There are fourteen branches of learning or *Vidyas* - four Vedas, Six Vedangas, Mimamsa (Purva Mimamsa and Uttara Mimamsa), Nyaya, Puranas and Dharma shastra. These fourteen along with Ayurveda, Dhanurveda, Gandharvaveda and Arthashastra become eighteen subjects for learning. All curriculum transaction was in Sanskrit language, as Sanskrit was the spoken language for a long time in this sub-continent.

Eighteen Shilpa-s or industrial and technical arts and crafts were mentioned with regard to the Shala at Takshashila. The following 18 skills/Vocational subjects are reported to be subjects of the study– (1) Vocal music (2) Instrumental music (3) Dancing (4) Painting (5) Mathematics (6) Accountancy (7) Engineering (8) Sculpture (9) Cattle breeding (10) Commerce (11) Medicine (12) Agriculture (13) Conveyancing and law (14) Administrative training (15) Archery and Military art (16) Magic (17) Snake charming (18) Art of finding hidden treasures.

For technical education in the above mentioned arts and crafts an apprenticeship system was developed in ancient India. As per the Upanishadic vision, the vidya and avidya make a person perfect to lead contented life here and liberation here-after.

Indian civilization has a strong tradition of learning of shastra-s, science and technology. Ancient India was a land of sages and seers as well as of scholars and scientists. Research has shown that India had been a Vishwa Guru, contributing to the field of learning (vidya-spiritual knowledge and avidya- materialistic knowledge) and learning centers like modern universities were set up. Many science and technology based advancements of that time, learning methodologies, theories and techniques discovered by the ancient sages have created and strengthened the fundamentals of our knowledge on many aspects, may it be on astronomy, physics, chemistry, mathematics, medicine, technology, phonetics, grammar etc. This needs to be essentially understood by every Indian to be proud citizen of this great country!

The idea of India like "Vasudhaiva Kutumbakam" quoted at the entrance of the Parliament of India and many Veda Mantra-s quoted by constitutional authorities on various occasions are understood only on study of the Vedas and true inspiration can be drawn only by pondering over them. The inherent equality of all beings as embodiment of "sat, chit, ananda" has been emphasized in the Vedas and throughout the Vedic literature.

Many scholars have emphasized that Veda-s are also a source of scientific knowledge and we have to look into Vedas and other scriptural sources of India for the solution of modern problems, which the whole world is facing now. Unless students are taught the recitation of Vedas, knowledge content of Vedas and Vedic philosophy as an embodiment of spiritual and scientific knowledge, it is not possible to spread the message of Vedas to fulfill the aspiration of modern India.

The teaching of Veda (Vedic oral tradition/ Veda recitation/ Veda knowledge Tradition) is neither only religious education nor only religious instruction. It will be unreasonable to say that Vedic study is only a religious instruction. Veda-s are not religious texts only and they do not contain only religious tenets; they are the corpus of pure knowledge which are most useful to humanity as whole. Hence, instruction or education in Veda-s cannot be construed as only "religious education/religious instruction."

Terming "teaching of Veda as a religious education" is not in consonance with the judgment of the Hon'ble Supreme Court (AIR 2013: 15 SCC 677), in Civil Appeal no. 6736 of 2004 (Date of judgment-3rd July 2013). The Vedas are not only religious texts, but they also contain the knowledge in the disciplines of mathematics, astronomy, meteorology, chemistry, hydraulics, physics, science and technology, agriculture, philosophy, yoga, education, poetics, grammar, linguistics etc. which has been brought out in the judgment by the Hon'ble Supreme Court of India. **Vedic education through establishment of Board in compliance with NEP-2020** 

The National Education Policy-2020 firmly recognizes the Indian Knowledge Systems (also known as 'Sanskrit Knowledge Systems'), their importance and their inclusion in the curriculum, and the flexible approach in combining various subjects. Arts' and Humanities' students will also learn science; try to acquire vocational subjects and soft skills. India's special heritage in the arts, sciences and other fields will be helpful in moving towards multi-disciplinary education. The policy has been formulated to combine and draw inspiration from India's rich, ancient and modern culture and knowledge systems and traditions. The importance, relevance and beauty of India's classical languages and literature is also very important for a meaningful understanding the national aspiration. Sanskrit, being an important modern language mentioned in the Eighth Schedule of Indian Constitution, its classical literature that is greater in volume than that of Latin and Greek put together, contains vast treasures of mathematics, philosophy, grammar, music, politics, medicine, architecture, metallurgy, drama, poetry, storytelling, and more (known as 'Sanskrit Knowledge Systems'). These rich Sanskrit Knowledge System legacies for world heritage should not only be nurtured and preserved for posterity but also enhanced through research and put in to use in our education system, curriculum and put to

new uses. All of these literatures have been composed over thousands of years by people from all walks of life, with a wide range of socio-economic background and vibrant philosophy. Sanskrit will be taught in engaging and experiential as well as contemporary relevant methods. The use of Sanskrit knowledge system is exclusively through listening to sound and pronunciation. Sanskrit textbooks at the Foundation and Middle School level will be available in Simple Standard Sanskrit (SSS) to teach Sanskrit through Sanskrit (STS) and make its study enjoyable. Phonetics and pronunciation prescriptions in NEP 2020 apply to the Vedas, the oral tradition of the Vedas and Vedic education, as they are founded upon phonetics and pronunciation.

There is no clear distinction made between arts and science, between curricular and extra-curricular activities, between vocational and academic streams, etc. The emphasis in NEP 2020 is on the development of a multi-disciplinary and holistic education among the sciences, social sciences, arts, humanities and sports for a multi-disciplinary world to ensure the unity and integrity of all knowledge. Moral, human and constitutional values like empathy, respect for others, cleanliness, courtesy, democratic spirit, spirit of service, respect for public property, scientific temper, freedom, responsibility, pluralism, equality and justice are emphasized.

The NEP-2020 at point no. 4.23 contains instructions on the pedagogic integration of essential subjects, skills and abilities. Students will be given a large amount of flexible options in choosing their

individual curriculum; but in today's fast-changing world, all students must learn certain fundamental core subjects, skills and abilities to be a well-grounded, successful, innovative, adaptable and productive individual in modern society. Students must develop scientific temper and evidence based thinking, creativity and innovation, aesthetics and sense of art, oral and written expression and communication, health and nutrition, physical education, fitness, health and sport, collaboration and teamwork, problem solving and logical thinking, vocational exposure and skills, digital literacy, coding and computational thinking, ethics and moral reasoning, knowledge and practice of human and constitutional values, gender sensitivity, fundamental duties, citizenship skills and values, knowledge of India, environmental awareness etc. Knowledge of these skills include conservation, sanitation and hygiene, current affairs and important issues facing local communities, the states, the country and the world, as well as proficiency in multiple languages. In order to enhance the linguistic skills of children and to preserve these rich languages and their artistic treasures, all students in all schools, public or private, shall have the option of learning at least two years in one classical language of India and its related literature.

The NEP-2020 at point no. 4.27 states that -"Knowledge of India" includes knowledge from ancient India and its contributions to modern India and its successes and challenges, and a clear sense of India's future aspirations with regard to education, health, environment, etc. These elements will be incorporated in an accurate and scientific manner

throughout the school curriculum wherever relevant; in particular, Indian Knowledge Systems, including tribal knowledge and indigenous and traditional ways of learning, will be covered and included in mathematics, astronomy, philosophy, yoga, architecture, medicine, agriculture, engineering, linguistics, literature, sports, games, as well as in governance, polity, conservation. It will have informative topics on inspirational personalities of ancient and modern India in the fields of medicinal practices, forest management, traditional (organic) crop cultivation, natural farming, indigenous sports, science and other fields.

The NEP-2020 at point no. 11.1 gives directions to move towards holistic and multidisciplinary education. India emphasizes an ancient tradition of learning in a holistic and multidisciplinary manner, including the knowledge of 64 arts such as singing and painting, scientific fields such as chemistry and mathematics, vocational fields such as carpentry, tailoring; professional work such as medicine and engineering, as well as the soft skills of communication, discussion and negotiation etc. which were also taught at ancient universities such as Takshashila and Nalanda. The idea that all branches of creative human endeavour, including mathematics, science, vocational subjects and soft skills, should be considered 'arts', has a predominantly Indian origin. This concept of 'knowledge of the many arts' or what is often called 'liberal arts' in modern times (i.e., a liberal conception of the arts) will be our part of education system.

At point No. 11.3 the NEP-2020 further reiterates that such an

education system "would aim to develop all capacities of human beings intellectual, aesthetic, social, physical, emotional, and moral in an integrated manner. Such an education will help develop well-rounded individuals that possess critical 21st century capacities in fields across the arts, humanities, languages, sciences, social sciences, and professional, technical, and vocational fields; an ethic of social engagement; soft skills, such as communication, discussion and debate; and rigorous specialization in a chosen field or fields. Such a holistic education shall be, in the long term, the approach of all undergraduate programmes, including those in professional, technical, and vocational disciplines."

The NEP-2020 at point no. 22.1 contains instructions for the promotion of Indian languages, art and culture. India is a rich storehouse of culture – which has evolved over thousands of years, and is reflected in its art, literary works, customs, traditions, linguistic expressions, artifacts, historical and cultural heritage sites, etc. Traveling in India, experiencing Indian hospitality, buying beautiful handicrafts and handmade clothes of India, reading ancient literature of India, practicing yoga and meditation, getting inspired by Indian philosophy, participating in festivals, appreciating India's diverse music and art and watching Indian films are some of the ways through which millions of people around the world participate in, enjoy and benefit from this cultural heritage of India every day.

In NEP-2020 at point no. 22.2 there are instructions about Indian arts.

Promotion of Indian art and culture is important for India and to all of us. To inculcate in children a sense of our own identity, belonging and an appreciation of other culture and identity, it is necessary to develop in children key abilities such as cultural awareness and expression. unity, positive cultural identity and self-esteem can be built in children only by developing a sense and knowledge of their cultural history, art, language and tradition. Therefore, the contribution of cultural awareness and expression is important for personal and social well-being.

The core Vedic Education (Vedic Oral Tradition / Veda Path / Veda Knowledge Tradition) of Pratishthan along with other essential modern subjects- Sanskrit, English, Mother tongue, Mathematics, Social Science, Science, Computer Science, Philosophy, Yoga, Vedic Agriculture, Indian Art, Socially useful productive work etc., based on the IKS inputs are the foundations/sources of texts books of Pratishthan and Maharshi Sandipani Rashtriya Veda Sanskrit Shiksha Board. These inputs are in tune with the NEP 2020. The draft books are made available in pdf form keeping in view the NEP 2020 stipulations, requirements of MSRVVP students and the advice of educational thinkers, authorities and policy of Maharshi Sandipani Rashtriya Veda Vidya Pratishthan, Ujjain. These books will be updated in line with NCFSE in future and finally will be made available in print form.

The Teachers of Veda, Sanskrit and Modern subjects in Rashtriya Adarsh Veda Vidyalaya, Ujjain and many teachers of Sanskrit and modern subjects in aided Veda Pathshalas of Pratishthan have worked for last two years tirelessly to prepare and present Sanskrit and modern subject text books in this form. I thank all of them from the bottom of my heart. Many eminent experts of the national level Institutes have helped in bringing quality in the textbooks by going through the texts from time to time. I thank all those experts and teachers of the schools. I extend my heartfelt gratitude to all my co-workers who have worked for DTP, drawing the sketches, art work and page setting.

All suggestions including constructive criticism are welcome for the improvement of the quality of the text books.

# आपरितोषाद् विदुषां न साधु मन्ये प्रयोगविज्ञानम्। बलवदपि शिक्षितानाम् आत्मन्यप्रत्ययं चेतः॥

(Abhijnanashakuntalam 1.02)

Until the scholars are fully satisfied about the content, presentation, attainment of objective, I do not consider this effort to be successful, because even the scholars are not fully confident in the presentation without feedback from the stakeholders.

#### Prof. ViroopakshaV Jaddipal

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#### FOREWORD

India has a rich tradition of mathematics. Indian philosophers and mathematicians have done excellent work in this field since ancient times. Since the beginning of the Vedic period, mathematics has been given the highest place. For example, Yajushajyotisham clearly supports the statement.

# यथा शिखा मयूराणां नागानां मणयो यथा । तद्वत् वेदाङ्गशास्त्राणां गणितं मूर्द्धनि स्थितम् ॥

(याजुषज्यौतिषम्, 4)

Meaning, just as the crest in peacocks and the gem in snakes have on the head (highest place), similarly mathematics has the highest place in all the Vedas.

In this book the ancient knowledge and achievements found in Vedic mathematics and Sanskrit literature are mentioned and used in consonance with modern mathematical techniques, in simplified manner. An attempt has been made to simplify the calculations through Vedic mathematics.

In the current global scenario, by harmonizing mathematics teaching with the changing environment, to provide the learning level of proficiency in mathematics to the Vedic students across India, as elucidated in the main principles contained of the National Education Policy 2020 such as discussions, analysis of examples and application has been implemented. Simultaneously, keeping the vision in mind, curriculum and text books have been designed to benefit the Vedic students.

The language used in the textbook is very simple and easy, which will help the students to understand easily. This textbook of Vedabhushan 1st (equivalent to 6th) year is almost equivalent to Class-6 Mathematics textbook of entire India. Many concepts in the textbook are explained along with the Vedic proofs of Sanskrit knowledge system citing the reference from Brahmasphuta Siddhanta, Shulbasutra, Aryabhatiyam, Lilavati and Bijaganitam etc so that Vedic students will be able to understand ancient mathematical concepts along with modern mathematics also will be able to experience the dignity of their Indian tradition. A total of 11 chapters in the textbook have been composed as per the requirement of Ved Bhushan first year curriculum of Veda schools. In Chapter 1, named as knowing the numbers, the concepts of types of numbers, numeration system, understanding of decimal number system have been described. In Chapter 2, playing with numbers, we will learn to find ascending and descending order, successor and predecessor of numbers, rules of divisibility, multiples, factors, least common multiple, highest common factor have been discussed. Integers are discussed in detail in Chapter 3. In Chapter 4, under Vedic Mathematics, addition using ēkādhikēna and ekanyūnena formulas, multiplication using Antar and antyayordaśake'pi formulas as well as writing tables using Vinakulam etc. have been explained in detail. Fractions have been presented in detail in Chapter 5 and decimal numbers in Chapter 6. Chapter 7 presents the description of different types of lines under the

concept of basic geometry. In Chapter 8, the classification of triangles along with plane and flat shapes under simple two-dimensional shapes has been described in detail. In Chapter 9, solid geometric shapes have been presented under three-dimensional shapes. The perimeter and area of Square and rectangular shapes are discussed in Chapter 10. Chapter 11 presents the description of ratio and proportion.

The textbook provides various activities to develop the skills of Vedic students in understanding mathematics as well as in retrieving facts. Which have been named as 'Do and Learn. To increase the level of understanding and proficiency of the students, important concepts and results have been given as "We Learned" at the end of each chapter.

Contribution of Indian mathematicians to mathematics has also been mentioned at the end of the textbook to make students appreciate the rich traditions of India and the contribution made by Indian.

By studying diligently the mathematical concepts available in the textbook, Vedic students will be able to prepare for competitive examinations. After reading this book, students should study NCERT textbook of class six and books related to the particular subject.

The author will be grateful to you for the positive suggestions sent for correcting the errors in the textbook.

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# Chapter – 1

# **Understanding Numbers**

According to our needs, counts the objects around us, such as number of children in a class, number of people living in a village, number of books in a library and number of tiles on a floor etc.

We can represent the above quantities using numbers. Now think and answer, how many different objects can be counted and represented in terms of numbers around you.

In this chapter we will revise the concepts of numbers which we have studied in our previous classes and explore further.

#### Formation of number-

In Vedic literature, we can come across various references about numbers in the form of Shlokas. One such S'lokas is given below.

त्रीणि शता त्री सहस्राण्यग्नित्रिशच देवा नव चासपर्यन्।

औक्षन् घृतैरस्तृणन् बर्हिरस्मा आदिद्धोतारं न्यसादयन्त॥

(Yajurveda- 33.07)

In the above Shlokas, taken from Yajur veda, one can find the reference about the number 3,339. Trinishata- three hundred, Tri sahastrani- three thousand, Trinshat cha and thirty, Navach- nine, Devah – deity Dev meaning sum, there is a mention of 3,339 deities.

Aaradhya and Lakhsmi are forming four- digit numbers using the digits 2, 4, 7, and 9. Aaradhya forms a four-digit number 4279 using the digits 2,4,7 and 9. 4279

Lakshmi: Oho ! This number is 4 thousand two hundred and seventynine.

Now, Lakshmi forms a 4-digit number using the same digits-9742



**Aaradhya:** Oho ! This is bigger than the number I have created and it is also the largest number that can be formed from these four digits.

Out of the number formed, by you and your friends which is the smallest number?

#### **Types of Numbers**

#### 1. Natural Number

Natural number is a whole number. Which is greater than 0. Natural number starts from 1 and goes on infinitely. (We call this group as natural number.)

**Example:** 1, 2, 3, 4, 5, ....

We can always find a natural number which is greater than a given number.

#### 2. Whole Numbers:

On adding zero to the group of natural numbers, we get the group of whole numbers. All the natural numbers are whole numbers.

**Example:** 0, 1, 2, 3, 4.....

Now think : Is all the whole numbers are natural number?

#### 3. Even Numbers:

चतस्रश्च मेऽष्टौ च मेऽष्टौ च मे द्वाद्रा......युहोनं कल्पन्ताम्॥ २५॥

(यजुर्वेद 18/25)

In the above-mentioned mantra, even numbers from four to fortyeight in multiples of four are mentioned.

A whole number that is completely divisible by 2 is called an even number. The last digit of an even numbers is either 0, 2, 4, 6 or 8.

Example: 2, 4, 6, 8, 10, 12....

#### 4. Odd Numbers:

#### एका च मे तिस्नश्च मे तिस्न श्च मे पञ्च च मे......यज्ञेन कल्पन्ताम् ॥

```
(Yajurveda-18/24)
```

In the above mantra we can find the reference about the odd numbers lying between 1 and 33.

Such whole numbers, which are not divisible by 2 exactly are called odd numbers. The last digit of an odd number is 1, 3, 5,7 or 9.

Example: 1,7,17,27,39.....

#### 5. Integers:

Integers are the numbers, which can beeitherare positive, negative or 0.

Integers are represented by the capital letter Z.

**Example:**  $Z = \{\dots, \dots, 0, 1, 2, 3, \dots\}$ 

#### 6. Prime Numbers:

The numbers which aredivisible by 1 and itself are called prime numbers. 1 is not a prime number. Hence, 2 is a least prime number

Example: 2, 3, 5, 7, 11, ....

- Note:
  - 1. 0 and 1 are notprime numbers
  - 2. There are only two factors for any prime number.
  - 3. All the prime numbers other than 2 are odd numbers.

#### Do and Learn:

Write all the Prime Numbers lying between 1 and 100.

#### Discuss: Why 1 is not a Prime Number?

#### Comparison of Numbers:

Form some more five-digit numbers other than the numbers given in the following table

Number	Number Name
58,476	Fifty-Eight Thousand four hundred seventy six
48,765	
45,978	
87,654	
67,845	

Aaradhy on observing the numbers given in the table said, "87,654 is the largest and 45,678 is the smallest".

We always compare numbers using the symbols (=, > and <).

15	=	15	18	<	20	0	<	10
17	>	16	121	<	212	12	<	21

#### Do and Learn -

1. Compare the following numbers.

19	20	48	69	120	24
69	70	12	17	119	30

2. In the following group of numbers, circle the greatest number and draw a rectangular box on the smallest number.

(i)	4536	4897	4976	4329
(ii)	2567	2387	7892	2934
(iii)	22567	25678	57289	92878
(iv)	68768	98762	12389	23456
(v)	4687	9348	8423	5678

#### **Reading Numbers:**

In Vedic literature, one can find the method to read and write the numbers, in ankanam wamto gati "अङ्कानां वामतो गति:" which is given below.

## शतंतेऽयुतंहायनान् द्वेयुगेत्रीणिचत्वारिकृण्मः इन्द्राग्नीविश्वेदेवास्तेऽनु मन्यन्तामहृणीयमानाः॥

(Atharvveda 8/2/21)

In Vedas, while writing or reading a number, we start from right most number and proceed towards left (reverse order).

In the above mantra, writing the numbers mentioned in the above mantra following the rules of ankanam wamto gati "अङ्कानां वामतो गति:"

Chatwari चतवारी	Trini (त्रिनि)	Dwe (द्वे)	Ayut (अयुत)	Shat <b>হা</b> র্
4	3	2	0000	00

The age of the universe is of four billion thirty-two crore (432 crores) years is divided into era.

Utsav and Khushi are trying to read the number 231,324



**Utsav-** How to read the number 231,324? Is it two hundred thirty-one thousand three hundred twenty-four?

**Khushi-** Yes, it seems that you are reading the number correctly, still let's confirm with our Guruji.





**Guruji-** The number 2,31,324 is read as two lakh thirty one thousand three hundred twenty four.

Form different six-digit number using digits of your choice. Then, share the numbers with your friends and compare.

In the Vedic literature, evidence for large numbers is found in the following mantra of the Atharva Veda.

# शतं सहस्रमयुतं न्यर्बुदमसंख्येय स्वमस्मिन् निविष्टम् । तदस्य घ्नन्त्यभिपश्यत एव तस्मादु देवो रोचत एष एतत् ॥

(Atharva Veda 10/08/24)

In the above mantra, the largest number of thousands, hundreds, tens and units are mentioned.

#### Activity:

- Complete the following table with the suitable digits.

Number	Lakh	Ten	Thousand	Hundred	Tenth	Unit	Number
		Thousand					(In
							Words)
3,52,027	3	5	2	0	2	7	three lakh
							fifty two
							thousand
							twenty
							seven
2,43,596							
7,13,412							
1,56,789							
2,34,567							

Now, let us form, read and compare more seven-digit numbers with our friends.

Number (In Figures)	Ten Lakh	Lakh	Ten Thousand	Thousands	Hundreds	Tens	Unit	Number (In Words)
57,42,683	5	7	4	2	6	8	3	Fifty Seven Lakh Forty Two Thousand Six
								Hundred Eighty Three
99,89,673								
23,43,584								
12,56,789			AF		XS	and and		
23,46,789								
43,46,129								
78,46,923								
53,47,197								

#### Table 1.3

**Do and Learn-** Complete the following tableby writing the digits corresponding to their place values. One has been done for you.

In this we will learn about 8-digit numbers –

///

Number (In Figures)	Crore	Ten Lakh	Lakh	Ten Thousand	Thousand	Hundred	Tenth	Unit	Number (In Words)
9,87,65,432	9	8	7	6	5	4	3	2	Nine Crore Eighty Seven Lakh Sixty Five Thousand Four Hundred Thirty Two
4,53,10,670				Enda	Y		5		
1,23,45,678									
8,97,67,341			SI			23			

#### Table 1.4

Activity- Arrange the students of the school in seven groups, each group having ten students. (Number of students in each group should be 10 instead of 7, as 10 cards should be distributed per group.) Distribute the cards 0 to 9 to the students of each group (1 card per student). Now ask the students to form various 7 – digit number to read and compare. **Remember:** 

#### Complete the following table; one has been done for you:

Largest Number	New number obtained on adding 1
Single digit –9	9 + 1 = 10(ten)
Two-digit-99	99 + 1 = 100 (hundred)
Three-digit-999	999 + 1 = 1000 (one thousand)
Four-digit-9999	9999 + 1 = 10000 (ten thousand)
Five-digit - 99,999	99999 +1 = 1,00, 000 (one lakh)
Six-digit - 9,99,999	9999999 + 1 = 10,00,000 (ten lakhs)
Seven-digit - 99,99,999	99,99,999+1= 1,00,00,000(one crore)

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#### The numbers 1 to 1 crore are written as follows:

Crore	Lak	hs	Thousands		Units		
Crore	Ten Lakh	Lakh	Ten	Thousa	Hundre	Tens	Units
			Thousa	nd	d		
			nd				
1,00,00,000	10,00,000	1,00,000	10,000	1000	100	10	1

Now, let us understand the relationship between units, tens, hundreds etc. Which are given below:

10 units = 1 ten 10 tens = 1 hundred 10 hundreds = 1 thousand 10 thousand = 1 ten thousand 10 thousands = 1 lakh (100 thousand) 10 lakhs = 1 ten lakhs 10 ten lakhs = 1 crore (100 lakhs)

#### Numeration System:

Since ancient times, Indians adopt base 10 system of numeration which is called Devnagari Method. In this method we use the digits 0, 1, 2,3,4,5,6,7,8 and 9. In this method, the position of zero has great significance in writing numbers. Using this method we can write any large number very easily.

For Example: 01 and 10 are different numbers though they have two same digits namely 0 and 1. In 01, zero does not have any meaning and is read as one, whereas 0 has significance in 10, which is read as ten.

#### Indian Numeration system:

In this system we have periods such as units, tens, hundreds, thousands, lakhs and crores. We use commas (,) to write large numbers.

We write commas before three digits initially and before two digits successively, starting from right and moving to-words left.

**Example:** The number 33025324 is written as 3, 30, 25, 324 in Indian Numeration System and it is read as three crore thirty lakh twenty-five thousand three hundred twenty-four.

#### <u>Do and Learn</u>

Rewrite the following numbers as per Indian System of numeration. (use commas) one has been done for you.

S. No.	Numbers	Number with	Number in words	
	(in digits)	commas		
1.	4512671	45,12,671	forty-five lakh twelve thousand	
			six hundred seventy-one	
2.	45672314		5	
3.	79231456			
4.	9234567			
5.	2345676			

#### International System of Numeration:

In the International System of Numeration units, tens, hundreds, thousands and further millions and billions are used. Commas are used to represent different periods. In this system of numeration moving from right to left, a comma is placed before every three digits as given below.

**Example:** The number 3324251342 is written as 3, 324,251,342 and it is read as three billion three hundred twenty-four million two hundred fifty thousand three hundred forty-two.

100 thousands	=	1 Lakh
10 Lakhs (1000 thousand)	=	1 million
10 million (100 Lakh)	=	1 Crore
100 Crores (10,000 Lakh)	=	1 billion

#### Do and Learn -

In the following numbers, write commas according to the International Numeration System and also write in words: -

S.No.	Numbers in	Number	Number in words
	digits	with	
		Commas	
1.	2345671	2,345,671	Two Million Three Hundred Forty-Five
			Thousand Six Hundred Seventy-One
2.	45674671		
3.	13456789		
4.	91756781		$\rightarrow$
5.	13456781		
6.	1345671	2	A A

#### Table 1.7

#### Reference about Number System in Vedic Leterature

In Rigveda, we can find references about mathematical words such as Ganak, Gana and Ganya etc, in the topics of Astronomy. By performing yagya at right time, good results were received and evil works were warded off. To find the right time to perform the yagya, one used the knowledge of Astrology which is based on movement of stars and planets.

 इमा मे अग्न इष्टका घेनवः सन्त्वेका च दरा च दरा च रातञ्च रातञ्च । सहस्रश्च सहस्रञ्चायुतश्चायुतश्च नियुतञ्च नियुतश्च ॥२॥ अर्बुदञ्च न्यर्बुदश्च समुद्रश्च मध्यञ्चान्तश्च परार्घश्च । एता मे अग्न इष्टका घेनवः सन्त्वमुत्रामुष्मिऌ्यौ के ॥३॥

#### (शुक्लयजुर्वेद काण्व शाखा 18/2-3)

The gratest contribution by the Vedas to the filed of mathamatics is the invention of numbers and decimal number system.

In the above-mentioned mantra, there is evidence about the numbers in powers of ten, namely ten, hundred, thousand and so on up to 1000 crores.

### असञ्चाता सहस्राणि ये रुद्रा अधि भूम्याम् । तेषाँ सहस्रयोजने व धन्वानि तन्मसि ॥

#### यजुर्वेद १६-५४

In the above-mentioned mantra, there is reference about thousands and infinite numbers.

In the following mantra taken from Taittiriya Samhita, there is a mention about decimal system of numbers and writing number names of numbers.

शताय स्वाहा सहस्राय स्वाहाऽयुताय स्वाहा नियुताय स्वाहा प्रयुताय स्वाहा-ऽर्बुदाय स्वाहा न्यर्बुदाय स्वाहा समुद्राय स्वाहा मध्याय स्वाहाऽन्ताय स्वाहा परार्धाय स्वाहोषसे स्वाहा व्युष्ठौ स्वाहोदेष्यते स्वाहोद्यते स्वाहोदिताय स्वाहा सुवर्गाय स्वाहा लोकाय स्वाहा सर्वस्मै स्वाहा।

(तैत्तिरीय संहिता- 7/2/20)

One can find the references about the decimal number system in the texts Yajur-Veda (17/2), Leelavati (1.2) and Taittiriya Samhita (4/4/41) other than Taittiriya Samhita-7/2/20.

The following definitions are given in Taittiriya Samhita as per the mantra mentioned above. -

Dacimal system-				
10 <sup>2</sup> = Shat (शत्)	10 <sup>3</sup> = Sahastra(सहस्र)	10 <sup>4</sup> = 1 Ayut (अयुत)		
$10^5 = \text{Niyut}$ (नियुत्त)	10 <sup>6</sup> = Prayut (प्रयुत)	10 <sup>7</sup> = Arbud (अर्बुद्)		
$10^8 = Nyarbud (न्यार्बुद्)$	10 <sup>9</sup> = Samudra (समुद्रा)	10 <sup>10</sup> = Madhya (मध्य)		
10 <sup>11</sup> = Ant (अन्त)	10 <sup>12</sup> = Parardha (परार्धः)	10 <sup>13</sup> = Ushas (उषाः)		
10 <sup>14</sup> = Vyushti (व्युष्टि)	10 <sup>15</sup> = Deshyat (देश्यत)	10 <sup>16</sup> = Udhyat (उध्यात्)		
10 <sup>17</sup> = Udit (उद्ति)	$10^{18} = $ Suvarg (सुवर्गः)	10 <sup>19</sup> = Lok (लोक)		

#### Common arithmetic operations

Common arithmetical operations such as addition or division are found in a very sophisticated form in the Vedic literature. In the Taittiriya Samhita, the names of 10 loka are given according to the decimal system of numbers. In this way, the list of numbers mentioned in Taittiriya Samhita is the evidence for not only the knowledge of decimal system of numbers, but also writing the numbers in scientific notation and inventing names for the large numbers are available.

#### Understanding Measuring Units

#### (i) Unit of Length

In the Sanskrit literature, the following verse is found for measurement of length in the works of Lilavati's Mathematics.

# यवोदरैरङ्गुलमष्टसंख्यैर्हस्तोऽङ्गुलैः षड्रुणितैश्चतुर्भिः । हस्तैश्चतुर्भिर्भवतीह दण्डः कोश्वः सहस्रद्वितयेन तेषाम् ॥

(लीलावती, परिभाषा : 5)

The words give us the relationship between the different measures of length which are given below.

One Angul = 1.763 cm. One hath = 45.72 cm
One dand = 1.5 to 2.0 m. One kosh = 3 to 4 km.

In general, we use centimetre (cm), meter (m), and kilometre (km) to measure lengths in modern days. Now, let us learn the relationship between the units of length.

10 mm = 1 centimetre (1 cm) 100 cm = 1 meter (1 m.) 1000 meter = (1 km)

**Deepika** - Can you tell me how many centimetres make 1 kilometre?

Manisha - Yes, I can. It is as given below.

1 kilometre = 1000 meters  
= 
$$1000 \times 100 \text{ cm}$$
 {1 meter = 100 cm}  
= 1, 00,000 cm

Deepika - Yes, absolutely correct.

(ii) Unit of Weight: -

In Lilavati's work, a verse describes the ancient scale to measure weight, which is given below.

```
तुल्या यवाभ्यां कथिताऽत्र गुञ्जा वल्लस्त्रिगुञ्जो धरणं च तेऽष्टौ ।
गद्याणकस्तदुद्वयमिन्द्रतुल्यैर्वल्लैस्तथैको धटकः प्रदिष्टः ॥
```

(लीलावती, परिभाषा : 3)

The meaning of the above verse gives the relationship between the different units of weight as given below.

1 karshas = 1 suvarna

1 maasha = 0.97 gram

1 karshas = 15.52 gram

In our daily life, we use standard weights of grams and kilograms to weigh any solid objects. The relationship between the units of weights is as follows

> 1000 milligram = 1 gram 1000 gram = 1kilograms

We can find, yet another definition for weights in Lilavati's Mathematics

```
दशार्धगुञ्जं प्रवदन्ति माषं माषाह्वयैः षोडशभिश्च कर्षम् ।
कर्षैश्चतुर्भिश्च पलं तुलाज्ञाः कर्षं सुवर्णस्य सुवर्णसंज्ञम् ॥
```

(लीलावती, परिभाषा: 4)

The above verse suggests the relationship between the different units of Weights as used by the specialists on weights, which are given below

five gunjjas (गुन्जा) = one maasha (माशा) sixteen maashas (माशा)= karsh (कर्ष) four karshas (कर्ष) = one pal (पाल) Golden karsh (स्वर्णकर्ष) = one suvarna.(सुवर्ण) 1 Karsh (कर्ष) = 1 suvarna.(सुवर्ण) 1 Maash (माशा) = 0.97 grams, 1 Karsh (कर्ष) = 15.52 grams (ग्राम)

You must have seen weights at the goldsmith's shop, there are weights for weighing smaller than a gram. These weights are in milligrams.

1 gram = 1000 mg

(iii) Measuring Units of Liquids: - Recall that theLiquids such as (milk, oil, petrol etc) are measured in litres or millilitres. Let us revise the relationship between litres and millilitres.

1 litre = 1000 ml

#### Points to remember: -

Dear students! In all the units of measurements, the words mille, centi and kilo, are used. One kilo metre / gram/litre means thousand

metre/ gram/ litre and it is the biggest unit that we use. Centi metre/ gram/ litre means hundredth part of metre/ gram/ litre, mille metre/ gram/ litre means thousandth part of metre/ gram/ litre and it is the smallest unit.



# EXERCISE 1.1

- 1. Select the correct answer for the following multiple-choice questions.
- (a) Using the digits 0 and 3, Form the smallest 4- digit number repeating both the digits for equal number of times is \_\_\_\_\_.
  (I) 3003 (II) 3030 (III) 3300 (IV) 0033
- (I) 3003
  (II) 3030
  (III) 3300
  (IV) 0033
  (b) 1 million =..... lakh
  (I) 10
  (II) 100
  (III) 1000
  (IV) 10000
  (c) 1 billion =..... million
  (I) 10
  (II) 100
  (III) 100
  (IV) 10000
- (d)  $1 \text{ lakh} = \dots \text{ ten thousand}$ (I) 1 (II) 10 (III) 100 (IV) 1000
- (e) 1 Kilometre = ..... meter (I) 10 (II) 1000 (III) 100 (IV) None of these
- (g) Using the International System of numeration put commas (,) in the number 43810138.
  - (I) 43,810,138(II) 43,81,01,38(III) 4, 38, 10,138(IV) 438, 10,138

(i) Using the Indian System of numeration, put commas in thenumber 4556132.

(I) 45, 56,132	(II) 4,55,6,132
(III) 4,556,132	(IV) 4,556,132
Which of the following is true?	
(I) 310 > 301	(II) 310 < 301
(III) 310 = 301	(IV) None of these

(j)

- 2. Using the digits 2, 5, 0 and 3. Formfive, 4- digit numbers.
- 3. Using the digits 5, 4, 6,7 and 8, form the smallest and the greatest 5-digit number?
- 4. Using the digits 3, 7, 5 and 4 make the smallest and the greatest three-digit number?
- 5. Using the digits 4, 8, 9, 0 and 1, make the largest five-digit number and also write its unit digit?
- 6. Using any one of the digits twice of 3, 6 and 5 form the greatest 4 digits number.
- 7. Using any one of the digits of 3, 0 and 7 twice, form the greatest 4 digits number.
- 8. Write all the prime natural numbers which are less tan 13.
- 9. Write all the prime natural numbers between 5 and 37 (both inclusive).
- 10. A press note says, in the year the number of books sold related to Vedic Literature 2022-23 is 8,40,000 and in 2023-24 is 9,00,400. Find the year in which more number of books is sold, and how much?
- 11. Ramprasad stiches five kurtas everyday. If he works on all the Sundays, find the number of kurtas that he stiches in the month of February 2024. (28 days).
- 12. Write the following numbers in their standard form (in numbers): -1. Five thousand nine hundred and ten
  - 2. Five lakh twelve thousand one hundred three
  - 3. Five thousand seven
  - 4. Two crore seventeen lakh thirty-one thousand three hundred five
  - 5. Twenty-three lakh fifteen thousand twelve

13. Compare the following numbers using the symbols (<, > and =): -

 1.
 5678
 5768

 2.
 6890
 9068

 3.
 234
 432

 4.
 567
 657

 5.
 70130
 70310

14. Fill in the following blanks:

- 1. 1 kilo gram = ..... gram.
- 2. 100 cm. =.....metres.
- 3. 1 Crore = ..... Lakh.
- 4. 1 litre =.....ml.
- 5. 100 lakhs =.....crores.

# We learned that-

- Number System can be classified into Natural numbers (1, 2, 3, 4, ....), Whole numbers (0, 1, 2, 3 ....), Even numbers (2, 4, 6, 8 ....), Odd numbers (1, 3, 5 ....), Integers (....., 3, -2, -1, -0, 1, 2, 3.....) and prime numbers (2,3, 5, 7,11,....)
- 2. While forming numbers from the given digits, the greatest number is formed by writing the digits in descending order (largest to smallest).
- 3. Commas are used in writing and reading numbers.
- 4. In the Indian System of numeration, the first comma is placed before three digits starting from the right and every two digits thereafter.
- 5. In the International System of numeration, commas are placed before every three digits starting from the right side.
- 6. Discussed about the Numeration System in Vedic literature.

# Chapter - 2

# Playing with Numbers

Dear Vedic students! We must have played many types of games such as cricket, football etc., in our daily life. In this chapter we will play with numbers and their digits. You must have read about numbers in the previous chapter. In this chapter we will revise the concepts of numbers and also further explore.

#### Subsequent numbers (successor):-

By adding 1 to the given number, we get its next number. The new number obtained is called the successor of the given number.

Example:	23	+	1	=	24
	24	+	1	=	25

Hence, 24 is the successor of 23, and 25 is the successor of 24.

# Previous Number (Predecessor):-

On subtracting 1 from a given number we get its previous number. The new number obtained is called the predecessor of the given number.

## Example:

19 –	1	=	18	
15 –	1	=	14	

Hence, 18 is the predecessor of 19 and 14 is the predecessor of 15.

# Ascending order:-

In Vedic literature, we find the reference about the ascending order of numbers in Taittiriya Samhita (तैत्तिरीयसंहिता) which is given below

# एकान्नविश्वात्यै स्वाहा नवविश्वात्यैस्वाहैकान्न.....वाताय स्वाहा।

(तैत्ति. संहिता. 7/2/11)

Ascending order (increasing) order of number is the arrangement of numbers in the given group from smallest to largest.

**Example 1:** Write the numbers 2434, 6892, 1234 and 321 in their ascending order.

Solution : Given numbers 2434, 6892, 1234 and 321

Arranging the numbers from smallest to largest, the numbers in their ascending order is-

321 < 1234 < 2434 < 6892

Example 2 : Write the numbers 1345, 3462, 2461 and 1465 in their ascending order.

Solution: Given numbers 1345, 3462, 2461 and 1465

Arranging the numbers from smallest to largest, the numbers in their ascending order is -

1345 < 1465 < 2461 < 3462

Descending (Decreasing) order:-

Descending order (Decreasing) order of number is the arrangement of numbers in the given group, from largest to smallest.

**Example1:** Write the numbers 1984, 2848, 92345 and 6841 in their descending order.

**Solution:** Given numbers 1984, 2848, 92345 and 6841

Arranging the numbers from largest to smallest, the numbers in their descending order is -

92345 > 6841 > 2848 > 1984

**Example 2:-** Write the numbers 2374, 7894, 1257 and 926 in their descending order.

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Solution: Given numbers 2374, 7894, 1257 and 926

Arranging the numbers from largest to smallest, the numbers in their descending order is -

7894 > 2374 > 1257 > 926

#### Divisibility Rules of Numbers:-

In mathematics, divisibility tests are divisibility rules help us to find the divisibility of numbers.

A number (p) is said to be divisible by another number (q), exactly, if the quotient is a whole number and the remainder is zero on division. If the remainder is any number other than zero, we conclude that the number (p) is not divisible by the given number (q).

Let us learn the method of identifying whether a given number is divisible by 2, 3, 4, 5, 6, 8, 9, 10 or 11 without actually dividing the numbers.

#### Divisibility Rule of 2: -

The numbers whose unit digit is 0,2,4,6 or 8 (multiples of two or even numbers) are divisible by 2.

#### Divisibility Rule of 3: -

If the sum of the digits of a number is a multiple of 3, the number is divisible by 3.

**Example:** Is the number 1236 divisible by 3 or not?

Solution:

Adding all the digits of the given number

1 + 2 + 3 + 6 = 12

12 is a multiple of 3 hence, 1236 is divisible by 3.

#### Divisibility Rule of 4: -

If the last two digits (tens and ones) of anumber is divisible by 4 are ending with 00, the number is divisible by 4.

**Example**: 4612, 3516, 9532, 200

#### Divisibility Rule of 5: -

If the unit digit of a number is 0 or 5, the number is divisible by 5.

**Example:** - 105, 1005, 2005, 2010

**Divisibility Rule of 6:-** If a number is divisible by both 2 and 3, the number is also divisible by 6

**Example:** 6, 18, 30, 36, 12

#### Divisibility Rule of 7: -

If the sum of product of the unit digit of the given number and 5, and the number formed by rest of the digit is divisible by 7, the given number is divisible by 7.

If the difference between the product of unit digit of the given number and 2, and the number formed by rest of the digit is divisible by 7, the given number is divisible by 7.

Example: 343

In 343, the unit digit is 3.

 $= 3 \times 5 = 15$ 

Adding to the number formed by the remaining digits

34 + 15 = 49, 49 is divisible by 7. Hence 343 is divisible by 7

#### Example:-

Is 256 divisible by 7 or not?

#### Solution

Unit digit of the given number is 6. Multiplying 6 by 2.

6×2 =12

Number formed by the remaining digits: 25

Subtracting 12 from 25 we get

25 - 12 = 13

13 is not divisible by 7. Hence 256 is not divisible by 7

# **Divisibility Rule of 8:**

In the numbers having 4 or more digits, if the last three digits of the number is divisible by 8 or 000, the number is divisible by 8.

```
Example: 99216, 82216, 10216, 73512, 5000
```

#### Divisibility Rule of 9:

If the sum of all the digits of a number is divisible by 9, the number is divisible by 9.

**Example:** 4608,

Adding all the digits of the number 4 + 6 + 0 + 8 = 18

18 is divisible by 9 hence, 4608 is divisible by 9.

#### Divisibility Rule of 10:

If the digit at the unit place of a number is 0, the number is divisible by 10.

**Example:** 10, 350, 450, 45670

#### Divisibility Rule of 11:

If the difference between the sum of the digits at the odd places and the sum of the digits at the even places from the right of a number is 0 or is divisible by 11, the number is divisible by 11.

**Example**: 308 8 + 3 = 11 0 11 - 0 = 11

 $61809 \quad 9 + 8 + 6 = 23 \qquad 0 + 1 = 1 \qquad 23 - 1 = 22$ 

Since, 22 is divisible by 11, 61809 is divisible by 11.

Which of the following numbers in the table are divisible by 2, 3, 4, 5, 6, 8, 9 and 10? Use the rules of divisibility and solve. (Complete the following table by writing yes or no.) against each number.

NT	Are divisible by							
Numbers	2	3	4	5	6	8	9	10
128	Yes	No	Yes	No	No	No	No	No
990								
1586								
275								
210								
724								

# Table 2.1

# **EXERCISE 2.1**

- 1. Select the correct option for the following multiple-choice questions.
  - (a) The difference between a number and its predecessor is

- (b) We have to subtract \_\_\_\_\_ from the number to get its predecessor.
  - (I) 1 (II) 2 (III) 3 (IV) 4
- (c) To find the successor of a number, we have to add \_\_\_\_\_\_to the number.
  - (I) 1 (II) 2 (III) 3 (IV) 4
- (d) 128 is divisible by \_\_\_\_\_.
  (I) 2 (II) 5 (III) 3 (IV) 10
- (e) 210is not divisible by \_\_\_\_\_.
  - (I) 2 (II) 5 (III) 4 (IV) 10
- Write the predecessor of each of the following numbers (i) 191
  (ii) 8003
  (iii) 3012
  (iv) 5734

3. Write the successor of each of the following numbers -

- (i) 310 (ii) 1156 (iii) 2574 (iv) 8752
- 4. Arrange the following numbers in their ascending order -

(i)	9824	7894	3456	2897
(ii)	3456	7899	3576	8924
(iii)	1345	5760	7823	9231
(iv)	3561	3662	3777	2421
(v)	1345	5676	2436	3767

5. Arrange the following numbers in their descending order.

(i)	3556	3546	2421	3212
(ii)	2451	2556	2776	6781
(iii)	2571	2921	3678	3576
(iv)	2421	<mark>34</mark> 51	2421	<mark>345</mark> 1
(v)	2451	3751	3891	3915

- 6. Are all natural numbers also whole numbers? Are all whole numbers also natural numbers?
- 7. What do you understand by divisibility rules?
- 8. State the divisibility rules of 3 and 5.
- 9. Starting with 345, write the next five consecutive whole numbers.
- Write three consecutive whole numbers immediately preceding 240.
- 11. Write True/False -
  - (i) Zero is a natural number.
  - (ii) The predecessor of 2 is 0.
  - (iii) 423 is divisible by 6.
  - (iv) 112346 is divisible by 11.
  - (v) 410 is a predecessor of 409.
  - (vi) 10 is not a predecessor of 11.

(vii) The predecessor of a two-digit number can be a single digit number.

#### Factors and Multiples

You have already learnt about the divisibility of a number in the previous exercise and have the initial knowledge of composite and prime numbers. Now, we will discuss about factors and multiples of a number.

#### Factors

The number which divides a given number completely without leaving any remainder is called its factor. Therefore, the factor of a number is its perfect divisor.

We know that the numbers which exactly divide 6 are 1, 2, 3 and 6. Similarly the numbers which exactly divide 10 are 1, 2, 5 and 10. Hence, the factors of 6 are 1, 2, 3, and 6 and the factors of 10 are 1, 2,5 and 10.

# Do and Learn-

Number	Factors
12	1,2,3,4,6,12
20	1,2,4,5,10,20
4	1,2,4
27	
48	
136	
96	

#### Complete the following table

From the above table we conclude:

- 1. One is the factor of all the numbers.
- 2. Every number is a factor of itself.
- 3. Every factor of a number divides it exactly.
- 4. The number of factors of a given number is finite.

#### Multiples

The results obtained on multiplying the given number with natural numbers (1,2,3,...), in order, we get its multiples.

**Example:** On multiplying 5 with natural numbers 1, 2, 3, 4 respectively, the products obtained are 5, 10, 15, and 20. These numbers are multiples of 5. Since natural numbers are infinite, the number of multiples of any number is infinite.

Do and Learn

Complete	the foll	owing	table
----------	----------	-------	-------

Number	Multiples of the Number
3	3, 6, 9, 12, 15,
6	6, 12, 18,,
12	12, 24, 36, 48,
18	85
20	
25	

From the above table we conclude:

- 1. Every number is a multiple of itself.
- 2. Number of multiples of any number is infinite.
- 3. Every multiple of a number is either greater than or equal to the number.

#### Least Common Multiple

Among the common multiples of the given two or more numbers, the smallest one is called least common multiple of the numbers. The least common multiple is also written as LCM.

The least common multiple of two or more given numbers is the smallest common multiple that divides the numbers exactly.

**Example:** Find the least common multiple of the numbers 12 and 30 **Solution:** Multiples of 12 = 12, 24, 36, 48, 60, 72, 84, 96, 108,

120, 132,...

Multiples of 30 = 30, 60, 90, 120, 150...

The common multiples of 12 and 30 = 60, 120 and so on Therefore, the least common multiple of 12 and 30 = 60

Methods of finding Least Common Multiple:

1. Method of Multiples2 Prime factorization methodFinding Least Common Multiple using method of multiples

We adopt the following steps to find the Least common multiple (LCM) of two or more numbers.

Step 1: write the multiples of the given numbers.

Step 2: Find the common multiples among the multiples.

Step 3: Find the least number among the common multiples.

The result of Step3 is the required LCM of the given number

**Example:** We have already found the least common multiple of 12 and 30 in the previous **Example**.

**Prime Numbers:** The numbers whose factors or 1 and itselfare called prime numbers.

Example:

2, 3, 5, 7,11... Remember that 1 is neither prime nor composite number.

## 2. Prime factorization method of finding Least Common Multiple

- (i) First find the prime factors of the given numbers.
- (2) Find the minimum factors of the prime factors.

(3) After writing these minimum factors, write all the other factors in their product form with maximum powers. Find the product of the expression to find the least common multiples of the numbers.

Example: Find the least common multiple of 25, 48 and 75?

Solution :	5	25	2	48	3	75
	5	5	2	24	5	25
	P	1	2	12	5	5
		1	2	6		1
			3	3		
			$\wedge$	1		
25	= 5	× 5		$= 5^2$		
48 =	= 2	× 2 ×	$2 \times 2 \times 3$	$= 2^4$	$\times 3^1$	
75	= 3	× 5 ×	5	= 3 <sup>1</sup>	$\times 5^2$	

Here the highest powers of the prime factors 2, 3 and 5 are 4, 1 and 2 respectively.

Hence, required least common multiple =  $2^4 \times 3^1 \times 5^2$ =  $16 \times 3 \times 25$ = 1200

The numbers 25, 48,75 divide 1200 exactly.

## Greatest common factor (Highest Common Factor):

Among the common factors of the given numbers, the highest one is called highest common factor of the given numbers. The highest common factor is also written in its sort form as HCF.

In other words, the greatest common factor of two or more numbers is the greatest number which divides the numbers exactly.

**Example:** Find the greatest common factor of the numbers 6 and 18

**Solution:** Factors of 6 = 1, 2, 3, 6

Factors of 18 = 1, 2, 3, 6, 9, 18

Common factors of 6 and 18 = 3, 6

Among the common factors, 6 is the highest common factor.

Hence, the HCF of 6, 18 is 6

Methods to find the Highest Common Factor

1. Method of Factorization 2. Prime Factorization Method

1. Method of finding HCF by Factor Method

**Example:** Find the HCF of 6, 12 and 18.

**Solution:** Factors of 6: 1, 2, 3, 6

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 18: 1, 2, 3, 6, 9, 18

Common factors of (6, 12, 18): 1, 2, 3, 6

Among the common factors, 6 is the highest common factor.

Hence, the HCF of 6, 12, 18 is 6

# 2. Prime Factorization Method of finding HCF

1.Express the numbers as the product of their prime factors.

2. The product of minimum common factors is the HCF of the given numbers

**Example**: Find the highest common factor of 144, 180 and 192, by the method of prime factorisation.

**Solution**: Expressing the given numbers as the product of their prime factors:

2	144	2	180		2	192
2	72	2	90		2	96
2	36	3	45		2	48
2	18	3	15		2	24
3	9	5	5		2	12
3	3			5	2	6
	1				3	3
						1

 $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^{4} \times 3^{2}$   $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^{2} \times 3^{2} \times 5$  $192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^{6} \times 3$ 

The minimum common factors of all the three numbers are 2, 2, 3

Hence, the highest common factor is  $2 \times 2 \times 3 = 12$ .

Thus, the highest common factor is 12. 12 divide all the numbers exactly.

# Problem based on LCM and HCF

**Example:-** Find the greatest number that divides 2930 and 3250, and leaves the reminders 7 and 11 respectively.

Solution: According to the problem

$$2930 - 7 = 2923$$

$$3250 - 11 = 3239$$

Hence, Finding HCF of 2923 and 3239, we get

HCF (2923, 3239) = 79

Thus, the required number is 79.

Example: An army commander arranges the soldiers in the rows of 12,

15, 18 and 20. He expects the formation to be a complete one. Find the least number of soldiers required.

Solution: Finding LCM (12, 15, 18, 20), we get 180

Hence, the least number of soldiers required is 180.

#### Exercise - 2.2

- 1. Select the correct option for the following multiple-choice questions. (a) Which of the following numbers is not a factor of 36? (II) 9 (III) 4 (I) 8 (IV) 12 Which of the following numbers is not a multiple of 6? (b) (I) 12 (II) 21 (III) 24 (IV) 36 Which of the following numbers is not a factor of 35? (C) (I) 1 (II) 5 (IV) 8 (III) 7 2 Write the 5 multiples of the following numbers. 16 (a) 12 (b) 17 (c) 100 (d) Write all the factors of the following numbers. 3. (c) 31 (d)40(a) 18 (b) 13 4. Write all the multiples of 8 which are less than 40. 5. Write all the factors of 24 which are less than 24. 6. Write the least common multiple of each of the following pairs -48,60 12, 15, 3 (a) (b) 12, 15, 45 (d) 45, 15, 10 (c) 7. Find the highest common factor of the following numbers -45 and 105 (a) 144 and 180 (b)
  - (c) 12 and 18 (d) 17 and 51
  - (e) 18, 54, 63 (f) 27, 81

- 8. Find the least common multiple 6, 8 and 12.
- 9. Find the greatest divisor which on dividing 2400 and 1810 leaves the reminders 6 and 4 respectively.
- 10. Find the least number that needs to be subtracted from 10,000 so that it divides 32, 36, 48, and 54 exactly.
- 11. Find the greatest number which on dividing it by 38, 45, and 52 leaves the reminders 2, 3 and 4 respectively
- 12. Find the least number that dividesit by 18, 24, and 42, and leave the reminder 1
- 13. Find the least number which leave the reminders 33, 59 and 98 on dividing. It by 52, 78, and 117 respectively.
- 14. The ratio between two numbers is 5: 6 if their least common multiple is 120, find their highest common factor.
- 15. The product of two numbers is 7168. If their highest common factor is 16 Find the numbers.
- 16. Find the two pairs of composite numbers such that their difference is2.
- Classify and write the numberwhich are less than 25 as prime and Composite number.

# We learned -

- 1. By adding 1 to a number, we get its successor.
- 2. By subtracting 1 from a number, its predecessor is obtained.
- 3. Arranging the numbers starting from smallest to largest is called ascending order of numbers.
- 4. Arranging the numbers starting from largest to smallest is called descending order of numbers.
- 5. The number which completely divides a number is called a factor of that number.
- The numbers obtained by multiplying a natural number
   (1, 2, 3, 4...), individually with the given number are called its multiples.
- The smallest multiple among the common multiples of two or more numbers is called the least common multiple of those numbers.
- 8. Out of the common factor of two or more numbers, the highest factor is called the highest common factor of those numbers.

# Chapter – 3 Integers

#### Dear student,

You are aware of natural numbers namely 1, 2, 3.... On including 0 to this group of natural numbers, the new group of numbers so obtained is called whole numbers 0,1,2,3.... The numbers which are less than 0 are negative numbers -1, -2, -3.... Now, we know that the numbers can be negative like -1, -2, -3.... The new group obtained on including the whole numbers  $\{0, 1, 2, 3...\}$  with negative numbers -1, -2, -3,...) is called group of integers. The group of integers is represented by I.

#### Representation of Integers on a Number Line:

Number line of integers has both positive and negative integers "meeting at 0. hence, 0 is neither positive nor negative. The number line of integers is drawn as shown below,

From the above picture, we can conclude, all the positive integer which are lying on the right side of zero is greater than all the negative integers which are lying on left of 0.

#### Example:

(i)	- 15 < 16	(ii)	-3 < 3	(iii)	17 >-17
(iv)	12 >-28	(v)	-21 < 212	(vi)	-72 <- 21

#### Important results of integer on a number line:

- Every positive integer is greater than every negative integer.
- Zero is smaller than every positive integer.
- Zero is greater than every negative integer.
- Larger the positive integer, smaller its corresponding negative integer.
- The negative of a negative integer is a positive integer.

-(-a) = a

• On a number line of integers, as we move from left to right the value of the integers increase, and as we move from right to left their values decrease.

#### Do and Learn:

#### Compare the following pairs of numbers:

(i)  $(100) \_ (-200)$  (100) > (-200) (100) is greater than (-200)

- (iii) (201) <u>(101)</u>
- (iv) (-150) \_\_\_\_(-200) \_\_\_\_\_

Let us now learn to add (+), subtract (-) of integers

**Note: -** We should observe the sighs  $(+, -, \times, \div)$  carefully while doing operations of the integers

The number that comes after each number is called its successor and the number that comes before it is called its predecessor. Write the successor and predecessor of the numbers in the table given below.



Number	Successor	Predecessor
-06	-5	-7
0		
12		
-09		
-19		

# Use of negative numbers:

 A parrot is flying at a height of + 30 m above sea level. Just below it there is a fish swimming 5 m below the sea level i.e. (- 5) m. Height is indicated by a positive sign and depth (down) by a negative sign. Figure 3.2





2. A temple is on a hilltop at a height of 100 m from the ground. And a trench which has a depth of 25 meters below the ground level (- 25) metres.

#### Do and Learn -

- 1. Write any 3 numbers less than  $0 = \dots$
- 2. 30 meters below the sea level =.....

#### Think:

Prashant represents a positive integer for the stairs to go to the terrace and a negative integer to go to the godown the ground level is denoted by 0, as shown below.



Figure 3.4

Has Prashant represented these integers correctly?

#### Sum of two positive integers:

In vedik literature, we find the rules of addition of positive integers in Brahmasphutasiddhanta, Kuttakadhyaya (ब्रह्मस्फूतसिद्धांत, कुट्टकाध्याय) which is given below.

धनयोर्धनमृणमृणयोर्धनर्णयोरन्तरं समैक्यं खम्। ऋणमैक्यं च धनमृणधनशून्योः शून्ययोः शून्यम्॥

(ब्रह्मस्फुटसिद्धान्त,कुट्टकाध्याय: 30)

According to the above mantra,

- the sum of two positive integers is positive and
- The sum of two negative integers is negative.
- Sum of positive and negative integers is the difference between them.
- Sum the integers with opposite signs is zero.
- Sum of zero and positive or negative integer is the same integer.
- The sum of two zeros is zero.

Other than Brahmasphutasiddhanta, we find the reference about addition of integers in page no.5 of beeja ganitam.

(1) Sum of two positive integers



(+4) + (+2) = +6

(2) Addition of two negative integers (-4) + (-3) = -7



When we add two positive integers using number line, and move towards right for two times. Hence, we are at right of zero which is positive side. Thus, the sum of two positive integers is always positive.

Similarly, when we add two negative integers, using a number line we move towards left for two times. As we are on the left of zero, which is the negative side, the sum of two negative integers is always negative (-).

#### 3. Addition of a positive and a negative integer

To add a positive and negative integer, without considering their signs find the difference between them, by subtracting the smaller one from the larger one and right the sign of the larger integer.



(iii) 15 + (-17)= 15 - 17= -2(iv) (-70) + (+100)= -70 + 100= +30

## Do and Learn

Complete the following table:

S. No.	Sum	Sign Positive/Negative	Result
1.	(-6) + (+2)		
2.	(-9) + (-1)		9
3.	(+3) + (-5)		
4.	(+2) + (+4)		Ĩ

Subtraction (difference) of integers:

In the Brahmasphutasiddhanta, the method of subtracting an integer from the other has been explained. The following verse gives us the method.

# ऊनमधिकाद्विशोध्यं धनं धनादणादधिकमूनम्। व्यस्तं तदन्तरं स्यादृणं धनं धनमृणं भवति॥

#### (ब्रह्मस्फुटसिद्धान्त,कुट्टकाध्याय: 31)

In the above verse, it is mentioned that on subtracting smaller positive integer from a larger positive integer, the result is positive. Similarly on subtracting the smaller negative integer from a larger negative integer, the result is negative. From an integer, when a larger

positive integer is subtracted the result is negative and when larger negative integer is subtracted the result is positive.

Other than, Brahmasphutasiddhanta, we can find the references in Bijaganitam (Dhanarnasankalan p. 7) also, above the method of subtraction of integers.

Before we start subtracting the integers, it is necessary to understand the product of the signs of the integers.

- $(+) \times (+) = (+)$  $(-) \times (-) = (+)$  $(-) \times (+) = (-)$  $(+) \times (-) = (-)$
- 1. Deference between two positive integers

$$(+7) - (+5)$$

=7 - 5 =2

(We know that  $(+) \times (-) = (-)$ )

2. Deference between two negative integers (-7) - (-5)= -7 + 5 = -2

**Note:** Negative of a negative integer is positive.

-(-a) = +a or a (where *a* is positive integer)

3. Deference between a positive and a negative integer:

(a)(-7) - (+15) using the rule: (-) × (+) = (-) = -7 - 15 = -22
(b) (15) - (-7) using the rule: (-) × (-) = (+) = 15 +7 =22
(c) (+4) - (-16) using the rule: (-) × (-) = (+)

$$=4 + 16$$
  
= 20  
(d) (+4) + (-5) using the rule: (+) × (-) = (-)  
=4 - 5  
= -1

# Do and Learn –

Subtract the following integer.

(i)	(+ 7) - (+ 4)	रावदानचाप्र
(ii)	(+ 7) - (- 4)	=
(iii)	(-7) - (-4)	=
(iv)	(+7) - (-4)	=

# BODMAS:-

The order of operations (BODMAS) is referred in the following verse

का करके पुनि भाग कर, फिर गुण लेह सुजान।

ता पिछे धन, ऋण कर, भिन्न रीति यह जान ॥

В	=	Brackets	:	(), { }, [ ]
0	=	For * of	:	() <sup>2</sup> or $\sqrt{00}$
D	=	Division	:	÷
Μ	=	Multiplication	भव	x
А	=	(Addition)	:	+
S	=	Subtraction	:	-

In the letter series "BODMAS", 'B' for Brackets, 'O' for of, 'D' for division, 'M' for multiplication, 'A' for addition and 'S' for subtraction are used for the analysis.

## Rules for removing brackets: -

There are three types of brackets as show below.

- parentheses represented by () small bracket, Curly brackets represented by { }, Square brackets represented by [ ] Removing the parentheses means we simplify the expression inside the parentheses.
- 2. If there is a positive (+) sign before the parenthesis, the parenthesis is removed without changing the sign of the resultant integer of the expression inside it.
- 3. If there is a negative (–) sign before the parenthesis, the sign of the resultant integer obtained is changed on removing the parenthesis.
- 4. If there is a number before the parenthesis, the parenthesis is removed by writing the product of the resultant integer of the expression inside the parenthesis and the number present outside it.

Example: 1.  $30 - 5 \times 2 \text{ of } 3 + (19 - 3) \div 8$ Solution:  $30 - 5 \times 2 \text{ of } 3 + (19 - 3) \div 8$  By BODMAS rule:  $= 30 - 5 \times 2 \text{ of } 3 + 16 \div 8$ (remove the parentheses ())  $= 30 - 5 \times 6 + 16 \div 8$ (means 3 of 2 to  $2 \times 3 = 6$ )  $= 30 - 5 \times 6 + 2$  (I.e. 16 / 8 = 2) = 30 - 30 + 2(That is, multiply  $5 \times 6 = 30$ ) = 32 - 30(addition) (30 + 2 = 32)= 2 (Subtraction) (32 - 30 = 2)Simplify the following – Example: (i)  $120 - 20 \div 2$ 

Solution:  $120 - 20 \div 2$  {Part} = 120 - 10 {subtraction} = 110

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(ii)  $28-5 \times 6+2$ Solution: 28-30+2 Or = 28+2-30 = 30-30=0(iii) 17 + (-3) + 7Solution: 17-3+7 Or = 17+7-3=24-3=21(v)  $4-(-5) \times 4$ Solution:  $4+5 \times 4=4+20=24$ 

## Exercise - 3.1

- 1) Represent the following Integers on a number line.
  - (i) -4 (ii) 3 (iii) -7
- Write the integer with proper signs (+, -) to represent the following situations.
  - (i) Water is heating to  $60^{\circ}$ C.
  - (ii) Depositing Rs.500 in a bank account.
  - (iii) Krishna incurred a loss of Rs.30 in selling a book.
  - (iv) 70 meters below sea level.
  - (v) Weight loss of 20 kg

3) Use the signs (>, < and =) to indicate the smaller and larger integers.

(i)4
$$-8$$
 $(IV) - 4$  $-6$ (ii) $-7$ 12 $(V) - 2$ 5(iii)0 $-1$  $(VI)$ 1 $-6$ 

#### 4) Evaluate the following?



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(-12) + (+8)(x) (+300) + (200) =(v) = 6) Fill in the blanks.  $-3 + \dots = 0$ (i)  $11 + (-11) = \dots$ (ii) 14 -.... = 16 (iii) (iv) 7 +..... = 0  $(-3) + \dots = -7$ (v)  $(-4) + \dots = -8$ (vi) (iv)  $(-3) + \dots = -7$ Fill in the blanks by writing the proper symbols (>, < and =) 7)  $(-7) + (-2) \dots (-2) + (-4)$ (i)  $10 + (-20) \dots (-20) - (-10)$ (ii)  $40 - (-10) \dots - 40 + (+70)$ (iii) Solve (using BODMAS rules) 8) 3 + 5 - 101. 2.  $3 \times 4 + 2$ = ..... 3. -3+4-3=.....  $-3 \times (-2) + 1$ 4. = ..... 5.  $-7 \times 2 + 2$ =..... 6.  $2 \times 18 \div 2$ =..... 7.  $1 + 9 \times 3 - 3$ = ..... 8.  $2 + 14 \div 7$ =..... 9.  $2 + 21 \div 7 - 3$ =.....  $3 - 35 \div 5 + 4$ 10. 9) Write T for true and F for false –

(i) The smallest negative integer is -1.

(ii) The smallest positive integer is 1.

- (iii) Negative of a negative integer gives positive integer.
- (iv) 0 is a negative integer.
- (v) -5 is greater than 5.
- (vi) +20 is greater than -3.
- (vii) -100 is less than -200.
- (viii) 100 is greater than 50.
- (ix) The successor of -4 is -3.
- (x) The predecessor of -5 is -4.

#### We learned:-

 When we need numbers with negative sign (-), we have to go left from zero on the number line. These are called negative numbers.
 For Example: - we use negative signs to represent, the depth of a ditch, temperature and loss in business.

2. The collection of the above numbers is called integers.

Where - Negative integer = -1, -5, ...

Positive integers = 3, 6,

Zero: 0 (It is neither a positive nor a negative integer)

- 3. The predecessor and successor of an integer are obtained by subtracting 1 from the integer and adding 1to the integer respectively
- 4. Use of signs (+, -) to add integers.
  - (i) If the signs of the integers are the same, add the integers and write the result with the same sign.

For Example - (i) (+4) + (+4) = +8(ii) (-4) + (-2) = -6

(ii) When we have the different sings, without considering the sings subtract the smaller one from the lager one and write the sigh of the larger one

For Example - (i) (+4) + (-2) = +4 - 2 = +2(ii) (-4) + (+2) = -4 + 2 = -2

5. Use of signs in subtraction of integers

(i) Negative of a negative integer is positive.

Like - (i) -(-4) = +4

(ii) The negative of a positive integer is always negative and the positive of a negative integer is always negative.

Like - (i) -(+4) = -4

(ii) 
$$+(-4) = -4$$

- (iii) The positive (+) of a positive number is positive. As - + (+ 4) = + 4
- 6. Studied the law of BODMAS.

The alphabet series BODMAS

- O = For of \* : ()  $2 \text{ or } \sqrt{(* = \text{power of})}$
- $D = Division : \div$
- M = Multiplication : x
- A = Addition : +
- S = Subtraction : -
# Chapter – 4

# **Vedic Mathematics**

Dear students India is famous due to the knowledge given by our Vedic sages and gurus. "Vedic Mathematics", compiled by Jagadguruswami Bharati Krishna TirthaJi, has many methods to solve complex arithmetic calculations easily and quickly in minimum number of steps.

### Ekadhikena (One more than the given)

A dot (•) is placed above the unit digit (first digit from the right) of the given number to represent one more than the number. This dot is called Ekadhikena mark.

### Example:

One more than the Ekadhik		New number
number	representation	
1 more than 3	3	4
1 more than 7	7	8
1 more than 12	12	13
1 more than 125 in 2	125	135
1 more than 245 in 5	245	246

### Ekadhiken-purvena

The word "Ekadhiken-Purvena" has two words "Ekadhik" and "purve. Observe and understand the following carefully

**For Example :-** (i) In the number 42, the previous digit of 2 is 4.

(ii) The previous digit of 4 in the number 743 is 7.

(iii) The previous digit of 1 in the number 134 is 0.

Hence, the number 134 can be written as 0134.

### Attention:

In the number 4732, when the digit previous to 2 has to be increased by 1, is represented as 4732. The value of the representation in modern system of numbers is 4742.

Thus,

One more than the digit previous to 3 in 4732 is represented as 4732 and its value is 4832.

Complete the following table.

Number	Representation of	New
	EkadhikenPurven	Number
Digit 6 in 16	16	26
Digit 5 in 325	325	335
Digit 2 in 275	Ó275	1275
Digit 0 in 2017	Ż017	3017
Digit 3 in 2123		
Digit 5 in 2257		
Digit 9 in 2697		
Digit 1 in 2217	X	2
Digit 5 in 2854	2854	
Digit 1 in 2127		
Digit 7 in 197	197	207

Addition of Numbers Using "Ekadhiken" "Purvena" Rule:

In this method, while adding the digits if the sum is 10 or more than 10, a dot are placed above the previous digit of the number. This process is followed successively.

**Example:** Find the sum of the numbers 18 and 36.

Solution:

#### Steps:-

- (i) Sum of unit digits = 8 + 6 = 14
- (ii) 14 is more than 10, we will write 4 below 6 and place a dot above 3(digit before 6).
- (iii) 1 + 3 (3 = 3 + 1)

```
= 1 + 4 = 5
```

Example: Add 7 rupees 70 paisa, 23 rupees 45 paisa and 38 rupees 50 paisa.

Solution:

Rupee Paisa
7.70
<u>23.45</u>
+38.50
69.65

### Steps-

- 1) 0 + 5 + 0 = 5. Write 5 at the hundredths place of the result.
- 2) 7 + 4 = 11 which is greater than 10. The previous digit of 4 in 23.45 is
  3. Hence, Place a dot above 3. 11 10 = 1, 1+5=6. Hence, write 6 at tenths place of the result.
- 3) 7 + 3 = 11 Which is greater than ten. Hence, place a dot above the digit 2 which is the previous digit of 3 in 23.45. Now, 11 10 = 1, 1+8 =9. Write 9 at the ones digit of the result.

4)  $\dot{2}$ + 3 = 6. Write 6 at the tens digit of the result.

### Exercise: 4.1

- 1. Select the correct option for the following multiple-choice questions.
- (a) The Ekadhikena of the number 12 will be.(I) 11 (II) 13 (III) 21 (IV) None of these
- (b) The representation of Ekadhikena Purvenaof 5 in the number 125 is.
  - (I) 125 (II) 125 (III) 125 (IV) None of these
- (c) The result of Ekadhikena Purvena of the digit 5 in 215 is.
  - (I) 225 (II) 216 (III) 325 (IV) None of these
- 2. Complete the following table:

Number	Number with Ekadhiken Sign.	new number
Ekadhik of 0	Ò	1
Ekadhik of 135	135	136
Ekadhik of 2 in 246	Ż46	346
Ekadhik of 4 in 134		
Ekadhik of 5 in 245		
Ekadhik of 7 in 178		
Ekadhik of 17		
Ekadhik of 8 in 2486	्रभव भवत्यवः	
Ekadhik of 2 in 3124		
Ekadhik of 1 in 3124		

3. Complete the following table using the rule Ekadhikenapurvena:

Number	EkadhikenPurvena Sign	New Number
Digit 5 in 325	325	335
<b>Digit 0 in 780</b>	780	790
<b>Digit 8 in 318</b>	318	
Digit 0 in 207	Ż07	
<b>Digit 7 in 273</b>		
Digit 8 in 284		

Digit 4 in 345		
Digit 3 in 135		
Digit 5 in 135		
Digit 1 in 135	Ö135	1135
Digit 2 in 245		
Digit 7 in 741		

4. Find the sum of following numbers by the **Ekadhiken-apurvena** rule.

(ii)	(iii)	(iv)	(v)
37	83	39	26
+ 35	+ 18	+ 14	+ 36
		0	
	(ii) <b>37</b> + <b>35</b> 	$\begin{array}{cccc} (ii) & (iii) \\ 37 & 83 \\ + 35 & + 18 \\ \hline & & \\ \end{array}$	(ii) (iii) (iv)  37 83 39  + 35 + 18 + 14

### Eknunena:

A dot (•) is placed (one less than the given digit) below the number which has to be reduced by 1. This dot is called Ekneunen's mark.

Eknunen: To decrease by one.

Number	Eknunen Sign	New Number
Eknunen of 3	3	2
Eknunen of 3 in 234	234	224
Eknunen of 4 in 134	134	133

**Eknunena Purvena** (One less than the previous): – The **Eknunena Purvena** has two words namely "Eknunena" and "Purvena". Which mean "one less than the previous". Observe the **Examples** given in the following table and complete it.

**Example**: In the table given below, write the symbol for Ekanunena Purvena and hence evaluate:

Number	Eknunena Purvena	New Number
	Sign	
Eknunena Purvena of 3 in 234	234	134
Eknunena Purvena of 6 in 246	246	236
Eknunena Purvena of 3 in 731	731	
Eknunena Purvena of 4 in 543	C	
Eknunena Purvena of 1 in 714		
Eknunena Purvena of 4 in 241	A.S.	
Eknunena Purvena of 4 in 342		
Eknunena Purvena of 2 in 782		772

Parammitra number -

Activity: Utsav's grandfather says that before I explain the second method of subtraction (difference), it is necessary to know the concept Parammitra.

- **Utsav:** What are these Param Mitra numbers?
- Dada ji: Two numbers whose sum is 10, are called mutually Parammitra numbers.

**Example**: 8 is the parammitra number of two, similarly, 7 is the parammitra number of 3.

Khushi: Well, the parammitra number of 5 will be 5 only.

Grandfather - Yes, absolutely correct.

**Do and learn:** Complete the following table by writing the parammitra numbers of the numbers given bellow.

Number	Parammitra Number	Number	Parammitra Number
1	9	4	
7		5	
6		2	
3		6	

**Grandfather:** Let us learn how to subtract the number by using Parammitra numbers and applying the rule of Eknunen purven.

Example: Subtract 26 from 85.

Solution:

85

- 26 **59** 

#### Instruction -

- (i) We cannot subtract 6 from 5. Since, 4 is the Parammitra number of 6, we add 4 with 5 to get 4 + 5 = 9, we write 9 in the result.
- (ii) In 85, the digit previous to the digit 5 is 8, place the Nunen sign.
- (iii) (\$=7) on subtracting 2 from 7 we get 7 2 = 5. Write 5 on the tens digit of the result.

Example: Subtract 287 from 546.

Solution:

### Instruction -

	(i)	7 cannot be subtracted from 6, so parammitra number of
546		7 is 3. Now write in the result, the sum of 6 and
<u> </u>		parammitra number 3. $(6 + 3 = 9)$
259	(ii)	Now write a dot below the digit (4) which is previous to

(iii) (4 = 3) 8 can not be subtracted from 3 so adding 3 and the

the digit 6.

parammitra number of 8 (2), to get 5. Now, we will write 5 in the tens digit of the result.

(iv) Put a dot on the digit 5 which is occurring before 4.

So (5 = 4). On subtracting 2 from 4, we get 4 - 2 = 2.

Write 2 in the hundreds place of the result.

### Exercise: 4.2

- 1. Select the correct option for the following multiple-choice questions.
  - (a) Ekanunena of 14 is \_\_\_\_\_. (I) 11 (II) 13 (III) 12 (IV) None of these
  - (b) The symbol to represent Ekadhikena purvena of 3 in 134 is\_\_\_\_\_.

(I) 134 (II) 134 (III) 134 (IV) None of these

- (c) New number obtained on doing Eknunen purven on 4 of the number 348 is \_\_\_\_\_.
  - (I) 338 (II) 248 (III) 347 (IV) None of these
- 1) Write the Eknunen of the following numbers.

(i) 234	(ii) 133	(iii) 15	(iv) 18

2) Write the Eknunen purven of the following numbers.

(i) Digit 4 in 248(ii) Digit 4 in 1345(iii) Digit 8 in 1280(iv) Digit 6 in 3467

- (v) Digit 1 in 3421 (vi) Digit 1 in 3217
- 3) Subtract using Eknunen Purvena rule.

(i)	(ii)	(iii)	(iv)	(v)
43	84	56	568	842
- 17	- 57	- 39	- 279	+ 384

### Deviation:-

In Vedic mathematics, generally all the calculations are done easily based on 10, 100, 1000 or the powers of 10.

If its base is subtracted from the number, then the remainder is called deviation. Therefore, the value less or more than the base is called deviation. A value less than the base is called negative deviation and a value greater than it is called positive deviation.

	Deviation	= Num	ber - B	ase
--	-----------	-------	---------	-----

### Do and learn:-

Number	Deviation		
9	10 How much less than1		
85	100 How much less than+15		
102	100 How much more than + 2		
113	100 How much more than		
97	100 How much less than		
7	10 How much less than		
114	100 How much more than		
89	100 How much less than		

### Vinkulam:

In vedik mathematics, writing the numbers in their negative forms are called Vinkulam numbers.

Here, in this method by converting a number greater than 5 (6, 7, 8, 9) into a smaller number, the calculations become smaller, simpler and easier.

**Like -** 8 is 2 less than 10.

8

So

 $= 10 + \overline{2}$  $= 1\overline{2}$ 

= 10 - 2

(2 is written in vinkulam as  $\overline{2}$ )

Thus,

Vinkulam number of	7	=	10 – 3
		=	$10 + \bar{3}$
		-	13

Let us learn to find ordinary numbers by doing the inverse of the above operation.

**Example:** Write  $1\overline{4}$  in its General Form of Number.

 $1\overline{4} = 10 + \overline{4}$ 

Solution :

= 10 - 4 = 6

So,  $1\overline{4} = 6$ , the general form of  $1\overline{4}$  is 6.

**Example:** Convert 64 into its Vinkulam form of number:

Solution:

### Instruction:

64	(i)	Keep the number 4 at the unit digit as it is and replace
òī		6 by its Parammitra number with Vinkulam Sign draw
=044		Vinkulam line on number 4 which is the Parammitra of
$=\overline{4}4$		6.

(ii) Put a dot on 0 which is the previous digit of 4

(iii) As  $\dot{0} = 1$ , write 1 in place of  $\dot{0}$ .

Example: Convert 067 to Vinkulam number:

Solution:

067

### Instruction -

- (i) Parammitra number of 7 is 3. Represent it by vinkulam number.
- =  $0\dot{6}\overline{3}$  (ii) Now put a dot above 6 which is the previous =  $07\overline{3}$  number of  $\overline{3}$ .
- =  $\dot{0}\overline{3}\overline{3}$ (iii) It has  $\dot{6} = 6 + 1 = 7$ ,  $67 = 7\overline{3}$  the parammitra=  $1\overline{3}\overline{3}$ number of 7 is 3 replace 7 by  $\overline{3}$ 
  - (iv) The previous number of 7 is 0, place a dot(Ekadhiken symbol) above 0.
  - (v)  $\dot{0} = 0 + 1 = 1$  now write 1 in place of  $\dot{0}$ .

### Converting Vinkulam number to general form of number: -

- (i) To convert Vinkulam number to normal number, take the positive value of Vinkulam number.
- (ii) We will write the Parammitra number of the number with Vinkulam sign (-).
- (iii) Put a nuena sign on the number before the number on which the line is drawn.
- (iv) If the Vinkulam number has three digits, then after converting the tens digit to normal, the units digit will change.

Example:

= 24	Instructions:									
	1. The negative value of four is replaced by its									
= 26	Parammitra number namely 6.									
	2. Place the Nuenena mark on the previous digit (2).									
= 16	3. (2 = 1) We will write 1 in the result.									

**Example**: Convert  $5\overline{32}$  in its general form.

### Instructions:

= 5 32	1. In the tens place, replace $\overline{3}$ by its Parammitra number
- 572	namely 7.
- 97 Z	2. Place the Nuenena mark on 5 which is digit previous to the
= 4 7 2	7 Namely 5 = 4.
- 1 79	3. In the unit digit replace $\overline{2}$ by its parammitra number namely 8.
= 4 / 0	4. Now place a nuenene mark on 7 which is previous digit of 8.
= 468	5. 7 = 6 Write 6 replacing 7.

### Vedic mathematical method of writing tables (Vinkulam method):-

In Vedic literature, we find various references about multiplication tables (multiples) in the form of mantras.

### Table of 10:

दशभ्यः स्वाहा विश्शत्यै स्वाहा त्रिश्शते स्वाहा चत्वारिश्शते स्वाहा पञ्चाशते स्वाहा षष्टयै स्वाहा सप्तत्यै स्वाहाऽशीत्यै स्वाहा नवत्यै स्वाहा शताय स्वाहा सर्वस्मै स्वाहा। (तैत्तिरीय संहिता : 7/2/18)

Table of 20 :

विश्शत्यै स्वाहा चत्वारिश्शते स्वाहा षष्टयै स्वाहाऽशीत्यै स्वाहा शताय स्वाहा सर्वस्मै स्वाहा। (तैत्तिरीय संहिता : 7/2/18)

In the above mantra, table of 10 (up to 100) and table of 20 are mentioned. Apart from this, references are also found in Taittiriya Samhita 7/2/15 (table of 4 up to 20) and Taittiriya Samhita (7/2/16) (Table of 5 up to 20).

Come, let us learn to write multiplication tables using Vinkulam numbers as mentioned in Vedic mathematics. By which the method of writing multiplication table becomes easier.

#### Method:-

- (i) Change the number whose multiplication table is to be written in Vinkulam form.
- (ii) Identify the tens and unit digits of Vinkulam number.
- (iii) The instruction is, in Vinkulam numbers, subtract on the right side and add on the left side with the respective digits of the Vinkulam number.

Let us write the tables according to the above method.

**Example**: Write the multiplication table of 9.

### Solution:

Converting 09 in its Vinkulam form we get,

 $9 = 10 - 1 = 1\overline{1}$ 

In  $1\overline{1}$  the unit digit  $\overline{1}$  means 1 has to be subtracted from the increased  $10^{\text{th}}$  digit namely 1.



 $(2+1) \rightarrow 36 \leftarrow (7-1)$   $(3+1) \rightarrow 45 \leftarrow (6-1)$   $(4+1) \rightarrow 54 \leftarrow (5-1)$   $63 \qquad 72 \qquad (Proceeding further in the same way)$  81 90

**Example:** Write the multiplication table of 8. **Solution:** 



In this way, we can write the multiplication tables.

Example: Write the multiplication table of 17 Solution: Converting 17 into its vinkulam form, General form  $17 = 1\overline{3}$ Vinkulam number :  $2\overline{3}$  $(2\overline{3} = 20 - 3 = 17)$ 17 34 (7-3) = 4(1+2)(3+2) 51 (4-3) = 1 $(5+2)7\overline{2} = 68$  (1-3=-2) Converting  $7\overline{2}$  in the general form, we get (6+2) 85 (8-3) = 5 17 = 20 - 3 = 23(8+2) 102 (5-3) = 2(10+2)  $12\overline{1} = 119$   $(2-3=\overline{1})$   $(12\overline{1} = 129 = 119)$ 136 (9 - 3 = 6)(11 + 2)(13+2) 153 (6-3=3)170 (3 - 3 = 0)(15 + 2)Multiplication using Vedik Mathematics:

 $\frac{12}{multiplicand} \times \frac{3}{multiplier} = \frac{36}{product}$ 

### (1) Multiplication using 'Antyaordasakepi' formula:

In Vedik Mathematics, this formula is very useful in multiplication of numbers:

(i) If the sum of the digits in the unit digits of the multiplicand and the multiplier is 10 and the rest of the digits are the same.

(ii) We multiply the unit digits and write the answer at the right end of the product.

(iii) We find the product of the same digit (rest of the digits) by its Ekadhiken and write the result on the left.

**Example:** Find the product of 24 and 26:

Solution:

### Instruction

24	(i) Sum of the unit digits: $4 + 6 = 10$ , the rest of the digits in
× 26	multiplicand and multiplayer are the same.

624

(ii)  $4 \times 6 = 24$ , write at the right end of the product.

- (iii) Multiplying the rest of the digit (2) with its Ekadhikena
- (3),  $2 \times 3 = 6$  writing the result on the left of the product.

Example: Find the product of 93 and 97:

Solution

#### Instruction

(i) Sum of the unit digits: 3 + 7 = 10, the rest of the digits in multiplicand and multiplayer are the same (9).

93	(ii) $3 \times 7 = 21$ , write at the right end of the product, multiplying
× 97	the rest of the digit (9) with its Ekadhikena (10) $9 \times 10 = 90$
9021	writing the result on the left of the product
	witting the result on the left of the product.

Hence,  $93 \times 97 = 9021$ .

Example: Find the product of 59 and 51

Solution:

### Instruction

(i)	Sum	of the	unit digits: 1	+ 9 =	10, finding	the	product	of 9	)
-----	-----	--------	----------------	-------	-------------	-----	---------	------	---

- 59 and 1, we get 9 which is written as 09 (Always write the result
- × 51 in its two-digit form).
- 3009 (ii) The same digit is 5, multiplying 5 by its Ekadhikena,  $5 \times 6$ = 30, writing the result at the left end.

Hence, 59×51=3009

**Example**: Find the product of 91 and 99 **Solution**:

### Instruction

(i) Sum of the unit digits: 9 + 1 = 10, finding the product of 9 and 1, we get 9 which is written as 09 (Always write the result in its two-digit form).

(ii) The same digit is 9, multiplying 9 by its Ekadhikena,99×10=90, writing the result at the left end.

Hence, 91×99=9009

#### 2. Multiplication by Vilokanam Sutra:

The best thing about Vedic maths is that you can calculate within seconds.

Multiplication by 11 :- Write the number to be multiplied by 11 in brackets and write one zero on both the sides. Start adding two numbers one by one moving from right to left. Whenever the sum of two numbers exceeds 10, the digit at the rightmost place is carried over to the next sum. As is usually done while adding numbers.

Example : Multiply 3252 by 11. Solution: 0 (3 2 5 2) 0

As per the instructions given in the above method, adding

$$= 0 + 3/3 + 2/2 + 5/5 + 2/2 + 0$$
  

$$3 5 7 7 2$$
  

$$= 35772$$

Hence, 3252 × 11 = 35772.

Example: Multiply 364279 by 11.

Solution: 0 
$$(3 \ 6 \ 4 \ 2 \ 7 \ 9) \ 0$$
  
 $0 + 3 / 3 + 6 / 6 + 4 / 4 + 2 / 2 + 7 / 7 + 9 / 9 + 0$   
 $3 \qquad 9 \qquad 10 \qquad 6 \qquad 9 \qquad 16 \qquad 9$   
 $= 4007069$ 

Hance, 364279 × 11 = 4007069.

# Exercise- 4.3

1. Select the correct option for the following multiple-choice questions.

	(a)	Conv	verting	g the cor	nmon ni	umber 26 t	o Vinkulam r	number,
		we g	et					
		(I) 1 <del>4</del>		(II) 4 <u>1</u>	(III)	34	(IV) None of	these
	(b)	Conv	verting	g t <mark>he</mark> con	nmon nu	mber 47 in	to <mark>Vinkulam</mark> r	umber,
		we g	et	ШΧ		XII		
		(I) 53		(II) 53	(III)	33	(IV) None of	these
	(c)	Vink	ulam	number	134 Con	verting to a	a normal num	ber, we
		get						
		(I) 76		(II) 067		(III) 076	(IV) None of	these
2.	Conv	vert t	he fo	ollowing	general	form of	numbers int	o their
	Vink	ulam	numb	er.				
	(I) 8		(II) 1	7		(III) 27	(IV) 48	
3.	Conv	vert th	e folle	owing Vi	inkulam	numbers ir	nto their gener	al form
	of numbers.							
	(i) 4 <del>5</del>	<del>.</del> )	(ii) 43	3		(iii) 2 <u>3</u>	(iv) 14	
	(iv) 8	334	(v) 7 <sup>2</sup>	$\overline{4}\overline{1}$				
4.	Write	e the n	nultip	lication t	ables for	the followi	ng numbers.	

(i) 12 (ii) 7 (iii) 6 (iv) 13

///

5. Find the product of the following: -

Antyaordashkepi Sutra:

(i)	(ii)	(iii)	(iv)	(v)
35	48	57	89	90
× 35	× 42	× 53	× 81	× 99

6. Multiply by the Vilokanam formula: -

(i)	45 x 11	(ii) 97 x 11	(iii) 45 x 11
(iv)	231 x 11	(v) 341 x 11	(vi) 461 x 11

### We learned:-

- 1. Ekadhikena means one more than.
- 2. One newnena means one less.
- 3. Ekadhikena Purvena means one more than the previous digit.
- 4. Ekneunena Purvena means one less than the previous digit.
- 5. If the sum of any two numbers is 10, then they are called complementary numbers or Parammitra numbers.
- 6. If a number is subtracted from its base, the remainder is called deviation. Deviation = Number Base
- 7. Vinkulam Writing negative numbers in their positive form.
- 8. Writing multiplication tables using Vinkulam.
- 9. Multiplying by the Antyaordashkepi formula.
- 10. Multiplying a number by 11 using Vilokanam Sutra.

# Chapter – 5

# Fractions

**Dear Vedic students-** You have learnt about different tips of numbers and different numeration systems in the previous chapters. In this chapter we will discuss and understand fractions in detail, the types of fractions and how to find equivalent fractions.

### Fractions

There are many references about fractions in Vedic literature.

### One such reference is given below

# त्र्यविश्च मे त्र्यवी च मे दित्यवाट् च मे दित्यौही च मे पञ्चाविश्च मे पञ्चावी च मे त्रिवत्सश्च मे त्रिवत्सा च मे तुर्यवाट् च मे तुर्यौही च मे यज्ञेन कल्पन्ताम् ॥ (यजुर्वेद 18/26)

In the above mantra we find the references about the fractions. In Yajurveda, the fractions such as Tryavi is referring to  $1\frac{1}{2}$  and panchavi meaning  $2\frac{1}{2}$  are found. In Yajurved mantra these fraction can be written as  $1\frac{1}{2}$  or  $\frac{3}{2}$  and  $2\frac{1}{2}$  or  $\frac{5}{2}$  We can find references about fractions in Atha Bhinnparikarmashtakam in page no. 35 of Lilavati 's mathematics.

In Rig-Veda, we find references about the operations on fractions in detail. According to a verse in Purushsukta- which is given below,

# "त्रिपादूर्ध्व उदैत्पुरुषः पादोऽस्येहा भवत्पुनः"

### (ऋग्वेद: 10/90/4)

The above mantra conveys that there are three layers in Heavens one out of these three layers, all pervading person (Virat Purush) has reappeared.

It is mentioned in the above mantra that during his reappearance  $\frac{3}{4}^{th}$  part is in heavens, from  $(\frac{1}{4})^{th}$  part the earth appeared. From this we can infer-

$$\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

When a quantity is divided into equal parts, to represent a part of the quantity we use fractions. In other words, a fraction is a part of the quantity

A fraction is expressed when a quantity is divided into several equal parts (parts) and some parts are subtracted from them. In other words, a fraction is "a number that represents a portion of a whole unit."

## अर्धप्रमाणेन पादप्रमाणं विधीयते

The meaning of the above statement is is  $\frac{1}{2}^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ . Similarly, अध्यर्ध पुरुषारज्ज़ुद्वीं सर्पादों करोति

The meaning of the above statement is  $[1\frac{1}{2}]^2 = 2\frac{1}{4}$ .



Let us understand the fractions using an **Example**. Abhishek divided a roti into five equal parts and ate two parts, represented by the shaded region in the adjoining figure

The amount of Roti eaten by Abhishek is given by  $\frac{\text{no.of parts eaten}}{\text{tatal no.of parts}} = \frac{2}{5}$ 

The above fraction is read as two upon five. The number 2 is in numerator and 5 is in the denominator of the fraction.

### Do and Learn:-

Write the fraction that represents the following figures.





### Do and learn:-

Write the numerator and denominator of the following fractions -

S.No.	Different	fraction	denominator
1.	3 5	3	5
2.	$\frac{14}{35}$	रविसाम	
3.	$\frac{2}{7}$		
4.	$\frac{15}{28}$		
5.	$\frac{18}{87}$		

### Types of Fractions -

### 1. Proper Fraction -

Proper Fraction: - fraction in which the numerator is less than the denominator.

# Example: $\frac{2}{7}$

Two upon seven is a proper fraction because two (numerator) is less than seven (denominator) (2 < 7).

```
Similarly, some of the proper fractions are - \frac{5}{7}, \frac{2}{3}, \frac{4}{7}, \frac{15}{18}
```

### **Proper fraction**

1. A proper (even) fraction is one in which the numerator is greater than the denominator. is known as a proper fraction.

Example. 2/7

# 2. Improper Fraction -

Improper Fraction: - Fraction in which numerator is greater than the denominator.

Example:  $\frac{7}{2}$ 

This is an improper fraction because the numerator (7) is greater than the denominator (2).

Similarly, some of the improper fractions are  $-\frac{15}{3}$ ,  $\frac{17}{12}$ ,  $\frac{53}{13}$ ,  $\frac{78}{57}$ 

### 3. Mixed fraction -

An improper fraction can also be represented as the sum of a whole unit and a fraction (a proper fraction). This is called a mixed fraction.

Example: 
$$\frac{5}{4} = \frac{4+1}{4} = \frac{4}{4} + \frac{1}{4}$$
  
Or

$$1 + \frac{1}{4}$$
 or  $1 \frac{1}{4}$ 

It is read as one and one quarter.

Let us learn how to express an improper fraction as a mixed fraction. For this we divide the numerator by the denominator to get the quotient and remainder. Then the mixed fraction is written as quotient remainder/divisor.

**Example:** Convert  $\frac{7}{3}$  into a mixed fraction.

Here 7 (the numerator) is greater than 3 (the denominator). Hence it is an improper fraction.

Converting the improper into a mixed fraction Dividing 7 by  $3\frac{7}{3}$ 

In the above division Dividend =7, Divisor =3, Quotient =2 and Reminder = 1.

Hence, the mixed fraction is quotient  $\frac{\text{Riminder}}{\text{Divisor}} = 2\frac{1}{3}$ 

The above can be done by the following method also.

 $\frac{7}{3} = \frac{3+3+1}{3}$  [ rewritin the numerator in terms of denominator]

$$= \frac{3}{3} + \frac{3}{3} + \frac{1}{3}$$
$$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$
$$= 2 + \frac{1}{3}$$

The mixed fraction  $2 + \frac{1}{3}$  is read as two (whole number) and one – third.

The above can be done by the following method also.

**Example:** Convert the following improper fractions into their mixed fraction:

Solution: Dividing 
$$3)25(8)$$
  
 $-24$   
1

Dividend 25, Divisor =3, Quotient = 8 and Remainder = 1 Hence, the mixed fraction is

Quotient 
$$\frac{Remainder}{Divisor} = 8\frac{1}{3}$$
  
 $\frac{19}{6}$   
Solution: Dividing  $6)19(3)$   
 $\frac{-18}{1}$ 

Dividend 19, Divisor =6, Quotient = 3 and Remainder = 1 Hence, the mixed fraction is

$$\frac{19}{6} = 3$$
 Wole number and  $\frac{1}{6} = 3\frac{1}{6}$ 

Dear children! Now, let us learn to convert a mixed fraction =  $\left( whol \, number \, \frac{numerator}{Denominator} \right)$  into an improper fraction. We add the numerator of the fraction with the product of the whole number and the denominator of the fraction, then the improper fraction is <u>(Whole number × Denominator) + Numerator</u>

Denominator

**Example:** Convert the following mixed fractions into their improper form:

(i) 
$$2\frac{3}{4} = \frac{(2\times4)+3}{4} = \frac{8+3}{4} = \frac{11}{4}$$
  
(ii)  $3\frac{2}{7} = \frac{(3\times7)+2}{7} = \frac{21+2}{7} = \frac{23}{7}$   
 $3\frac{2}{7} = \frac{23}{7}$ 

Do and learn:-

Identify and name the proper fraction, improper fraction and mixed fraction in the following fractions.

(1) $\frac{2}{7}$	(2) $2\frac{1}{7}$	(3) $\frac{17}{12}$	(4) $\frac{3}{7}$
()	()	()	()
(5) $\frac{12}{17}$	(6) $\frac{13}{7}$	$(7) \frac{9}{5}$	(8) $2\frac{1}{5}$
()	()	() a (*	()

### **Equivalent Fractions**

Two fractions are equivalent fractions, if their values are same. One can obtained equivalent fractions by either multiplying are dividing both numerator and denominator by the same a non – zero number.

**Interesting fact -** In the following verse of Lilavati's Mathematics, the method of finding equivalent fractions has been explained.

# अन्योन्यहारभिहतौ हरांशौ राश्योः।

# (लीलावती, भिन्नापरिकर्माष्टकम्,1अ)

In this sentence, Bhaskaracharya explains the method of multiplying numerator and denominator of fractions to know their equivalence. If the cross product of numerator and denominator of two fractions are equal, they are equivalent fractions.

Complete the following table. The first two rows are filled for you.

Equivalent	Product 1=	Product 2 =	Product is
fractions	numerator of 1×	numerator of 2 $\times$	product1=product
	denominator of 2	denominator of 1	2?
$\frac{1}{4} = \frac{4}{16}$	1 × 16 = 16	$4 \times 4 = 16$	Yes
$\frac{5}{4} = \frac{125}{100}$	5 × 100 = 500	$125 \times 4 = 500$	Yes
$\frac{3}{7} = \frac{18}{42}$			
$\frac{2}{5} = \frac{60}{150}$			•
$\frac{1}{7} = \frac{8}{56}$			

In the above table, product1= product 2. Such products are called cross product are scissor product of two fractions.

Is there a pair of equivalent fractions in who scissor or cross products are not equal?

This rule helps us to find whether given two fractions are equivalent fractions are not.

**Example:** Find the fraction equivalent to  $\frac{3}{7}$  whose denominator is 63. **Solution**: We know that-

$$\frac{3}{7} = \frac{1}{63}$$
$$3 \times 63 = \underline{\qquad} \times 7$$

But,  $63 = 7 \times 9$ 

Therefore,  $3 \times 7 \times 9 = \_ \times 7$ 

$$3 \times 9 \times 7 = \times 7$$

$$27 \times 7 = \times 7$$

After comparing

27=\_\_\_\_

Therefore,  $\frac{27}{63}$ 

Hence,  $\frac{3}{7} = \frac{27}{63}$ 

### Like Fractions -

Fractions with the same denominator are called like fractions.

Thus,  $\frac{2}{17}$ ,  $\frac{5}{17}$ ,  $\frac{16}{17}$ ,  $\frac{12}{17}$  are like fractions here. Are the fractions  $\frac{7}{13}$  and  $\frac{7}{15}$ , like fractions?

No they are not like fractions as there denominator are different.

### Comparison of like fractions

Compare two like fractions.

# **Example**: $\frac{2}{7}$ and $\frac{5}{7}$

Let us represent both the fractions pictorially.

$$\frac{2}{7}$$

Observe that in both the fractions the whole is divided into 7 equal parts.

To represent  $\frac{2}{7}$  and  $\frac{5}{7}$ , we shade 2 and 5 parts respectively.

It is clear from the representations that the shaded area of  $\frac{5}{7}$  is greater than the shaded area of  $\frac{2}{7}$ .

Hence,  $\frac{2}{7} < \frac{5}{7}$ .

Thus, it is clear that between two like fractions, the fraction with the greater numerator is the greater fraction.

Among  $\frac{6}{11}$  and  $\frac{4}{11}$ ,  $\frac{6}{11}$ , is greater fraction because 6 > 4

Comparison of two fractions with same numerator but with different denominator

Consider two fractions  $.\frac{5}{8}$  and  $\frac{5}{10}$ . Representing the fractions pictorially

In the above to representations, observe that the shaded area in the representation of  $\frac{5}{8}$  is more than that the shaded area represents  $\frac{5}{10}$ . Hence, we can conclude that  $\frac{5}{8} > \frac{5}{10}$ .

Further, observe that in the fractions having same numerator and different denominators, the fraction with smaller denominator is greater **Try:** 

1. Which of the following fraction is bigger?

a) 
$$\frac{7}{10}$$
 Or  $\frac{8}{10}$ ) b  $(\frac{12}{13}$  or  $\frac{11}{13}$ )) c  $\frac{10}{17}$  or  $\frac{13}{17}$ )

2. Write the following fractions in ascending order -

 $\frac{1}{7'} \frac{5}{7'} \frac{3}{7'} \frac{4}{7} \quad b \quad \left(\frac{3}{11'} \frac{5}{11'} \frac{2}{11'} \frac{1}{11'} \frac{7}{11'} \frac{10}{11'} \frac{9}{11}\right)$ 

### **Comparison of Unlike Fractions**

Fractions with different denominators are called unlike fractions.

### Example:

$$\frac{1}{2}, \frac{3}{7}, \frac{5}{8}$$
 etc.

Consider two unlike fractions  $\frac{1}{2}$  and  $\frac{1}{7}$ , which are represented pictorially.

These fractions represent then pictorially as shown below



Which one is greater  $\frac{1}{2}$  or  $\frac{1}{7}$ ? To solve this, we adopt the following steps. Step 1: Find the LCM of the denominators.

LCM 
$$(2, 7) = 14$$

Step2: Find the equivalent fractions of the given fractions such that their denominators is the LCM obtained in Step 1

$$\frac{1}{2} = \frac{1 \times 7}{2 \times 7} = \frac{7}{14}$$
$$\frac{1}{7} = \frac{1 \times 2}{7 \times 2} = \frac{2}{14}$$

Step 3: Compare the numerators.

Comparing numerators, we get 7>2. Hence,

$$\frac{1}{2} > \frac{1}{7}$$

### **Interesting facts:**

In two unlike fractions having same numerators, the fraction with greater denominator is smaller.

Which fraction is greater in  $\frac{3}{5}$  and  $\frac{3}{7}$ ?

Solution:

Both the above fractions have the same numerator here the smaller denominator is 5. Hence, the fraction with smaller denominator namely  $\frac{3}{5}$  is greater.

### Exercise: 5.1

- 1. Select the correct option for the following multiple-choice questions.
- Which one of the following is a proper fraction? (a) (II)  $\frac{5}{2}$  (III)  $\frac{2}{1}$  (IV) None of these  $(I)\frac{8}{10}$ Which one of the following is not a proper fraction? (b)(II)  $\frac{5}{12}$  (III)  $\frac{7}{11}$  (IV) None of these  $(I)^{\frac{12}{10}}$ Mark any one of the signs (<,>,=) appropriately: (c) (II)> (III) = (IV) None of these (I) < (d) Mark any one of the signs (<,>,=) appropriately:  $\frac{5}{2}[]\frac{3}{7}$ (III) = (IV) None of these (I) < (II)>Which of the following is the largest fraction? (e)  $(I)\frac{12}{10}$ (II)  $\frac{5}{10}$  (III)  $\frac{7}{10}$  (IV)  $\frac{17}{10}$ Which of the following is the smallest fraction? (f) (I)  $\frac{12}{7}$  (II)  $\frac{55}{7}$  (III)  $\frac{17}{7}$  (IV)  $\frac{37}{7}$ Converting the mixed fraction  $3\frac{2}{7}$  into the improper fraction, we (g) get  $(I)\frac{22}{7}$  (II)  $\frac{13}{7}$  (III)  $\frac{12}{7}$  (IV) None of these Converting improper fraction  $\frac{71}{10}$  into a mixed fraction, we (h) get..... (I)  $7\frac{1}{10}$  (II)  $6\frac{40}{7}$  (III)  $7\frac{10}{1}$  (IV) None of these 2. Write the shaded part of the given figure as a fraction. (1)(3) (2)  $\sim$ (.....) (.....) (.....)



(4)

3.

4.

5.

6.

7.

8.

9.

10.

11.

12. Find the equivalent fractions of  $\frac{4}{5}$ , whose

- 1. Denominator is 202. Numerator is 20
- 3. Denominator is 404. Numerator is (27)

13. check whether the following pairs of fraction are equivalent are not.

(1)  $\frac{13}{26}, \frac{1}{2}$  (2)  $\frac{3}{5}, \frac{4}{3}$  (3)  $\frac{15}{12}, \frac{5}{4}$  (4)  $\frac{10}{8}, \frac{25}{20}$ 

14. Aaradhya and Animesh practice yoga for  $\frac{4}{5}$  and  $\frac{10}{20}$  hours every day. Who practices yoga for more number of hours?

### We learned:-

- 1. A fraction is a number that represents a part of a whole.
- 2. In the fraction  $\frac{2}{7}$ , 2 is called the numerator and 7 is called the denominator.
- 3. In a proper fraction, the numerator is smaller than the denominator.
- 4. In an improper fraction, the numerator is greater than the denominator.
- 5. When an improper fraction is written as a whole number and a proper fraction, then it becomes a mixed fraction.
- 6. We can represent the fractions pictorially.
- 7. To convert an improper fraction into a mixed fraction, we find the quotient and the remainder on division, and write in the form of

Quotient Remainder Divisor

8. To convert a mixed fraction into an improper fraction, we follow two steps.

Mixed fraction =  $\frac{Quotient \times Divisor + Remainder}{Divisor}$ 

# Chapter 6 Decimals

### Dear children!

You must have observed the prices of a medicine, a litre of petrol or a gas cylinder. Which is shown below:



In the above figure the cost of the medicine is Rs 48.75 which means Rs 48 and 75 paisa. Similarly, the price of petrol is 83.22, Which means 83 rupees 22 paisa, and cost of a gas cylinder is 1007.84 which mean 1007 rupees and 84 paisa, which means 83 rupees 22 paisa. In the above-mentioned prices, we observe a dot before the last two digits which represents the decimal point. Here we will discuss about decimals.

### Decimal number:

What is the length of Aaradhya's pencil?

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# Place value of decimal:-

The value of a digit in a number depends on its place value.

# In the number 425

4 is in hundreds place so the place value of 4 is  $4 \times 100 = 400$ 

2 is in ten's place so the place value of 2 is  $2 \times 10 = 20$ 

Now let us write the place value of some decimal number.

In a number the place value of a digit, as we

move from left to right, decreases by  $\frac{1}{10}$ .

Decimal Number	Hundred	Tens	Unit	tenth hundred	hundreds
124.5	1	2	4	5	
551.41	5	5	1	4	1
124.32					
321.2			/	7	
345.6					

5 is in the unit's place so the place value of 5 is  $5 \times 1 = 5$ 

Here place value of 5 = .....

Similarly, on interchanging the place of 5, 4, and 2 we will get a different

Place value of  $2 = \dots$ 

Place value of 4 = .....

### Reading of Decimal Numbers:

value of the number.

You must have seen the decimal places usually in the prices of articles and weights of objects. Such as cost of medicines, petrol and currency exchange rates of dollars etc.

The amount Rs 34.57 is read as 34 rupees and 57 paisa (in case of currency we never call out the digits after the decimal point individually). Similarly, the conversion rate of dollar in rupees, if it is 62.025 rupees, then is read as, rupees sixty-two point zero two five. (Currency rates are not written correct to thousandth place, but for calculations we do consider it).

Write the decimal number in words.

(1)45.38	=
(2)125.78	=
(3) 57.35	=
(4) 16.138	=

### **Remember:**

We never read the decimal numbers collectively. Always replace the decimal point between once and tenths digits of the number.

We read 17.45, as seventeen point four five instead of seventeen point forty-five.

- Usually, the digits after the decimal points in a decimal number are called out individually for **Example** 3.45 is read as 3 point four five, 63.47 is read as sixty-three point four seven etc.
- In case of prices of commodities, the decimal point separates the rupees and paisa. Recall that 100 paisa = 1 rupee the last two digits in the price signify paisa which is read together. For **Example**, rupees 83.44 is read as eighty-three rupees forty-four paisa instead of eighty-three rupees and four paisa.

### Representation of a Decimal on a Number-line:

Now, dear students, let us learn to represent decimal numbers on a number line.

A decimal number has two parts namely whole number part and decimal part as shown below:



We adopt the following steps to represent a decimal on a number line.

- **Step 1-** Draw a number line and mark points at equal distances from each other and label them as 0, 1, 2...
- Step 2- Locate the whole number part on the number line.
- **Step 3-** Divide the distance between the whole number part and its successor in ten divisions if the number of digits after the decimal point is one.
- **Step 4-** Now, move as many divisions as the digit in a decimal part to locate the decimal number.

The representation of 0.9 on a number line will be:

0.9 1.8 2.5 (-------) 1.8 2.5 (-------) 1.8 2.5 (-------) 1.8 2.5 (-------) 1.8 2.5 (-------) 1.8 2.5 (-------) 1.8 2.5 (-------) 1.8 2.5 (-------) 1.8 2.5 (-------) 1.8 2.5 (-------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (------) 1.8 2.5 (-----) 1.9 2.5 (-----) 1.9 2.5 

**Note:** If the number of digits is 2, divide the whole number part of the decimal and its successor into 100 divisions.

### Do and learn -

Represent the decimal numbers 0.7, 1.8, 2.6 on the number line.

Write the decimals in their corresponding expended form

Observe the following {Tenths means 10<sup>th</sup> part of unit.

## Example:

Hence,  $\frac{1}{10} = 0.1$  }

i) 7 units and 5 tenths

Solution: 7 units and 5 tenths

$$= 7 + \frac{5}{10}$$
  
= 7.5

ii) 3 hundreds 8 tens 5 units 7 tenths

Solution:  $300 + 80 + 5 + \frac{7}{10}$ = 385 7
iii) 7 hundreds 3 units and 5 tenths 8 hundredths

Solution:  $700 + 3 + \frac{5}{10} + \frac{8}{100} = 703.58$ 

### Do and learn: -

Write the following in decimal form.

(1) 7 unit 5 hundredths	=	7.05
(2) 8 units 8 hundredths	= .	
(3) 3 tens 4 units and 5 tenths	= .	
(4) 9 units 7 tenths 6 hundredths	ŧ.	
(5) 9 tens 7 units 7 tenths	= .	

# Converting the decimal numbers as decimal fractions:

The fractions with denominators as powers of 10 are called decimal fractions.

Example:  $\frac{6}{10}, \frac{83}{100}, \frac{567}{1000}, \frac{163}{100}$ 

To convert a decimal number into its corresponding decimal fractions, we adopt the following steps.

Step 1 Count the number of digits in the decimal part of the number.

Step 2 Write the given number without decimal point as numerator.

Step 3 in the denominator; write 1 followed by as many zeros as the number of digits in the decimal part of the number.

While converting a decimal number into a fraction, we get decimal fraction. In decimal fractions, the denominator is always powers of 10. To convert a decimal number into its corresponding decimal fraction, we write the decimal number without decimal point in the numerator, and write in the denominator 1 followed by as many zeros as the number of digits after the decimal point in the decimal number.

Let us understand using an

Example – Consider 25.6.

In 25.6 The number of digits in the decimal part is 1. Hence,

(1) 25.6 = 
$$\frac{25.6}{10}$$

The number of digits in the decimal part is 2.

(2) 35. 47 = 
$$\frac{35.47}{100}$$

Uses of decimals:

**Money** - We know that 100 paisa = Re. 1

So 1 paisa = Rs. 
$$\frac{1}{100}$$
 = Rs. 0.01

Thus,

75 paisa = Rs. 
$$\frac{75}{100}$$
 = Rs. 0.75  
25 paisa = Rs.  $\frac{25}{100}$  = Rs. 0.25  
8 paisa = Rs.  $\frac{8}{100}$  = Rs. 0.08

How many rupees will be there in 135 paisa?

135 paisa = Rs. 
$$\frac{135}{100}$$
 = Rs. 1.35  
= It will be 1 rupee 35 paisa

### Do and learn

Write the following in their decimal forms

- 1) 3 rupees 10 paisa2)2 rupees 75 paisa3) 12rupees 17 paisa4)Rs 27 rupees 5 paisa
- 5) 10 rupees 10 paisa6)15 rupees 19 paisa

# Length -

1. Khushi wants to measure length of the top surface of her study table in metres. She has 50 cm. long ruler. She found that the length of the top of the table is 186 cm. How many meters are there in this length? We know that 1 cm =  $\frac{1}{100}$  m = 0.01 m.

Hence, the length of the top surface of the table is

Converting 186 cm into meters

186 cm = 100 cm + 86 cm $= \frac{100}{100} \text{ m} + \frac{86}{100} \text{ m}$ 

1m + 0.86 m = 1.86m

### Points to remember

Conversion of units of length

10 mm	= 1 cm	
100 centimetre	= 1 meter	
1000 meters	= 1 kilometre	
1 millimetre	= 1/10 centimet	re = 0.1 centimetre
1 centimetre	= 1/100 meter	= 0.01 meter
1 meter	= 1/1000 km	= 0.001 kilometre

# Do and Learn

1. Write 8 mm in centimetre using decimal?

- 2. Write 8 cm 6 mm in centimetre using decimal?
- 3. Write 53 m in kilometres using decimals?
- 4. Using decimals, write 340 m. in km.
- 5. Write 2006 meter in kilometre?

### Weight -

### Let us learn the concept of weight using the following Example

Shubham bought 500 grams of bananas, 250 grams of chillies, 700 grams of tomatoes, 500 grams of apples, 100 grams of onions and 300 grams of carrots. What is the total weight of the vegetables that he bought?

# Solution:-

Let us add the weights of all the vegetables: -

500 grams + 250 gr. + 700 gm. + 500 gm. + 100 gm. + 300 gm. = 2350 grams.

1000 gm. = 1 kg We know that  $1 \text{ g.} = \frac{1}{1000} \text{kg} = 0.001 \text{ kg}$ Hence, Thus 2350 g. = 2000 grams + 350 gr.  $=\frac{2000}{1000}kg + \frac{350}{1000}kg$ Or = 2 kg + 0.350 kgOr

# Do and Learn

Convert 458g into kg using decimals? 1.

= 2.350 kg

How will you write 9 grams into kg using decimals? 2.



# Exercise - 6.1

1. Select the correct option for the following multiple-choice questions.

(a)	13 kg 20 Gra	am =		•••••		
	(i) 13.02 kg.	(i	ii) 13.2 Kg.			
	(iii) 3.002 Kg	g_ (i	iv) 1302 gra	ms		
(b)	Standard for	rm of - 3	$0 + 4 + \frac{5}{10} + \frac{5}{10}$	$\frac{3}{100}$ is	·	
	(I) 30.53	(II) 34.35 (I	III) 34.53 (1	IV) Non	e of these	
(c)	Standard for	rm of - 1	$50 + \frac{3}{100}$ is	The	<u>.</u>	
	(I) 150.3	(II) 150.300 (I	III) 150.03	(IV	7) 150.003	
(d)	Standard fo	rm of-	500 +	$\frac{3}{10}$ is	2	
	(I) 500.03	(II) 50.30	(III) 5	0.003	(IV) 500.3	
(e)	Five hundr	ed and seve	en hundred	ths is v	written in the f	form of
	- 25-			$\bigvee$		
	(I) 500.007	(II) 50.007	(III) 50	0.07	(IV) None of	these
(f)	Compare th	e numbers u	sing <, > an	.d =. 4.6	5 4.08	
	(I) <	(II) >	(III) =	= (IV	) None of these	
(g)	Compare th	e numbers u	sing <, > an	.d =. 12	2.126 12.176	
	(I) <	(II) >	(III) =	= (IV	) None of these	
2.	Write the nu	umbers in the	e following	table -		
	(i) 1 hune	dredth 3	tenth	4 te	enths	
	(ii) 3 tens	7	units	5 te	enths	
	(iii) 2 hun	dreds 8	tens	1 u	init 2	tenths
hu	ndreds (100)	tens (10)	units (1)	tenths	Number to	be
	1	3	0	4	one hundred	thirty
					point four	

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3. Write the number names of the following decimal numbers.

(i) 23.4 (ii) 37.81 (iii) 135.7

4. Write the following decimal numbers in their expanded form.

(i) 23.4 (ii) 37.81 (iii) 135.7

- 5. Write each of the following in decimal form.
  - (i) fourteen point nine
  - (ii) six hundred point seven
  - (iii) one hundred and five point eight
  - (vi) Seven tenth
  - (v) Eight hundredths
  - (vi) Three tenths Two hundredths
- 6. Write the standard form of the following decimal expansions.

(1)	$\frac{3}{10}$	(2)	$4 + \frac{8}{10}$	(3) $300 + 80 + 8 + \frac{1}{10}$
(4)	$80\frac{7}{10}$	(5)	$30 + \frac{7}{10}$	(5) $50 + \frac{7}{10} + \frac{5}{100}$

- 7. Write the following decimal numbers in fractional form.
  - (i) 2.5 (ii) 13.7 (iii) 21.25 (iv) 6.7

Addition (+) and subtraction (-) of decimal numbers: -

- 1. Decimal point of both the numbers should be one below the other.
- 2. If the number of digits after the decimal point in one number is less than the other, we write as many zeros as required in the decimal number which has less number of digits, so that the number of digits after the decimal point in both the decimal numbers is the same. With this, it will be easy for us to add or subtract the decimal numbers.

Example: Add (32.64 and 2.41) Solution:

The given decimals are like decimals.

**Remember:** While adding decimal numbers, write the decimal points one below the other and write the digits accordingly.

Ensure that the number of digits after the decimal points in both the numbers is the same by writing zeros at the end of the number whose number of digits after the decimal point is less, at the same time the value of the decimal numbers does not change.

Do and learn:-

(1) 43. 75 more 12.12	(2) 0.75 + 1.25 + 2.4
(3) 2. 4 + 2.7 + 0.3	(4) 1.3 + 1.4 + 2.7
(5) 2. 5 + 1.7 + 2.1	(6) 5.1 + 8.2 + 1.6

Subtraction of decimal numbers: -

Let us learn how to subtract decimal numbers using an Example

(1)	Subtract 3.2 from 7.4.	(2) Subtract 3.61 from 8.42.
Solu	tion:	

(1)		7.4	(2)		8.42	
	_	3.2		_	3.61	
		4.2	-		4.81	

(3) Subtract 7.821 from 13.4.

13.400

Hance 13.4 – 7.821 = 13.400 – 7.821 = 05.579

Doa	and learn:-	
Sub	tract the following	
(1)	3.4 (2	) 7.81
	- 1.25	24
(3)	Subtract 3.412 from 7.3.	(4) Subtract 3.12 from 15.712.
	7.3	15.712
	- 3.412	- 03.120
	44	172

# Comparison of decimal numbers:-

In comparing two decimal numbers, firstly, we start comparing the whole number parts (which are lying at the left of the decimal point). If the whole number parts are same, we compare the digits at 10<sup>th</sup> place. Even if these digits are same, we compare the digits at 100<sup>th</sup> place and thereafter.

# Let us understand with an Example.

Which number is bigger? 2.6 or 2.14? Here we observe that the digits on the whole number parts of both the decimal numbers are same, so we first compare the tenth digit and followed by the hundredth digit in decimal numbers.

In 2.6 and 2.14, there are 6 tenths in 2.6 and 1 tenth in 2.14.

```
In tenths, 6 > 1
2.6 > 2.14
```

Similarly,

(1) 0.3 < 0.4 (2) 1.34 > 1.30

# Do and learn

Compare the following numbers -

(1) 1. 34	1.347	(2) 2.47	_2.42
(3) 0.34	0.47	(4) 4.56	_5.1
(5) 7.81	8.71	(6) 9.2	3.125
(7) 4.12	5.1	(8) 10.1	12.12

# Exercise - 6.2

1) Add the following decimal numbers?

(a)	7.4	(b) 7.1
	+ 8.5	+ 1.9
(c)	8.1	(d) 6.5
	+ 2.4	+ 2.7
	90	
(e)	3. 47	(f) 4.57
	+ 3.4	+ 14.802

2) Subtract the following decimal numbers?

(a)		7.4	(t	)	2.4
		3.5	<b>.</b> 99व	9E	1.7
					1000
(c)		8.5	(0	ł)	4.5
	_	2.7	_	_	3.2
(e)		9.8	(f	)	14.57
		5.4			- 14.32

3 Subtract the following number:

4

(1) 3.5 from 9.4	(2) 1.05 from 9.5
(3) 2.5 from 8.3	(4) 7.2 from 9.7
(5) 0.254 from 3.104	(6) 2.51 km from 5.06 km
Compare the following deci	mal numbers?
(a) 4.7 4.7	(b) 4.73 <u>4.734</u>
(c) 3.43 4.34	(d) 3.412 3.413

- (e) 4.213 \_\_\_\_\_ 4.12 (f) 5.77 \_\_\_\_\_ 7.3
- 5. Madhav has Rs. 22.50. He bought a pencil costing Rs. 8.25. How much will he have now?
- 6. Ram has 15 m 25 cm cloth. He uses 9 m 15 cm of cloth to stitch three kurtas. Find the remaining length of cloth that he has.



# We learned:-

- On dividing a block into 10 equal parts, each part will have 1-tenth (1/10) part (one tenth) of the block. We can write this in decimal form as 0.1.
- To convert a decimal number into a fraction, write the numerator as decimal number without decimal point and the denominator as 1 followed by as many zeros as the number of digits after the decimal point in the decimal number.
- On dividing a block into 100 equal parts, each part is 1-hundreedth (1/100) part of the block. We can write this as 0.01 in decimal form.
- 4. Decimal numbers can be represented on a number line.
- 5. Two decimal numbers can be compared. The comparison is made starting from the whole number part of the number (that is, the digits to the left of the decimal point). If the whole parts are same, the digits in the tenth place are compared and if they are also the same then compare the next the digits at hundredths place. This sequence goes on and on.

# Chapter - 7 Ratio and Proportion

Dear Students, in our daily lives we compare two similar quantities very often. To compare we subtract, divide or compare their sizes. In this chapter, we will study about comparison of quantities by division. Let us understand by some **Example**s.

# Comparison by subtraction (difference) -

**Example:** Avinash and Diwakar memorized the chapters of Gita. Out of the chapters of Gita, Avinash memorized 12 chapters and Diwakar 4 chapters. We can say Avinash memorized 12 - 4 = 8 chapters more than Divakar.

### Comparison by division -

Let us take another **Example**. The cost of a book is Rs.30 and a pen is Rs. 10. If we fine the difference between their costs, we get Rs.20 (30 - 10 = 20). We can compare the prices by division also. If we compare by division, it will be as follows:

$$\frac{\text{Cost of the book}}{\text{Cost of the pen}} = \frac{30}{10} = \frac{3}{1}$$

We can say that the cost of a book is three times the cost of a pen. In such circumstances, the comparison of similar quantities by division is called ratio.

### Ratio -

The following line is found in the context of writing the ratio, in the Bhaskaracharya's book on algebra.

एकः पदार्थस्तत् सजातीयद्वितीयपदार्थेन यदु गुणितः स एव सम्बन्धो निष्पत्तिर्वा। यथा अ, व अनयोः सम्बन्धः<sup>अ</sup>, वा अः व एवं लिख्यते ।

(बीजगणितम्, परिशिष्टम्मृ. 243)

The ratio is obtained when we divide one quantity by another similar quantity.

We use the concept of ratio to compare two or more similar quantities. The ratio gives the relationship between the two similar quantities, and is represented by numbers.

If two similar quantities x and y, the ratio between them is represented by x:y.

The first term of the ratio is called antecedent and the second term is called consequent. In the expression x: y, x is called antecedent and y is called subsequent.

Subsequent  $\uparrow$ x : y

Antecedent

We represent the ratio by symbol ':'. The ratio x: y, is read as x ratio y or x is to y.

Let us understand with an Example – the cost of a pen is Rs.10 and the cost of an eraser is Rs2, the cost of the pen is how many times than the cost of the eraser?

It is obvious that the cost of the pen is five times the cost of the eraser.

In the above **Example**, we compared the two similar quantities (costs) to find 'how many' times a quantity costs with respect to the other, by division. This comparison is called ratio. We can say that-

Ratio: 
$$\frac{10}{2}$$
,  $\frac{5}{1}$ , 5: 1

**Note**: If we multiply or divide the numbers in a ratio, by the same any non-zero number the value of the ratio does not change.

**Example:** If Anshul has Rs.60 and Himanshu has Rs.30, find the ratio between the amounts of moneybetween (a) Anshul and Himanshu, (b) Himanshu and Anshul.

### Solution:

(a) Find how many times Anshul has in comparison to Himanshu.

Ratio =  $\frac{\text{Amount of money with Anshul}}{\text{Amount of money with Himanshu}} = \frac{60}{30} = \frac{2}{1}$ 

Thus, Anshul has twice the amount of money than Himanshu. (b)Find how many times Himanshu has in comparison to Anshul. (c)

Ratio =  $\frac{\text{Amount of money with Himansu}}{\text{Amount of money with Anshul}} = \frac{30}{60} = \frac{1}{2}, 1:2$ 

Thus, Himanshu has half the amount of money than Anshul.

#### Equivalent ratio

The equivalent ratio of any ratio is obtained by either multiplying or dividing the numbers in ratio, by the same any non- zero number.

### Equivalent Ratio by Multiplication -

**Example**: Write two equivalent ratios of 6 :4.

Hence, 6:4, 12:8 and 18:12 are equivalent ratios.

### Equivalent Ratio by division –

Example: Write two equivalent ratios of 32:64.

Solution: Ratio 32:64  $\frac{32}{64} = \frac{32 \div 2}{64 \div 2} = \frac{16}{32} = 16:32$  $\frac{32}{64} = \frac{32 \div 4}{64 \div 4} = \frac{8}{16} = 8:16$ 

Hence, 32:64, 16:32 and 8:16 are equivalent ratios.

### Ratio in different situations:

Gautam and Vikas started a business by investing money in the ratio of 4: 5. After one year, the total profit earned by them was Rs.45, 000. At the time of sharing the profit, Gautam said, "let us divide the profit equally". For which Vikas replied, "The profit should be divided as for the ratio of money invested".

If the profit is to be shared in ratio of amount invested, in ratio 4: 5, there are two parts namely 4 parts and 5 parts.

The total number of parts = 4 parts + 5 parts = 9 parts...

Hence, out of 9 parts of the profit, Gautam should get 4 parts and Vikash should get 5 parts.

Gautam's share of profit =  $\frac{45000 \times 4}{9}$  = 20000 Vikas' share of profit =  $\frac{45000 \times 5}{9}$  = 25000

Thus, Gautamand Vikas share the profit amount of  $_{\rm Rs}$  20,000 and Rs 25,000 Respectively.

Note:-

The ratio 4: 5 can be expressed in the fractional form as  $\frac{4}{5}$ .

Also,  $\frac{20000}{25000} = \frac{4}{5}$ .

**Example -** Divide Rs 100 between Ajay and Divyakant in the ratio of 2 : 3?

# Solution:

In the ratio 2:3, the total number of parts = 2 + 3 = 5 parts

The ratio suggests, if the amount is Rs.5, Ajay will get Rs.2 and Divyakant will get Rs.3.

Since, the total amount is Rs.100

Ajay' share  $\frac{2}{5} \times 100 = \text{Rs.40}$ , and

Divyakant's share  $\frac{3}{5} \times 100 = \text{Rs.60}$ 

Exercise - 7.1

- 1. Select the correct option for the following multiple-choice questions.
- (a) Hari Kishan's monthly salary is Rs.80,000 and Lakhan 's monthly salary is Rs.40,000. Hari Kishan's salary is how many times Lakhan's salary.

(I) 1 times (II) 3 times (III) 2 times (IV) 6 times

(b) In a Vedic school, the number of students who study Yajurveda and Atharvaveda is 30 : 20 respectively. The ratio of the number of students who study of Atharvaveda and Yajurveda is-.

(I) 2: 3 (II) 3: 2 (III) 5: 2 (IV) 2: 5

(c) Ram's height is 120 cm. And Shyam's height is 150 cm. The ratio of the height of Ram to that of Shyam is-.

(I) 4: 5 (II) 5: 4 (III) 9: 2 (IV) 9: 5

- (d) Ratio 40 grams to 1 kilogram =..... (I) 4:5 (II) 2:5 (III) 5:10 (IV) 1:25
- (e) The sides of a triangle are in the ratio 1:2:3 and its perimeter is 36 cm. the sides of the triangle are \_\_\_\_\_\_.
  (i) 1cm, 2cm and 3cm (ii) 2cm, 4cm and6cm (iii) 4cm, 8cm and 12cm (IV) 6cm, 12cm and 18cm

 25 girls and 35 boys participated in the training of Vedic Mathematics, then find the ratio of the following?

- (a) Number of girls to number of boys.
- (b) Number of boys to total number of trainees.
- In the programme of tree plantation, the students of class 6 planted
   50 saplings in which 8 saplings were of neem, 20 were of mango,
   10 were of tulsi and 12 were of banyan, and then find the ratio of
   the following.
  - (a) The ratio of the number of neem plants and banyan plants
  - (b) Ratio of number of tulsi plants to that of Mango plants.
  - (c) Ratio between the number of mango plants and the total number of plants
- 4. Find the Ratio of the following pairs in their simplest form.
  - (i) 25 : 70
    (ii) 35minutes : 70 minutes
    (ii) 72 : 24
    (iv) 48 : 12
- 5. Find two equivalent ratios of each of the following.
  - (i) 5:3 (ii) 3:7 (iii) 5:4 (iv) 4:3
- The ratio of the age of A and B is 4 : 3. The sum of their ages is 70.
   Find the ages of A and B.
- Rahul and Neha invested in a textile business in the ratio of 5 : 4.At the end of the year, there is a profit of Rs.72,000. Find the profit

share of each, if the profit is shared in the ratio of their investments.

- 20 fruits are divided between Kavita and Deepika in the ratio 3 : 2.
   Find the number of fruits that each willget.
- 9. The ratio between monthly salaries of A, B, and C is 2:3:5. If the salary of C is Rs. 12000 more than salary of A, find the yearly salary of B.
- 10. Write the ratio 75:125 in its simplest form.

#### Proportion –

If two ratios are equal to each other, the ratios are said to be in proportion.

The following line is found in the context of writing the quantities in proportion, in 'Bhaskaracharya's' work on algebra.

> यदि चत्वारो राशयः सम्बन्धिनो भवेयुस्तदा आद्यन्त्योर्घातः द्वितीयतृतीयराश्च्योर्घाततुल्यो भवेत् । कल्प्यन्ते राशयः अ, व, क, ड तदा अः व = कः ड अर्थात् <sup>अ</sup> = <sup>क</sup> पक्षौ व ड अनेन गुण्यते तदा अ ड = व क॥

# (बीजगणितम्, परिशिष्टम् पृ. 245)

The above line says, when two ratios are equal, they are said to be in proportion.

The proportion is represented by either ' ::' or '=' sign.

If two ratios a: b and c : d is equal, the ratios are in proportion and it is represented by

*a*: b :: *c* or *a* : 
$$b = c : d$$

In the above, the first term 'a' and the last term 'd' are referred as outer quantities (extreme terms). The second term' b' and third term 'c' in a proportion are called inner quantities (mean terms)

Product of Outer terms (extreme terms) = Product of inner terms (mean terms)

Example: 0 uter terms 10:15:2:3  $\psi$   $\psi$ Inner terms

In the above proportion, 10 and 3 are outer terms, 15 and 2 are inner terms.

Product of outer terms = Product of inner terms

10 × 3	=	15 × 2
30	=	30

Hence, the numbers are in proportion.

**Example:** The cost of 3kg of grapes is Rs.180. and the cost of 4 kg. Of watermelon is Rs.200. Check whether the weights and prices are in proportion or not.

**Solution**: The ratio of the weights of grapes and watermelon is 3 : 4. The ratio of the prices of grapes and watermelon = 180 : 200 or 9:10.

If they are in proportion, 3:4: 9:10.

Finding the product of the outer terms we get,  $3 \times n10 = 30$ 

Finding the product of inner terms we get,  $4 \times 9 = 36$ 

Since,  $30 \neq 36$  the numbers 3, 4, 180 and 200 are not in proportion.

The numbers 3, 4, 180 and 200 are not in proportion.

**Example**: check whether the following quantities or in proportion or not.

(a) 8, 6, 48, 36

**Solution**: 8 : 6 :: 48 : 36

If the quantities are in proportion,

Product of extremes = product of means terms.

=> 8 × 36 = 48 × 6

=> 288 = 288

Hence 8, 6, 24 and 36 are in proportion.

### Exercise 7.2

- 1. Select the correct option for the following multiple-choice questions.
- (a) Which of the following is true?

(I) 15:40 :: 10:30(II) 16:48 :: 25:75(III) 4:6 :: 3:4(IV) 2:10 :: 3:12

- (b) Which of the following is false ?
  - (I) 25 Gram : 30g :: 40kg : 48kg \_
  - (II) 81: 91 :: 24 hours : 27 hours
  - (III) 32 m : 40m :: 6 min : 12 min
  - (IV) 25 km: 60 km :: 10 m : 24 m
- (c) Which of the following statements is not true?

(I) 4:7=5:9 (II)  $5 m: 25m = 12g:60g_{-}$ 

- (III) 30:80 = 6:16 (IV) 12:36 = 14:42
- 2. Which of the following quantities are in proportion?
  - (1) 30, 20, 18, 12
  - (2) 16, 20, 4, 5
  - (3) 14, 18, 63, 81
  - (4) 14, 35, 2, 5
- 3. Check the proportion and write true or false.
  - (1) 15: 45:: 20: 60
  - (2) 20 : 22 :: 32 : 16
  - (3) 12 : 15 :: 24 : 40
  - (4) 18 : 16 :: 45 : 20

- 4. A car travels 40 km in 2 hours. and a bike covers 120 km in 6 hours. Check whether the distances and time are in proportion are not.
- 5. Are the given ratios 30 cm : 36 cm And 10 m : 12 m are in the proportion
- 6. Are the given ratios 25 Gram: 30 grams and 40 kg : 48 kg are in the ratio.
- 7. A train covers 100 km in 2 hours. And a motor cycle travels 120 km in 6 hours. Check whether the distances and time are in proportion are not.
- 8. Ramesh plucks 4.5 kg of flowers from his garden in 5 hours and Suresh plucks 3 kg of lotus flowers from his pond in 1 hour, are the weight of flowers and the time in proportion?

# Unitary Method :-

In the Sanskrit literature a verse is found about unitary method, in Aryabhatiyam,, which is given below.

# त्रैराशिकफलराशिं तमथेच्छाराशिना हतं कृत्वा। लब्धं प्रमाणभाजितं तस्मादिच्छाफलमिदं स्यात्॥

# (आर्यभट्टीयम्,गणितपाद : 26)

The above verse conveys, by multiplying the quotient obtained on dividing the total amount by number of quantities with the desired quantity we get the desired amount.

# $Desired amount = \frac{Given amount \times Desired quantity}{Givan quantity}$

In mathematics the method of finding the cost of one article and hence finding the cost of given number of articles is called unitary method

To find the cost of one article, we use

# $Price of one artical = \frac{Price of number of articles}{Number of articles}$

**Example**: If the cost of 2 flower garlands is Rs.30, find the cost of 5 such garlands?

**Solution**: given: the cost of 2 flower garlands = Rs.30.

The cost of each flower garland

$$\frac{30}{2} = 15$$

Hence, Cost of 5 garlands =  $5 \times 15 = \text{Rs.75}$ 

Thus, the cost of 5 such flower garlands is Rs.75.

**Example**: If the cost of 5 chairs is Rs.500, how many such chairs can be bought for Rs.1000?

**Solution**: given: The cost of 5 chairs = Rs.500.

 $\therefore \text{ Cost of 1 chair} = \frac{500}{5} = 100$ 

Then number of such chairs can be bought for Rs.1000 =  $\frac{1000}{100}$  = 10

Hence 10 chairs can be bought.

# Points to Remember -

When the cost of many articles is given and cost of one article is ask, we divide the cost of many articles by number of articles.

Many to one we divide

When the cost for one article is given and cost of many articles is asked, we multiply the cost of one article by number of articles. One to many we multiply

# Exercise-7.3

- If the cost of 10 liters of milk is Rs.180, what will be the cost of 3 liters of milk ?
- 2) If the cost of 6 books is Rs.240, what will be the cost of 15 books?
- 3) If the cost of 3 dozen pencils is Rs.120, for Rs.200. How many dozen of pencils can be bought ?
- 4) If the rent for 4 months is Rs.1600, how much will be the rent for one year?
- 5) Weight of 4 books is 2 kg then what will be the weight of 9 such books?
- 6) The cost of 105 newspapers is Rs. 315, for Rs.123. How many newspapers can be bought?
- 7) The cost of 3 kg of flowers is Rs. 150, for Rs.250, How many kilograms of flowers can be bought?
- 8) Bhola bought 5kg of mangoes for Rs.200. what is the cost of 8kg of mangoes?
- 9) Aardhya bought 20 lotus flowers for Rs.100 to offer them to his personal deity. What is the cost of each flower?
- 10). Ashish buys 5Kg of modak for Rs 1000 during Ganpati festival to offer as prasad. To buy 3kg of modak, how much money he needs to pay?
- 11). The train fare for 3 passengers to travel from Kolkata to New Delhi is Rs.1461. find the train fare for 2 passengers?
- 12). Find the cost of 25 copies, if each dozen costs Rs 108?
- 13). The cost of 4 upvastras and 2 shawls are equal. If the cost of each upvastra is 100, find the cost of each shawl.

- 14). If 5 men can complete a work in 12 days, find the time taken by 12 men to complete the same work.
- 15). If 24 men can complete a work in 12 days, find the number of men required to complete the same work.by 6 men.

### We learned

- 1. To compare similar quantities, we generally use the methods of comparison by difference of quantities.
- 2. A comparison made by division is called a ratio. In other words, two quantities are compared to find how many times one with respect to the other is. This comparison is called 'ratio' and is denoted by the symbol " : ".
- 3. The equivalent ratio of any ratio is obtained by multiplying or dividing the antecedent and subsequent by the same non-zero number (any).
- 4. In different situations the ratios can be the same.
- 5. If two ratios are equivalent if their corresponding fractions are equivalent.
- 6. A ratio can be reduced to its simplest form.
- 7. Four numbers are said to be in proportion, if the ratio of the first two numbers is equal to the ratio of the last two numbers.

a : b :: c : d Or 
$$\frac{a}{b} = \frac{c}{d}$$

That is,

Product of outer term = Product of inner term

$$a \times d = b \times c$$

8. In unitary method, we find the cost of each article and to calculate the cost of many articles, we multiply the cost of one article and the given number of article.

# Chapter - 8

# Basic geometric concepts

In India, ancient mathematicians were attracted to words geometrical construction while designing altars to perform yagnas. In Vedik literature, various geometrical designs of Havan Kund, Mundaps and various structures are mentioned in shulb sutra. The evidence for creative designs of geometrical can be found from the constructions of rajbhvans, temples, dams and cities, art and sculpting to modern day architecture, clothes designing, engineering etc. from these evidences, we can conclude that geometry is practiced in India since ancient times. The word geometry is made up of "geo" and "metry". Geo means land and metry means measuring i.e., 'measuring land'.

In the following mantra taken from Rigveda, we can observecuriosity show to construct the altars to perform yagnas

> कासीत् प्रमा प्रतिमा किं निदानम्, आज्यं किमासीत् परिधिः क आसीत्। छन्दः किमासीत् प्रउगं किमुक्थं यद् देवा देवमयजन्त विश्वे॥

> > ऋग्वेद्) - 10/130/(3

It means, when all the angels (Devatas) completed the Yagna, what was the limits of the altar? What are the measuring instruments used? What was the perimeter of the altar? What was the mantras meters and uktha?

In the above mantra, the following questions related to primary concept of geometry are asked by a curious scholar to Rishi. Post completion of Yagna:

(a) What are the map(limits) of altar?

(b)What are the geometrical instruments used to measure?

- (c) What is the perimeter of the altar?
- (d) What are the mantras chanted and pronouncements called out?
- (e) What are the sacrifices offered in the yanga by the sage?

We observe a various type of shapes around us. List out the shapes that you are familiar with and identify the formation of such shapes

In this chapter, we will learn about the different geometrical instruments required to draw various shapes.

Now, let us understand the names and uses of the following geometrical instruments.

# 1. Ruler (scale) or straight edge

cm 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

9 Output of the state of the

The ruler (scale) found in your geometry box has centimeter marks along one edge and inches along the other edge.

# Application -

To draw line segments and measure their lengths

# 2. Compasses

# Description -

In compasses, there are two arms. Out of which one arm, has pointed, And the other arm has a pencil holder.

# Application –

1. To draw equal lengths, but not to measure the lengths of line segments.





# 2. To draw arcs and circles of given radius.

# 3. Divider

# Description –

It has two pointed ends.

# Application –

To compare lengths of two lines segments, keep the pointed ands of the divider at the end points of the line segments and compare.

# 4. Set Square (Gunia)

# Description-

There are two triangular instruments, in which the angles at the vertices of one are 45°, 45°, 90° and in the other the angles are 30° 60° 90°.

# Application -

To draw perpendicular lines and parallel lines, we use.

# 5. Protractor (protractor)



# Description -

A semicircular instrument with 180° divisions marked on it.

There are two markings one above the other having  $0^{\circ}$  and  $180^{\circ}$  on the right and left.

# Applications -

Drawing and measuring angles

To draw and measure angles we always consider the marking on the right starting from  $0^{\circ}$  of protractor



We can also measure and draw angles lying between 180° and 360° using a protractor.

#### The Dot: -

Mark a dot (.) on the paper with the pointed end of the pencil. The sharper the tip of the pencil, smaller the mark will be. An almost invisible and subtle mark will give you the impression of a dot. A point determines a position or location of the intersection of two lines or curves. In other words, a point is a geometric figure that has no length, width, and thickness.

If you mark some points on the paper, to distinguish the points one from the other we name them using English letters (In capitals) such as A, B, C and D

If three or more points lie on the same line, then the points are called collinear. If the points or not collinear, they are called noncollinear.

Mark four points on a paper with the pointed end of a pencil, name them as PQR and S.



**Example:** How many points are marked in the given figure?

**Solution**: There are 5 points in the above given figure.

(Points A, B, Q, P and S)



Line segment -

The shortest path joining two points is called as a line segment. The name of line segment is written using the name of the points joined with small line segment above them.  $(\overline{AB})$ 

### Hence,

(a) A line segment has two definite and points.

(b)A line segment has definite length.

We can measure the length of a line segment using a ruler. The line segment having A and B as end points is represented by  $\overline{AB}$  or  $\overline{BA}$ .



 $\overline{AB}$  Or  $\overline{BA}$ . Both represent the same line segment.

Example:

Draw a line segment of length 3.7cm using a ruler.

# Example:

Name the end points of the line segments shown below

The above figure has two line segments  $\overline{AB}$  and  $\overline{BC}$ .

### Line -

When a line segment is extended infinitely on both sides, we get a line. In other words, a line has No end points

Since, a line extends infinitely, it has no definite length

В

We get a line on extending a line segment, infinitely on both sides. If a line segment has end points A and B, an extended on both directions, the line so obtained is represented as  $\overrightarrow{AB}$  or  $\overrightarrow{BA}$ .



A line has infinitely many points on it. To represent a line, we require to points which are lying on it. A line is represented by writing the points with an arrow head above them ( $\overrightarrow{AB}$ ). At times, we represent the lines as *l*, *m*, *n* or *p*.



In the above figure, PQ is a straight line represented by  $\overrightarrow{PQ}$  and also by l**Note: 1.** Two points always lie on a line.

2. Three points lying on a line are called collinear points. **Example:** Write the names of the following lines.



In the above given figure there are three lines named as

# ÄP, QRAndBC



Anant: I know that we represent a line with

the symbol of double arrow.  $(\leftrightarrow)$ 

Ray: Have you ever played archery game of (and bow and arrow)?



Observe that when an arrow is shot, it falls down after travelling certain distance. Think of an arrow traveling continuously. The path of such an arrow depicts a ray.

A ray starts from a point, called starting point. Since, it follows straight path, it is a part of a line. Which move on continuously in a direction.

In the following figure



- (a) 'A' is the starting point.
- (b) 'P' is another point on the ray.
- (c) The ray is represented by  $\overrightarrow{AP}$ .

# Think:

Is it possible to measure the length of a ray?

Example: Identify the starting point and any other point (ifany) in the

following figure

# Example: B C

In the above figure, B is the starting point of the ray  $\overrightarrow{BC}$ . and point C is another point on the ray  $\overrightarrow{BC}$ .

Intersecting Lines: -

If two lines meet or cut each other at a point, they are called intersecting lines. The point, at which they meet or cut each other, is called an intersecting point.



In the above figure, two lines are represented by  $l_1$  and  $l_2$ . These two lines meet or cut each other at the point P.

Hence, lines  $l_1$  and  $l_2$  are intersecting lines and P is called intersecting point.

Some of the Examples for intersecting lines are given below.







English alphabet X



In the above figure  $\ell_1$ ,  $\ell_2$ ,  $\ell_3$  and  $\ell_4$  we can observe that four lines are meeting each other at the point P.

In the above figure, we can observe that four lines are meeting each other at the point P.

On observing the above figure representing four intersecting lines, we can conclude:

- 1. Two lines can meet at only one point.
- 2. Two or more lines can meet at a point.

# Parallel Lines:-

Any two lines which are at equal distances from each other, and do not meet each other even on extending (on both sides) infinitely are called parallel lines. In other words, any two lines which do not intersect each other are called parallel lines.



In the above figure, two lines  $P_1$  and  $P_2$  are parallel lines as they do not intersect each other.



### train tracks

window bar



# Sarthak:-

Observe that the distance between two rails of railway track and the distances between any two successive grills of the window is the same.



# Do and Learn:-

In the following figure, mark the parallel lines



### **Concurrent** lines

When three or more lines pass through a point, they are called concurrent lines. The point at which concurrent lines meet is called the concurrent point (point of concurrent). In other words when two or more lines meet at a point, the meeting point can be called as concurrent point. They are called concurrent lines.

All the intersecting lines are concurrent lines. In the following figure , AD, BE and CF are concurrent lines.



#### Do and Learn:-

Identify the concurrent lines in the following figure.



Identify the concurrent lines in the following figures.



# Exercise – 8.1

Write the definition of line and line segment.

2) In the following figures write the names of the line segments.



- 3) Draw a line segment of the given lengths using a scale.
  (1) 3.5 cm
  (2) 4.3 cm
- 4) Measure the given line segments with a scale and write their measurements.



- 5) Define intersecting lines and parallel lines.
- 6) Fill in the blanks.

(Numerous, parallel lines, intersecting lines, line segments, rays, concurrent, ending)

- 1. When two different lines intersect each other, they are called \_\_\_\_\_\_ lines.
- 3. \_\_\_\_\_ lines can be drawn from a point.
- 4. A line segment has a starting point and \_\_\_\_\_point.
- When two or more lines meet at a certain point, they are called a \_\_\_\_\_\_\_\_
   \_\_\_\_\_\_\_\_\_

- 6. The path starting from a point and extending infinitely in one direction is called \_\_\_\_\_\_.
- 7) Identify the following shapes and write the name against them.

(Line, line segment, ray, parallel line, intersecting line, concurrent line)



**Angle :-** An angle is formed by two rays starting from the same point. The point where both the rays meet is called the vertex of the angle. The angle is represented by " $\angle$ ".



Suppose, if two rays  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$  make an angle AOB, then we also write it as  $\angle AOB$ .
# Do and learn

## Using the following figure, fill in the blanks



Classification of Angles based on their Measurements:-

1. Acute angle

An angle that is greater than zero and less than 90° is called an acute angle.



# Right Angle-

Angles which measure 90° exactly are called right angles.



## Obtuse Angle

Angles whose measure lie between 90° and 180° are called obtuse angles.



### Straight Angle:

Angles that measure 180° exactly are called straight angles. In other words, two opposite rays meeting each other form straight angle.



#### **Reflex Angles:**

Angles whose measurement lie between 180° and 360° are called reflex angles.

#### Note:

Angle around a point is 360°.

Angle included by a circle is 360°.

#### Introduction to Protractor

An instrument is required to measure and compare the angles. This instrument is called protractor. We can find this instrument in geometry boxes; they are in semi-circular in shape.

Looking at the figure, we can find two measurements at every mark of the protractor namely inner and outer measurements. The angle that represents the line at right angle is marked as 90°.

The rest of the angles line between 0° and 180° are marked in the clockwise (in the direction of the movement of the hands of a clock) and

anti-clockwise (in the opposite direction of the movement of the hands of a clock). Directions.

Let us learn how to draw angles using a protractor.

**Remember**: As we can draw angles in both clock and anti-clockwise directions, we have two measurements on a protractor.

#### Construction of angles

Dear Vedic students, now, we learn how to draw angles using a protractor.

**Example:** To construct an angle of 60° with the help of a protractor. **Solution** :

Step1 draw a ray AB

A

**Step 2-** Place the midpoint of the protractor at A of the base line in such a way that its  $zero(0^\circ)$  mark coincides with the ray ( $\overline{AB}$ ).

B

**Step3**-From B move towards 60° in the anti-clockwise (to your left side) direction, Mark the point C at 60° of the protractor.



**Step – 4,** Join point A to C. Now,  $\angle$  CAB = 60° is formed.

# P is

#### Do and learn

Draw the angles of the measurement given below.

(1)  $\angle ABC = 120^{\circ}$  (2)  $\angle MNO = 50^{\circ}$ 

**Example –Draw** an angle of 40° in the clockwise and anti-clockwise direction.

In the clockwise direction

Figure	steps		
A	<b>Step1:</b> Recall that 40° is an acute		
	angle		
B O	<b>Step 2:</b> Draw a ray $\overrightarrow{OB}$ , as shown		
	in the figure.		
A 10 80 90 100 170 720 CCCC	Step3: keep the midpoint of the		
100 100 100 100 100 100 100 100 100 100	protractor at O.		
19-99-99-11-11-11-11-11-11-11-11-11-11-1	Step 4: Without moving position		
	of the protractor, adjust it such		
	that its base line coincides with		
19	OB.		
	<b>Step 5:</b> Look at the 0° mark which		
	coincides with base line.		
	<b>Step 6</b> : Moving in the direction of		
	the hands of a clock from B, locate		
173 A	40° on the protractor and mark		
(अरुव भ	the point A above it.		
	<b>Step7</b> : Remove the protractor, join		
	the points O and A.		
	Now, angle $\angle AOB = 40^{\circ}$		

///

///

In	the	Antic	lockw	ise	dire	ction:
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Figure	Steps			
A X	<b>Step1:</b> Recall that 40° is an acute angle			
	<b>Step 2</b> : Draw a ray $\overrightarrow{OB}$ , as shown in the			
	figure.			
ОВ	Step3: keep the midpoint of the			
	protractor at O.			
A A A A A A A A A A A A A A A A A A A	Step 4: Without moving position of the			
	protractor, adjust it such that its base line			
o B Coldel	coincides with OB.			
14	Step 5: Look at the 0° mark which			
	coincides with base line.			
	Step6: Moving in the opposite direction			
10 /	of the hands of a clock from B, locate 40°			
	on the protractor and mark the point A			
90	above it.			
Te	<b>Step7:</b> Remove the protractor, join the			
H H	points O and A.			
	Now, angle $\angle AOB = 40^{\circ}$			

111

///



	(4)	An angle which is greater than $90^\circ$ but less than $180^\circ$ is called							
	(5)	Whe	en two rays	meet at a po	int, the	n	is f	ormed.	
	(6)	The	point at wł	which the two lines meet is called a					
4)	Selec	ect the correct option for the following objective type questions.						questions.	
	(1)	1) How many degrees is a right angle?							
		(i)	$180^{\circ}$	(ii) 90°	(	(iii) 45°		(iv) 60°	
	(2)	The following is an acute angle.							
		(i)	90°	(ii) 60°	वाप	(iii) 100°	,	(iv)95°	
	(3)	The following is an obtuse angle.							
		(i)	120°	(ii) 13	35°				
		(iii) (	(i) and (ii)	(iv)	none o	of these			
(4) Which is the measure of a straight angle?									
		(i)	90 °	(ii) 100 °	(iii) 18	80° (iv	7)0°		

#### We learned

- 1. Point determines a position; it is generally marked with the English Capital letter such as 'A'.
- 2. The shortest path joining two points is called a line segment. The line segment joining two points A and B, is represent as  $\overrightarrow{AB}$ .
- 3. When a line segment AB is extended infinitely on both sides, we get a line  $\overline{AB}$ .
- 4. A ray is a part of a line which starts from a point and extends without any end.
- 5. When two different lines meet or intersect each other at a point, they are called intersecting lines.
- 6. When two lines do not intersect each other i.e. do not cut, they are called non intersecting lines. If the distance is the same, they are calledparallel lines.

- 7. If two or more lines pass through the same point, the lines are called concurrent lines.
- 8. An angle is formed between two rays starting from the same point.
- 9. With the help of protractor, we can draw angles in clockwise and anti-clockwise direction.
- 10. We use protractor to measure angles in degrees. The measure of a right angle is 90° and the measure of a straight angle is 180°. An angle whose measure is less than a right angle (90°) is called an acute angle. An angle whose measure is more than 90° and less than 180° is called an obtuse angle.

#### Class Activity:-1

Take two matchsticks; join them at their ends to form a shape  $\bot$ . This Shape forms the Right angle which is equal to 90°. We will use this figure as a tester angle. Shape



- (i) Side of ∠ AOB is placed on OB. Since ∠ AOB is smaller than the right angle, it will be an acute angle.
- (ii) Keep the tester on the arm of the angle ON in such a way that the vertex of the tester comes exactly at O.
   Since ∠ MON is greater than the right angle. it will be an obtuse angle.

Measure the following angles given below with a right-angle tester and find whether these angles are acute, obtuse, or rightangle.



Students are expected to observe and discuss these angles in the class.

**Abhishek** :- How many rays are there in each angle ?

**Shubham** :- There are two rays in every angle.

Abhishek :- But their starting points are the same.

**Guruji** :- Yes, students! If two rays emerge from the same starting point, then an angle is formed and that starting point is called the vertex of the angle.



#### Class Activity:- 2

Measuring angle with the help of protractor

Measure the angles  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$  and  $\angle 4$  formed on the paths of Varadraj and Pratyay(as shown below).



## Class Activity:-3

**Class activity 3-** Classify the measured the angles using the angle tester as acute, right, obtuse or straight angles. Now use a protractor to measure the angles, classify them and complete the following table

Measure of straight angle	Angle 1	Angle 2	Angle3	Angle 4
Measure of acute angle				
Measure of right angle				
Measure of obtuse angle				

According to this activity, we found that all acute angles measure less than 90° and all obtuse angles measure more than 90° and less than 180°.

Students discuss in the class by looking at the angles.

- **Vishal** : acute angles Measure between  $0^{\circ}$  to  $90^{\circ}$  angles.
- **Aasha** :- an angle of 90° is called a right angle.

**Abhishek** : - What is the 180° angle called?

Sheetal :- An angle of 180° is called a straight angle (straight angle).

**Guruji** :- Yes, it is correctly discussed – if the angles or smaller than 90°, they are called acute angles. If theangles are greater than 90° but less than 180°, theyare called obtuse angles. If it is 90°, it is called right angles and angles of 180° are called straight angles.

# Chapter - 9

# Simple Two-Dimensional Shapes

Dear Students, we can observe many things around us. Some of them have flat surfaces while some have uneven surfaces. Given below are the pictures of some plane figures, look at them carefully.



From the above figure, we can understand that a plane is a flat surface, which can be extended to infinitely all around. Thus, the plane extends infinitely all around. It is assumed that the plane has infinite length and width. But there is no thickness.

Such a figure which can be drawn using pencil starting from a point without cutting anywhere and without lifting the pencil, is called a simple figure.



From the point P, without lifting the pencil and without cutting the lines, you can draw the whole figure passing through P, Q, R and S

respectively. So it is a simple figure. On the contrary, the shape which cannot be drawn without lifting the pencil is a complex shape.

As:



## Open and Closed figures:

In each of the figures given below there is a mouse. Find out the figures in which the mouse can come out.



In figures (ii), (iv) and (viii) there is no way by which the mouse can come out, so these are closed figures, those figures which end at their initial point are called closed figures and those figures which do not end at their initial point are called open figures.



In this picture, though one side is closed, the figure is not a closed one, because the starting and ending points of the figure are not the same.

> In the above picture, we can easily return back to the starting point. Hence, it is a closed figure but the points of intersections are many. Hence, it is a

In this figure, even though one part is closed, it is not a closed figure, because we cannot reach the initial starting point.

But in this figure we can easily reach the starting point. Therefore it is a closed figure but intersects it at many points. So it is a complex figure. (Repeated statement)

• Is the diagram representing letter P, a

A closed figure has three regions.

- (1) Interior
- (2) Exterior
- (3) Boundary









#### Do and learn:-

Fill in the blanks with the points which are located in the interior region, exterior region and on the boundary of the following closed figures.



(1) The points that are lying in the interior region:

(2) The points that are lying in the exterior region:\_\_\_\_

(3) The points that are lying on the boundary\_\_\_\_

# Polygon:-

In the word polygon, there are two words, namely 'poly' meaning many and 'gons' meaning sided figure. In other words, polygon is a closed figure having three or more number of sides.

In Vedic literature, there is mention of geometrical shapes describing the position of fire.

# अस्माकम् अग्ने अध्वरं जुषस्व सहसः सूनोत्रिषधस्थ।

(ऋग्वेद् 5/4/8)

# यज्ञस्य केतुं प्रथमं पुरोहितम् अग्निन्नरस्त्रिषधस्थे समिन्धते।

(तैत्तिरीयसंहिता : 4/4/4/3)

In Rigveda and Taittiriya Samhita, the three positions of agni (fire) are described as 'Trishadhastha' the three forms of alter of agni (fire) namely– (1) Garhapatya, meaning circular in shape, (2) Ahvaniya chaturbhuj meaning quadrilateral or square and shape (3) Dakshinagni (दक्षिणाग्निः) meaning the shape is in semi - circle or half of the moon.



shrowtkarma Using the match sticks, form the following shapes on a card sheet.



**Ganesh:-** No, we require minimum three match sticks to form a closed figure.

From the above conversation, it is evident that to form a closed figure we require three are more sides. The polygon that has minimum number so sides is a triangle which has three sides.

In Lilavati's mathematics, we can find the method of drawing closed figure while explaning about them, in form of sloka which is given below.

# धृष्टोद्दिष्टमृजुभुजं क्षेत्रं यत्रैकबाहुत: स्वल्पा। तदितरभुजयुतिरथ वा तुल्या ज्ञेयं तदक्षेत्रम्।।

(लीलावती गणित, अक्षेत्रलक्षणसूत्रम् 208)

#### Meaning:-

A space (containing any polygon such as triangle, quadrilateral etc.), the space can not be exist if one side is grater than sum of all the other sides. In other words, such space cannot be found.

A closed figure (triangle, quadrilateral, etc.) cannot be formed if the sum of one side is less than or equal to the sum of the other sides, that is, the sum of one side in any closed figure is less than the sum of the other sides.

Using a geoboard and raber bands, we can make many closed figure



#### Two dimensional shapes:

All the shapes lying on a plan, and having length and breadth are called two dimensional shapes. Since, these shapes have length and breadth (two dimensions), they are also called 2D shapes. We can draw 2D shapes on a paper or blackbords etc. the Examples for 2D shapes:

triangles, circle, square and rectangle. Let us understand two dimensional shapes in detail.



Triangle:

The following line about the triangle is found in the buddhi Vilasini of Lilavati's Mathematics.

# तिस्रोऽस्रयः कोणा यस्येति त्र्यस्रम् ।

# (लीलावती गणित, अथ क्षेत्रव्यवहार, बुद्धिविलासिनी व्याख्या)

Meaning, a closed figure made up of three non-collinear points is called a triangle. In a triangle, there are three sides, three vertices and three angles.

- 1. The sum of the three interior angles of a triangle is 180°.
- 2. A triangle is the polygon with the least number of sides.
- 3. In any triangle, the sum of the lengths of any two sides is greater than the third sides.



Where can we find the objects in the shapes of a triangle, in our daily lives?



#### **Types of Triangles:**

Triangles can be classified based on sides and angles as follows.



Classification of Traingles Based on their Sides:

## 1. Equilateral Traingle:

Traigales in which, all the three side are equal in length, are called equilateral traingle.



In the figure, AB, BC, and CA are three sides of the traingle.

 $\overrightarrow{C}$  AB = BC = CA.

**Note:** Every internal angle in an equilateral traingle is 60°(degrees).

2. Isosceles Triangle:

In triangles, which have two sides are equal in lenth and the third side is deffrent, are called isosceles triangles. The angles which are oposite to the equal sides of an isosceles tringle, are equal.



## 3. Scalene Triangle:

The triangles in which all the three sides are not equal to each other in length, are called scalene triangles. Since, the three sides of scalene triangle are different, the internal angles of a scalene triangle are also different.



In the figure the length of the sides AB, BC and AC are different (not equal to each other). Hence,

 $AB \neq BC \neq AC$ 

Classification of Triangles Based on their angles:

Students in class, drew different types of traingles.



When I measure the internal of the trainle that I have cut, are all acute



In the traingles that I have, all the three internal angles are different, but one of the angles is a right angle and the other two angles are acute angles.





In the traingles that I have, one of the angles is an obtuse angle and the other two angles are acute angles.

# (i) Acute Angled Triangle:

The triangle in which all the three internal angles are acute angles or less than 90°, is called acute angled triangle.



In the given figure the angles are  $\angle A = 60^{\circ}, \angle B = 50^{\circ}$  and  $\angle C = 70^{\circ}$ . All the angles are acute, Hence, the triangle acute angled triangle.

# (ii) Right-angled Triangle:

The triangle in which one of the internal angles is a right angle, is called a right angled triangle



In the figure among the internal angles,

 $\angle$  B is a right angle(90°), hence, the triangle is a right-angled triangle.

# (iii) Obtuse angled Triangle:

The triangle, in which one of the internal angles is an obtuse angle, is called an obtuse angled triangle.

In the figure



 $\angle$  B is an obtuse angle, hence, the triangle is an obtuse angled triangle.

# Quadrilateral –

The following line about the quadrilateral is found in the of buddhi vilasini of Lilavati'sMathematics.

# चतस्रोऽस्रयो यस्येति चतुस्रम् ।

(लीलावती गणित, अथ क्षेत्रव्यवहार, बुद्धिविलासिनी व्याख्या)

**Meaning**, a closed figure made up of four noncollinear points on a plane is called a quadrilateral.

A quadrilateral has four sides, four angles and fourvertices. In a quadrilateral, sum of all the four internal angles is 360°. Quadrilaterals are two-dimensional figures. There are two diagonals in a quadrilateral. The given figure is made up of four points A, B, C and D. In the quadrilateral AC and BD are two diagonals.



You must have seen quadrilateral shapes in daily life-



This means that the shape of

the biscuit I eat in the morning

Pawan and Ganesha have made the shape of a quadrilateral which has equal sides.



**Pavan:** When I measure all the internal angles of the figure, I found out to be equal further; the opposite angles are also equal.

Ganesh: In the figure which I have made which is a quadrilateral, has

equal angles and equal sides?

**Guruji:** Yes! Both are quadrilaterals because they have four sides and four angles. But this is a special quadrilateral. The quadrilateral that is formed by Pavan is a rectangle and Ganesh has formed a square. **Square:** A quadrilateral which has four equal sides and four right angles (90°) is called a square.



In the figure, AB = BC = CD = DA and  $\angle A = \angle B = \angle C = \angle D = 90^{\circ}$ . In this, all the four angles and all the four sides are equal. Such quadrilaterals are called squares.

# **Rectangle:**

The quadrilaterals, in which the opposite sides are equal in length and all the four angles or right angles (90°) are called rectangles.



In the figure  $\angle M = \angle N = \angle O = \angle P = 90^{\circ}$  and, side MN equals side PO and side MP equals side NO. Hence, MN = PO and MP = NO, the opposite sides are equal in length.

# Do and Learn:

Fill in the blanks by using the following figures -

#### Triangle:

Number of sides:.....

Number of angles:.....

Number of vertices: .....

## Quadrilateral:

Number of sides:..... Number of angles:.....

Number of vertices:.....

## Exercise – 9.1

- 1. Select the correct option for the following multiple-choice questions.
  - (a) A triangle whose three sides are not equal is called a..... triangle:
    (I) equilateral
    (II) isosceles
    (III) scalene
    (IV) none of these
  - (B) A triangle with two sides equal is called a ..... triangle.
    (I) Equilateral (II) Isosceles
    (III) Scalene (IV) None of these
  - (c) A triangle in which one angle is obtuse is called a..... triangle.
    (I) acute angle
    (II) obtuse angle
    (III) right angle
    (IV) none of these
  - (d) A figure having four sides is called .....
    - (I) Triangle(II) Quadrilateral(III) Scalene triangle(IV) noneof these
- 2. State the open and closed shape in the figures given below.



(iv)



(v)

3. Using the figure given below answer the following questions –



7. Write the names of all the triangles that can be formed in the following figures and write the number of triangles so formed.



8. Count and write the number of triangles, circles and quadrilaterals in the following figure.



7. Count and write the number of squares and rectangles in the following figure.



No. of Squares..... No. of Rectangles.....

- 10. Fill in the blanks: -
  - (1) The sum of the three internal angles of a triangle is .....
  - (2) The sum of the four internal angles of a quadrilateral is .....
  - (3) A triangle has ..... angle, vertex and side.
  - (4) All the sides of a square are .....
  - (5) All the angles of a rectangle are .....
  - (6) In a rectangle, opposite sides are .....to each other
  - (7) On the basis of sides, the types of triangles are .....
- 11. Answer in one word -
  - 1. What is the name of the triangle whose all three sides are equal?

- 2. What is the name the triangle whose two sides are equal and one side is different?
- 3. What is the name of the quadrilateral whose all sides are equal?
- 4. What is the name of the triangle whose one angle is obtuse angle and the other two angles are acute?
- 5. Which is the name of the quadrilateral whose opposite sides are equal?

#### We learned:-

- The inner region of closed figures is called its interior region, the outer region is called its exterior region and the edges are called its boundary.
- 2. Area and perimeter are determined only for the closed figures.
- 3. A closed figure bounded by three or more than three sides is called a polygon. A polygon which has three sides is called a triangle.
- 4. A closed figure bounded by three points is a triangle. On classifying the triangles on the basis of their sides are Equilateral, scalene and isosceles triangles. Classification of triangles on the basis of angles are acute angle, right angle and obtuse angle triangle.
- 5. A closed figure surrounded by four points is called a quadrilateral.
- 6. A quadrilateral in which opposite sides are equal and each angle is right angle (90°), called a rectangle.
- A quadrilateral in which each angle is equal to right angle (90°) and each side is equal to every other side called a square.

# Chapter – 10

# **Understanding Three-Dimensional Shapes**

#### **Dear Students**

In our daily lives, we observe many planar and solid objects. In our last chapter, we learnt about planar shapes. You are aware that square, circle, rectangle and triangle are two-dimensional shapes. Can you draw the above two – dimensional shapes? In this chapter, we will learn about the three- dimensional shapes.

In our daily lives we observe many solid objects and shapes. All this objects differ in their shapes. Are all the solid objects flat?



Ice – cream is in conical shape.

It means that, the ice-cream that I eat is in conical shape!



This is a can, which is cylindrical in shape.

This is a box. This shape is called cuboid.

Identify five objects which are in spherical shape.

#### Three Dimensional shapes:

The all solid shapes which have length, breadth and height or depth are three dimensional shapes. Every solid shapeoccupies space. Since these shapes have three dimensions namely length, breadth and height, these are called three - dimensional shapes. Cubes, cuboids, spheres, cylinders, cones, prisms etc., are classified as three- dimensional shapes.



This is magical square box, which is cubical in shape.

This is the shape of a pyramid.

#### Face, Edge and Vertex:

We can identify the faces, edges and vertices of three-dimensional shapes very easily. The meaning of the three words namely face, edge and vertex is



#### Face:

The part of the solid shape that can be seen by you is called its face. Faces are always flat.

#### Edge:

The two flat faces of a solid shape while meeting forms a line segment. This line segment called its edge.

#### Vertex:

The meeting point of three edges (line segments) is called vertex of the solid shape.

Now let us understand about the three – dimensional shapes in detail.

#### Cube:

The shape such as dice and magical cube, have square faces. Hence, length, breadth, and height are equal and these are called cubs. The three – dimensional projection of square is a cube. In a cube there are 6 faces, 8vertices and 12edges. All the 6 faces are identical (in shape and size). All the faces are square in shape.



#### Cuboid:

Match box, suitcase and rectangular boxes have rectangular faces. Every flat surface is face is rectangular in shape. Hence, their edges are not equal in length. The three-dimensional projection of a rectangle is cuboid. In a cuboid, there are 6 faces, 8 vertices and 12 edges, similar to a cube.







## Cylinder:

Have you aver seen an iron pipe, vessel to store water are grains? Such shape having two surfaces out of which one is curcular and other surface is curved, is called sylinder.cylinder is a three - dimensional soild shape.



# Sphere:

A ball, football and lemon are similar and shape. The surface are all these, are curved. Hence, a sphere is a solid which has only one curved surface and all the points on the surface are at equal distance from the centre of the sphere. Three – dimensional projection of a circle is a sphere. Are all the coins, cylindrical in shape?



Think: Are cylinders and spheres, similar in shape?

Cone:

You must have seen shapes of the birthday caps and clown caps etc.; these shapes are three–dimensional. Such three–dimensional shapes are called cones. A cone has a vertex and abase connected by a curved surface. If the base of a cone is circular it is called a right circular cone.



- 1. Select the correct option for the following multiple-choice questions.
- (a) The line segment on which two support faces meet is called .......(I) Face (II) Edge (III) Vertices (IV) None of these
- (b) In the following, \_\_\_\_\_\_is a projection of cuboid?
  (I) Square (II) Rectangle (III) Cube (IV) None of these
- (c) Which of the following is a three-dimensional figure of a triangle?(I) prism (II) Rectangle (III) Cube (IV) None of these
- (d) Which of the following is a three-dimensional figure of a rectangle?
  - (I) Cube (II) Cuboid (III) Cone (IV) None of these
- (e) Which of the following is a three-dimensional shape of a sphere?(I) Circle (II) Rectangle (III) Square (IV) None of these

2. Identify the following pictures and write their names -



- 3. Answer the following -
  - 1. Name two objects which are cuboidal in shapes.
  - 2. Write the names of any two fruits which are spherical in shapes.
  - 3. What is the shape of your geometrybox?
  - 4. Name any two objects which are cylindrical in shape.
- 4. Write the differences between cylinder and a sphere.
- 5. What is the difference between a cube and a cuboid?
- 6. What do you understand by three dimensional shapes?
- 7. What do you understand from face, edge and vertex of a threedimensional figure?
- 8. Write True / False -
  - 1. Cuboid and cube have 8 vertices.
  - 2. All edges of a cuboid are equal.
  - 3. The entire surface of a sphere is curved.
  - 4. The surface (face) of a cuboid is square.
  - 5. All three-dimensional figures are solid.
- 9. Fill in the blanks -
  - (Circular, spherical, rectangular, square)
  - 1. Both the ends of the cylinder are .....

- 2. Each face of a cuboid is .....
- 3. The shape of a ball is .....
- 4. Each face of a cube is .....

# 10. Match the following-



///

# We learned -

- Such solids which have length and width, height or depth, are called three dimensional (3D) figures.
- 2. We can easily identify faces, edges and vertices in three dimensional figures.



- 3. Each face of a cube is a square. A cube has 6 faces, 8 vertices and 12 edges.
- 4. Each face of a cuboid is a rectangular in shape; a cuboid has6 faces, 8 vertices and 12 edges.
- 5. The figures whichhave two circular surfaces and a curved surface are called cylinders.
- 6. The shapes whose entire surface is curved like-football, ball etc. are all spheres.

# Chapter – 11

# Perimeter and Area

Dear students! We have studied about different shapes in the previous chapters. You must be able to remember about closed and open shapes. Observe the following figures carefully.



In the above given figures (i) and (v) are closed figures, but (ii), (iii) and (iv) are open figures. Recall that there are three parts of the closed figure, as shown below-



The measure of all four sides of the closed figures given above is the perimeter. In this chapter, we will understand the concepts related to perimeter and area.

Remember that- Perimeter and area are found only for closed figures
**Perimeter** – In Vedic literature, we can find a mention about perimeter and periphery in a mantra of Yajurveda.

# सप्तास्यासन् परिधयस्त्रिः सप्त समिधःकृताः। देवा यद्यज्ञं तन्वाना अबभ्रन् पुरुषं पञ्चम्॥

(यजुर्वेद. 31/15)

In the commentary of the above Yajurveda's mantra, the seven seas have been considered as the periphery in the Purushamedha Yagya. Perimeter has two words namely "Peri" and "meter". Peri means all around the sides and meter stands for measurement. The complete meaning is, the sum of the length of all the sides is called its perimeter. In other words, 'the length of the boundary of a closed figure is called its perimeter'. For calculating the perimeter of a closed figure, it is necessary to have the same units of all lengths (eg - cm, meter).

Let us consider the figure given below for understanding perimeter.



In this figure we start from point O and move in the direction of the arrow. The distance covered till it reaches back to the point O is called the circumference or perimeter of the figure.

Perimeter of the given figure = 5 cm + 10 cm + 2 cm + 5 cm + 8 cm

#### = 30 cm

Let us calculate the perimeter of square and rectangle.

**Perimeter of a square -** The perimeter of a square is equal to the sum of the lengths of its four sides. If we know the length of a side of a square, we can find its perimeter. Recall that all the four sides of a square are equal in length. To find the perimeter of a square, multiply length of one side of the square by 4



Perimeter of square = 4 × length of a side
Example: Find the perimeter of the following figure.
Solution: The above given figure is a square.
Thus, perimeter of square = AB + BC + CD + DA

```
= 3 \text{ cm} + 3 \text{ cm} + 3 \text{ cm} + 3 \text{ cm}
```

 $= 4 \times 3 \text{ cm}$ 

= 12 cm

Example: Find the perimeter of a square whose one side is 5 cm.Solution: We know

Perimeter of square =  $4 \times \text{length one side}$ 

 $= 4 \times 5$  cm

= 20 cm



Perimeter of square ABCD = sum of the four sides of the square

$$= AB + BC + CD + DA$$
$$= 5 cm + 5 cm + 5 cm + 5 cm$$
$$= 4 \times 5 cm$$
$$= 20 cm$$

#### Perimeter of rectangle

The perimeter of a rectangle is equal to the sum of the length of its four sides. Opposite sides of a rectangle are equal to each other, hence to find the perimeter, multiply the sum of the length and width of the rectangle by 2.

Perimeter of a rectangle = 2 (Length + Breadth) Example: Find the perimeter of the following figure.



Solution: The above figure is a rectangle. Perimeter of rectangle = PQ + QR + RS + SP= 8 cm + 4 cm + 8 cm + 4 cm = 2 × (8 cm + 4 cm) = 2 × 12 cm = 24 cm

**Example**: Find the perimeter of the rectangle, whose length is 10 cm and the breadth is 3 cm.



= 2 × (10 cm + 3 cm) = 2 × (13 cm) = 26 cm

## Exercise – 11.1

- Select the correct option for the following multiple-choice questions.
   (a) Derivation of the ensurement
- (a) Perimeter of the square = .....
  - (I)  $4 \times \text{side}$  (II) 4 + side
  - (III) side  $\times$  side (IV) none of these
- (b) Perimeter of the rectangle = .....
  - (I) length + breadth (II) 2 (length + breadth)
    - (III) Length × Breadth (IV) 2 (Length × Breadth)

- (c) The perimeter of an equilateral triangle is 15 metres, then the measure of the side of the triangle will be .....
  - (I) 1 m (II) 3 m (III) 5 m (IV) 6 m
- 2. Find the perimeter of each of the following rectangles -



3. Find the perimeter of each of the squares given below -



- 4. Find the perimeter of a rectangle whose length and breadth are given below.
  - (1) 6 cm, 4 cm
  - (2) 3 cm, 2 cm
  - (3) 7 cm, 5 cm
  - (4) 12 cm, 15 cm
- 5. Find the perimeter of a square whose length of one side is given below.
  - (i) 4 cm (ii) 6 cm (iii) 7 cm
- 6. Find the distance covered by Rajat if he goes around a square park of side 10 m for four times.
- 7. Nandani makes 7 rounds of her rectangular courtyard of length 5 m and breadth 3 m. Then find the distance covered by Nandani.

- 8. Find the perimeter of the square whose side of length is 10 cm.
- 9. If the perimeter of a square is 48 cm, find the length of each side of the square.

#### Area -

Observe the closed figures given below, all these figures occupy some space on a plane. Which of the following figures occupy more space?



In the above given closed figures, the space occupied by the figures on a plane is called that area. Can you tell? Which of the above figures has more area?

In Lilavati Mathematics, the following verse is found regarding the formula for finding the area of a quadrilateral with equal hypotenuse.

## समश्रुतौ तुल्यचतुर्भुजे च तथाऽऽयते तद्भुजकोटिघात:।

## (लीलावती गणित, क्षेत्रव्यवहार: 225)

According to the above mantra, in a quadrilateral (square and rectangle etc.) with equal hypotenuse, the area is obtained by multiplying the length (arm or base) and breadth. In the shape of a square and a rectangle, the product of length and breadth gives their area.

#### Unit of area

Since, two identical units are multiplied, unit of area is a square unit and it is written as  $(unit)^2$ .

Such as:

 $cm. \times cm = square cm or (cm)^2$ 

 $m \times m =$  square meter or  $(m)^2$ 

**Area of rectangle -** To find the area of a rectangle, we multiply the length and breadth of the rectangle.

#### Area of rectangle = length × breadth

**Example**: The length of a rectangle is 5 cm and its breadth is 4 cm. Find the area of the rectangle.

**Solution**: Given, length of the rectangle = 5 cm.

Breadth = 4 cm

4 cm.

5cm.

We know that

The area of rectangle = length  $\times$  breadth

$$= 5 \text{ cm} \times 4 \text{ cm}$$

= 20 sq. cm or 20 (cm)<sup>2</sup>

Do and learn:-

The length of my house in the village is 20 m, and its breadth is 10 m, what will be its area?

#### Area of square -

We can easily recognize the shape of a square. The length and breadth of a square are of the same size.

Formula: - Area of square = side  $\times$  side

```
Or
```

Area of square =  $(side)^2$ 

## Example:

The length of the side of a square is 12 cm. Find the area of the square.

Solut	ion:	Side	of the	square	e = 12	cm.					
		Area of sq	uare	= side	e × sid	e					
				= 12 c	$m \times 12$	2 cm					
				= 12 >	< 12 cn	n					
				= 144	sq. cm	115					
		Or		144 (0	$(m)^2$						
				Exerc	ise – 1	11.2					
1.	Select	the cor	rect o	option	for	the	follov	ving	mult	iple-cl	hoice
	quest	ions.		_/							
(a)	Area	of rectangl	e =		•••••						
	(I) ler	i <mark>gth + b</mark> rea	dth /		(II) 2 (	leng	th + b	readt	h)		
	(III) L	ength × Br	eadth		(IV) 2	(Ler	ngth ×	Breac	lth)		
(b)	Area	of the squa	re =		·····	•••••					
	(I) 4 ×	side 😸			(II) 4 -	+ sid	e				
	(III) s	ide × side			(IV) n	one	of thes	se			
2.	Find	the area of	the re	ectangl	e who	se le	ngth a	ind b	readtl	n are g	given
	below	7.									
	(i)	length = 60	cm, br	eadth =	= 3 cm	•					
	(ii)	length = $12$	2cm, b	readth	a = 4 cr	n.					
	(iii)	length $= 36$	cm, br	eadth :	= 8 cm	•					
3.	Find	the area of	the sq	uare of	f the si	de g	iven b	elow.			
	(i)	3 cm	(ii)	5 cm			(iii)	10 ci	m		

///

///

- 4. The length of a hall is 20 meters and the width is 12 meters. If a carpet is completely spread in it, then find the area of the carpet.
- 5. A square playground whose length is 15 metres. Then find the area of the playground.
- 6. The area of rectangular field is 396 sq. meter. If its length is 22 meters, find the breadth and the perimeter of the field.
- The length and breadth of a rectangle is 12 and 5 cm respectively.
   Find the area and perimeter of the rectangle.
- 8. A length and breadth of a rectangular lawn is 50 m and 28 meters respectively. Find the total cost of laying grass bed, if the cost of laying grass bed per square meter is Rs27.

#### We learned

- 1. The distance travelledon going around a closed figure is called its perimeter.
- 2. (i) Perimeter of the rectangle =  $2 \times (\text{length} + \text{breadth})$ 
  - (ii) Perimeter of square =  $4 \times \text{length of one side}$
- 3. Perimeter and area can be found only for the closed figures.
- 4. The space occupied by a closed figure on a plane is called its area.
  - (i) Area of rectangle = length  $\times$  breadth
  - (ii) Area of square = side  $\times$  side
- 5. To find perimeter and area, all the units of the sides should be the same.
- 6. Unit of area –

Like: (i) cm.  $\times$  cm = square cm or (cm)<sup>2</sup>

(ii)  $m \times m =$  square meter or  $(m)^2$ 

#### Introduction and Contribution of Indian Mathematicians

#### Aryabhata-I (Aryabhatt -I)

In the field of mathematics and astronomy, Aryabhata-I is famous as a prodigious visionary. He was born in Kusumpur (Patna) in 476 AD. At the young age of 23, he wrote an important treatise on mathematics called 'Aryabhatiyam', which is named after him. Its composition method is scientific and the language is concise. 'Aryabhatiyam' is divided into four parts (padas)- 1. Geetikapad 2. Ganitapad 3. Kalakriyapad 4. Golpad. It has total 121 verses. In its first two padas, mainly Arithmetic, Algebra, Geometry and Trigonometry have been described, in which the method of writing numbers with alphabets, General and Quadratic Equations, Kutka's method, Trerasik rule, Square root, Cube root, Trigonometry. Area, cone, circle-circumferencediameter proof i.e. value of Pi ( $\pi$ ) etc are included. In the remaining two feet (Kalakriyapad and Golpad), astronomical principles such as the daily rotation of the earth, calculation of era, year, month, day etc. and the rules of planetary motion have been rendered. The scientificity of the number writing method in Aryabhatiyam is mentioned as follows-

## वर्गाक्षराणि वर्गेऽवर्गेऽवर्गाक्षराणि कात् ङमौ यः। खद्विनवके स्वरा नव वर्गेऽवर्गे नवान्त्यवर्गे वा।।

स्वर	अ	इ	उ	ऋ	ਲ੍	ए	ऐ	ओ	औ
वर्ग	10 <sup>0</sup>	<b>10</b> <sup>2</sup>	<b>10</b> <sup>4</sup>	10 <sup>6</sup>	10 <sup>8</sup>	<b>10</b> <sup>10</sup>	<b>10</b> <sup>12</sup>	<b>10</b> <sup>14</sup>	<b>10</b> <sup>16</sup>
अवर्ग	10 <sup>1</sup>	<b>10</b> <sup>3</sup>	10 <sup>5</sup>	<b>10</b> <sup>7</sup>	10 <sup>9</sup>	<b>10</b> <sup>11</sup>	<b>10</b> <sup>13</sup>	<b>10</b> <sup>15</sup>	10 <sup>17</sup>

(Aryabhatiyam, Dashagitika Pad - 2)

वर्ण	अङ्क								
क	1	च	6	ट	11	त	16	ч	21
ख	2	छ	7	δ	12	थ	17	দ্দ	22
ग	3	ज	8	ड	13	द्	18	ब	23
घ	4	झ	9	ढ	14	ध	19	ਮ	24
ख़	5	ञ	10	ण	15	न	20	म	25
य	र	ਲ	व	হা	ष	स	ह	4	
30	40	50	60	70	80	90	100	A	

### The values of Swars are as follows-

The concise method of finding square root of a number is formed in in Ganitpaad's in fourth poem. The succinct value of  $\Box$  is found in Ganitpaada's, shloka no. 10 of

'Aryabhattiyam'. Which is as follows:

 $\overline{eq}_{2} = \overline{eq}_{2} + \overline{q}_{2} + \overline{q}_{2} = \overline{eq}_{2} \times \overline{q}_{2} + \overline{q}_{2} \times \overline{q}_{2} = 2 \times 10^{4} + 30 \times 10^{4} + 4 \times 10^{6}$ 

 $= 2 \times 10000 + 30 \times 10000 + 4 \times 1000000$ 

= 43,20,000

Aryabhata has told the number of rotation of Sun in one era through Khyughri as 43 lakh 20 thousand. In this method, the largest number is given in a few letters.

The value of  $\pi$  :

### चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणां । अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ।।

Means, a great Indian mathematician Aryabhata (476 – 550) BC in  $\pi$  He gave an approximate value of  $\pi$  = (circumference)/diameter = (62,832)/20,000 which is approximately equal to 3.1416.

The given value of  $\pi$  is considered to be universally valid. The succinct method of cube root is described in verse number (5) of Ganitapada. Aryabhata has given the formula of the rule of trees in the 26th verse of Ganitapada. Aryabhata has told in the book Golpad (गोलपाद) that the earth revolves on its axis. It was Aryabhata who told that the Earth revolves around the Sun in 365 days, 6 hours, 12 minutes and 30 seconds.

Aryabhata believed that the moon and other planets shine with the light of the sun. Aryabhata also explained the solar eclipse and lunar eclipse. Aryabhata told that the main reason for the eclipse is due to the shadow of the moon on the earth falling on the earth and vice-versa. He told that a solar eclipse occurs when the Moon comes between the Earth and the Sun. During the solar eclipse, as the moon comes between the earth and the sun, we cannot see the sun. Similarly, during lunar eclipse, as the earth comes between the moon and the sun, we cannot see the moon.

There are four commentaries on 'Aryabhatiyam'. First of all, Bhaskar-I (629 AD) wrote Mahabhaskariya and Laghubhaskariya commentaries on "Aryabhatiyam". Suryadevajjwa has written 'Aryabhata Prakash' commentary. Parmeshwar (1430 AD) has written 'Bhatdipika'. Neelkanth Somayaji (1443-1543 AD) wrote 'Aryabhatiya Bhashya'.

#### Prabodh Chandra Sengupta (1876-1962):

He was Professor of Mathematics at Bethune College, Calcutta. He wrote several articles highlighting Indian mathematics and astronomy, comparing the distinctive nature of the Greek and Indian methods and how the Indian methods differed from the Greek methods. He published a translation of Aryabhatiyam (1927) and Khandakhadyaka (1934) with detailed articles and illustrations. Later he edited Khandakhadyaka with Prathudaka's commentary (Calcutta 1941). He also wrote on Ancient Indian Chronology (1947).

#### Attippatto A. Krishna Swami Ayyangar (1891–1953):

He was educated at Pachaiyappas College, Madras and later worked as a professor at Maharaja's College, Mysore. In his many published works on mathematics, he covered many technical aspects of the Indian mathematical tradition. In a series of articles published between 1929–42, he showed that the cyclic process always leads to Solutions of quadratic nature and showed how this method differs from successive fractions. He pointed out that this concept had been overlooked by Andreweil, who thought that the cyclic system of the Indians was only an 'experimental fact' and that Fermit and Lagrange give general proofs for continued differential equations.

## Appendix

Checking the answers of large numbers of numbers with the help of a code

#### Beejank system:

We know that 1, 2, 3, 4, 5, 6, 7, 8, 9 are used in beejank number system. To find the beejank number of a number, the sum of the digits of that number is done till one digit is obtained.

#### Joint check:

The answer will be correct if the sum of the digits of the number = the algebraic number of the answer.

Example: Find the sum.

Solution:

1815 0

Check: The sum of the numbers

 $9 + 3 + 5 \rightarrow 17 \rightarrow 1 + 7 = 8$ 

Answer key  $9 + 2 + 0 + 6 \rightarrow 17 \rightarrow 1 + 7 + 8$ 

That is, the answer is correct. Both the beejanks are equal.

#### Exercise 1

1. Add the following and check the answer using the beejank.

1.	86 + 84	3.	1545 + 1455
-			

2. 518 + 336 4. 3978 + 2312

#### 2. Subtraction check

In this, the beejank of the number minuend  $(3 + 2 + 5 \rightarrow 10 \rightarrow 1)$  the beejank of the subtrahend  $(4 + 5 + 6 \rightarrow 5 \rightarrow 6)$   $(1 + 6 \rightarrow 7)$  The beejank of the subtrahend number is equal to (7).

#### Example 1

test



## Checking:

- (1) Beejank of minuend + beejank of answer  $\rightarrow$ 1 + 6  $\rightarrow$  7
- (2) Beejank of the subtrahend  $\rightarrow 7$

Both the beejanks are equal. That is, the answer is correct.

Exercise 2

1. Subtract the following questions and check the answer using beejank rule.

1. 96-82	3.	3545-1455
----------	----	-----------

2. 718-336 4. 5978-2312

## 3. Multiplication Check

The beejank of the first number  $\times$  beejank of the second numbers = the beejank of the product obtained.

## Example

### Check:

- 1) The beejanks first number × the beejank's of second number = The beejanks obtained product  $\rightarrow$  the beejanks number of 8× 4 =  $32 \rightarrow 5$
- 2) Answer key  $\rightarrow$  5



#### Exercise 3

- 1. Find the product and check the answer with using beejank rule.
  - 1. $806 \times 745$ 3. $2032 \times 60$ 2. $701 \times 499$ 4. $3891 \times 20$

Example	$4857 \div 1$	14	
	346 q	uotient	Check :
	)		Beejank of dividend = (beejank of
divisor	14)4857	←dividend	quotient x beejank of divisor) +
	- 42		beejank of remainder
	0650		$6 \longrightarrow (4x5) + 4$
	56	_	$\rightarrow 20 + 4 \rightarrow 24$
	097		$\rightarrow 6$
	84		That is, the answer is correct.
	13	Remainder	-172

## Exercise 4

1. Divide and check the answer using beejank rule.

1) 625 ÷ 125	4)	12005 ÷105	2)	89063 ÷35
5) 26025 ÷1000	3)	96324 ÷205	6)	82845 ÷300

///

#### Question – 01. Put ( ) mark against the correct option – $10 \times 2 = 20$ शून्य (0) है / Zero (0) is -पूर्ण संख्या प्राकृत संख्या (आ) (अ) Natural Number Whole Number (ई) विषम संख्या सम संख्या (इ) **Even Number** Odd Number संख्या 125 को पूर्णतः विभाजित करने के सम्बन्ध में सही विकल्प का चयन करें –

आदर्श्त प्रश्नपत्र/ Model Que. Paper : I/ गणित /

वेदभूषण प्रथम-वर्ष / Vedabhushan First Year/

कक्षा – 6वीं/ प्रथमा - I / Class 6th / Prathama - I

विषय - गणित

Choose the correct option for dividing the number 125 completely.

- 125, 5 से पूर्णतः विभाजित है परन्तु 25 से नहीं। (अ) 125 is exactly divisible by 5 but not by 25
- 125 केवल 15 से ही पूर्णतः विभाजित होता है। (आ) 125 is exactly divisible by 15 only.

प्रश्न - 01. सही विकल्प के सामने (√) चिह्न लगाइए –

1.

2.

- 125, 5 एवं 25 दोनों से पूर्णतः विभाजित है। (इ) 125 is exactly divisible by both 5 and 25
- (ई) 125 केवल 15 से ही पूर्णतः विभाजित होता है।

125 is exactly divisible by 15 only.

- 'BODMAS' के सन्दर्भ में कौन-सा कथन सही है -3. Which statement is correct with respect to 'BODMAS'?
  - 'BODMAS' के नियमानुसार सर्वप्रथम कोष्ठक को हल किया जाता है। (अ)

 $10 \times 2 = 20$ 

According to the rule of 'BODMAS' the bracket should be solved first.

- (आ) 'BODMAS' के नियमानुसार भाग के बाद गुणन कि प्रक्रिया कि जाती है। According to the fule of 'BODMAS', the process of multiplication is done after division.
- (इ) दोनों (अ) एवं (आ) / Both (A) and (AA)
- (ई) इनमें से कोई नहीं / None of these
- 4. 'एकाधिकेन पूर्वेण' सूत्र का अर्थ है -

The meaning of sutra 'Ekadhiken Purvena' -

(अ) पूर्व के अङ्क को एक अधिक करना।

Adding one more to the previous digit

- (आ) एक अधिक करना। / Adding one more
- (इ) दोनों (अ) एवं (आ) / Both (A) and (AA)
- (ई) इनमें से कोई नहीं / None of these

### 5. निम्न में से कौन-सा कथन सही है-

Which of the following statement is correct -?

(अ) प्राकृत संख्या 0 से प्रारम्भ होती है।

Natural numbers start from 0.

(आ) पूर्ण संख्या () से प्रारम्भ नहीं होती है।

Whole numbers do not start with 0.

(इ) 5 पूर्ण संख्या है परन्तु प्राकृत संख्या नहीं है ।

5 is a whole number but not a natural number.

(ई) सभी प्राकृत संख्याएँ, पूर्ण संख्या भी होती हैं ।

All natural numbers are whole numbers.

 $\Pi L$ 

6. भिन्न के सम्बन्ध में सही विकल्प का चयन करें-

Select the correct option respect the fraction -

- (अ) सम भिन्न में अंश बड़ा एवं हर छोटा होता है।
   An even fraction in the numerator is large and the denomination is small.
- (आ) विषम भिन्न में अंश छोटा एवं हर बड़ा होता है।

An improper fraction in the numerator is small and the denominator is large.

- (इ) मिश्रित भिन्न, एक पूर्ण संख्या एवं सम भिन्न से मिलकर बनती है।
   A mixed fraction is made up of a whole number and an even fraction.
- (ई) मिश्रित भिन्न, एक पूर्ण संख्या एवं विषम भिन्न से मिलकर बनती है।
   A mixed fraction is made up of a whole number and an improper fraction.
- 7. एक सौ पाँच द्शमलव आठ संख्या का विस्तार रूप होगा-

The expanded form of the number one hundred five decimal eight will be -

- $(\Im) \quad 105.08 = 100 + 10 + 5 + 1/10 + 5$
- (3T) 105.8 = 100 + 10 + 5 + 8/10
- $(\mathbf{\bar{s}})$  105.08 = 100 + 00 + 5 + 8/10
- (ई) इनमें से कोई नहीं
- 8. निम्न में से कौन-सा कथन सही है-

Which of the following statement is correct -?

(अ) 10 पेन एवं 4 पेंसिल का अनुपात, 5 : 4 है।

The ratio of 10 pens and 4 pencils is 5 : 4. is.

- (**आ**) 10:4 :: 5:2
- (इ) विनकुलम संख्या 23=33

Vinculum number 23<sup>-</sup>=33

 $(\mathbf{\hat{s}})$  102 = 50 + 8 + 44

## 9. निम्न में से कौन-सा कथन सही है-

Which of the following statement is correct -

 (अ) जब दो रेखाएँ एक-दूसरे को प्रतिच्छेद करती है, तब वे प्रतिच्छेदी रेखाएँ नहीं कहलाती है।

When two lines intersect each other, then they are not called intersecting lines.

 (आ) जब दो रेखाएँ एक-दूसरे को प्रतिच्छेद नहीं करती है, तब वे समान्तर रेखाएँ कहलाती है।

When two lines do not intersect each other, then they are called parallel lines.

- (इ) जब दो रेखाएँ एक-दूसरे को प्रतिच्छेद करती है, तब वे प्रतिच्छेदी रेखाएँ कहलाती है। When two lines intersect each other, then they are called intersecting lines.
- (ई) दो़नों (आ) एवं (इ) / Both (a) and (e)
- 10. निम्न में से कौन-सा कथन सही नहीं है-

Which of the following statement is not correct -?

- (अ) जिस त्रिभुज की तीनों भुजाएँ बराबर होती है, उसे समकोण त्रिभुज कहते हैं।
   A triangle whose all three sides are equal is called a right angled triangle.
- (आ) जिस त्रिभुज की दो भुजाएँ बराबर होती है, उसे समद्विबाहु त्रिभुज कहते हैं।

A triangle whose two sides are equal is called an isosceles triangle.

- (इ) जिस त्रिभुज की तीनों भुजाएँ बराबर नहीं होती है, उसे विषमबाहु त्रिभुज कहते हैं। A triangle whose all three sides are not equal is called an equilateral triangle.
- (ई) जिस त्रिभुज की तीनों भुजाएँ बराबर होती है, उसे समबाहु त्रिभुज कहते हैं।
   A triangle whose all three sides are equal is called an equilateral triangle.
- प्रश्न 02. निम्नलिखित युग्मों पर विचार कीजिए -/Consider the following pairs -



उपर्युक्त युग्मों के आधार पर सही विकल्प का चयन कीजिए-

Select the correct option based on the above pairs -

- (आ) (1) (ਬ), (2) (अ), (3) (ङ), (4) (ग), (5) (ख)
- (হ) (1) (ঘ), (2) (अ), (3) (ग), (4) (ङ), (5) (ख)
- (ई)(1) (ख), (2) (अ), (3) (ङ), (4) (ग), (5) (घ)

प्रश्न - 03. रिक्त-स्थानों की पूर्ति कीजिए – / Fill in the blanks  $5 \times 2 = 10$ 

- 1. 10 मिलियन = ..... हजार।
  - 10 million =..... thousand.
- किन्हीं दो अंकों का योग का यदि 10 हो, तो वे एक-दुसरे के..... होते हैं ।
   If the sum of any two numbers is 10, then they are..... of each other are.
- 3. ऋजुकोण की माप..... है । Measure of right angle is.....
- भुजाओं के आधार पर त्रिभुज..... प्रकार के होते हैं।
   On the basis of sides, triangles are of...... types.

There are..... vertices in a cuboid.

प्रश्न - 04. सत्य / असत्य कथन पर विचार कीजिए –

Consider the true / false statement -

1. 2 की पूर्ववर्ती संख्या '0' है।

The preceding number of 2 is '0'.

शून्य, प्रत्येक ऋणात्मक पूर्णाक से बड़ा होता है ।

Zero is greater than every negative integer.

- 3. 12.127 = 12.176
- एक समकोण को सरल कोण भी कहते है ।

A right angle is also called a straight angle.

 $5 \times 1 = 5$ 

 $\Pi L$ 

5. प्रत्येक ऋण पूर्णांक, धन पूर्णांक से छोटा होता है ।

Every negative integer is smaller than the positive integer.

उपर्युक्त कथनों को पढ़कर सही विकल्प का चयन कीजिए –

Read the above statements and choose the correct option

(अ) (1) सत्य, (2) असत्य, (3) सत्य, (4) असत्य, (5) सत्य

(1) True, (2) False, (3) True, (4) False, (5) True

(आ) (1) असत्य, (2) असत्य, (3) सत्य, (4) असत्य, (5) सत्य

(1) False, (2) False, (3) True, (4) False, (5) True

(इ) (1) सत्य, (2) सत्य, (3) असत्य, (4) असत्य, (5) सत्य

(1) True, (2) True, (3) False, (4) False, (5) True

(ई) (1) असत्य, (2) सत्य, (3) असत्य, (4) असत्य, (5) सत्य

(1) False, (2) True, (3) False, (4) False, (5) True

प्रश्न - 05. अति लघूत्तरीय प्रश्न / Very short answer question –  $10 \times 2 = 20$ 

1. 2, 345, 671 की अन्तर्राष्ट्रीय पद्धति से शब्दों में लिखिए।

Write the number 2,345,671 in words using the international system.

2. 240 से ठीक पहले के तीन कमागत पूर्ण संख्याएँ लिखिए।

Write three consecutive whole numbers just before 240.

3. 144, 192 का लघुत्तम समापवर्तक लिखिए।

Write the least common factor of 144, 192.

	DMAS क नियम का प्रयोग कर)/Solve (Using by BOD
Rule) -	$(-3) \times (-2) + 1$
	द्रश्यवीवेचाप्र
2	
विनकुलम् संख्या !	5 <sup>-</sup> 3 <sup>-</sup> 2 को सामान्य संख्या में बदलिए। /
Vinkulam nu	mber 5 <sup>-</sup> 3 <sup>-</sup> 2 to general number.
	र मवर्ण
	र्भ मवर्ण
35 मिनट एक घ	रि का कौन सा भिन्न है । /35 minutes is what fraction

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- 3.412 इल करें/Solve - 15.712 + 03.120 3 दर्जन पेन्सिल का मूल्य 120 रु. है, तो 200 रु. मे कितने दर्जन पेन्सिल खरीदी ज है ? 3 dozen pencils cost 120 Rupee, then how many dozen pencil be bought in 200 Rupee?	हल भर/ 501Ve-	7.3
हरू करें/Solve - 15.712 + 03.120 3 दर्जन पेन्सिल का मूल्य 120 रु. है, तो 200 रु. मे कितने दर्जन पेन्सिल खरीदी ज है ? 3 dozen pencils cost 120 Rupee, then how many dozen penci be bought in 200 Rupee?		- 3.412
हल करें/Solve - 15.712 + 03.120 3 दर्जन पेन्सिल का मूल्य 120 रु. है, तो 200 रु. मे कितने दर्जन पेन्सिल खरीदी ज है ? 3 dozen pencils cost 120 Rupee, then how many dozen penci be bought in 200 Rupee?		
हल करें/Solve - 15.712 + 03.120 3 दर्जन पेन्सिल का मूल्य 120 रु. है, तो 200 रु. मे कितने दर्जन पेन्सिल खरीदी ज है ? 3 dozen pencils cost 120 Rupee, then how many dozen penci be bought in 200 Rupee?		
+ 03.120 3 दर्जन पेन्सिल का मूल्य 120 रु. है, तो 200 रु. मे कितने दर्जन पेन्सिल खरीदी ज है ? 3 dozen pencils cost 120 Rupee, then how many dozen penci be bought in 200 Rupee?	हल करें/Solve -	15.712
3 दर्जन पेन्सिल का मूल्य 120 रु. है, तो 200 रु. मे कितने दर्जन पेन्सिल खरीदी ज है ? 3 dozen pencils cost 120 Rupee, then how many dozen penci be bought in 200 Rupee?		+ 03.120
3 दर्जन पेन्सिल का मूल्य 120 रु. हैं, तो 200 रु. में कितने दर्जन पेन्सिल खरीदी ज है ? 3 dozen pencils cost 120 Rupee, then how many dozen penci be bought in 200 Rupee?		
हे ? 3 dozen pencils cost 120 Rupee, then how many dozen penci be bought in 200 Rupee?	3 दर्जन पेन्सिल का मूल •	य 120 रु. है, तो 200 रु. मे कितने दर्जन पेन्सिल खरीदी ज
3 dozen pencils cost 120 Rupee, then how many dozen penci be bought in 200 Rupee?	हें?	E A A
be bought in 200 Rupee?	3 dozen pencils c	ost 120 Rupee, then how many dozen penci
	be bought in 200	Rupee?
र स्वत्यक		
रिषेषं मवत्यक	k /	
राष भवत्य		Di the
		्रिय भवत्य
	अक 5 स विमाज्यता क	ानयम । लाखए ।
अक 5 स विभाज्यता क नियम लिखिए ।	Write the rules of	t divisibility by the number 5.

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प्रश्न - 06. लघूत्तरीय प्रश्न/ Short Answer Questions- 5 × 3 = 15 144 एवं 180 का महत्तम समापर्वक ज्ञात कीजिए। 1. Find the greatest common factor of 144 and 180. पिता और पुत्र की आयु का अनुपात 4 : 3 है दोनों की आयु का योग 70 है, तब पिता एवं पुत्र 2. की आयु ज्ञात कीजिए। Ration of age of father and son is 4:3 and the sum of the ages of both is 70, then find the age of father and son. त्रिभुज में कितनी भुजाएँ, कितने कोण एवं कितने शीर्ष होते है ? ज्यामिति की आकृति से 3. सम्बन्धित वेद मन्त्र लिखिए।

How many sides, angles and vertices are there in a triangle? Write the Veda mantra related to the shape of geometry?

ात वेद मन्त्र	
	्रेजीताक
घन एवं घनाभ में कोई	तीन अन्तर लिखिए।
Write any three	differences between cube and cuboid.
	X
वेद का कोई एक मन्त्र	लिखिए, जिसमें संख्या का बोध होता हो ?
A mantra of the	e Vedas write, in which there is cognition of
number?	

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प्रश्न - 07. दीर्घ उत्तरीय प्रश्न / Long Answer Type Questions –  $4 \times 5 = 20$ 

1. त्रिभुज से आप क्या समझते हैं, त्रिभुज के वर्गीकरण को समझाइये।

What do you understand by triangle, explain the classification of triangle?



2.

3. भिन्न किसे कहते हैं, भिन्न के प्रकार को समझाइये।

What is a fraction, explain the types of fractions.

 द्विविर्म	ाय आकृति से सम्बन्धित वेद मन्त्र लिखित तथा निम्न दी गई आकृति के चित्र बनाः
Writ	e Veda mantras related to two dimensional figures and giv
below	v Draw a picture of the figure -
(i)	त्रिभुज/Triangle (ii) चतुर्भुज/Quadrilateral
(iii)	पंचभुज/Pentagon (iv) षष्टभुज/Hexagon
	Pari son
	राष भवत्य

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सम्बन्धित वेद मन्त्र -

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